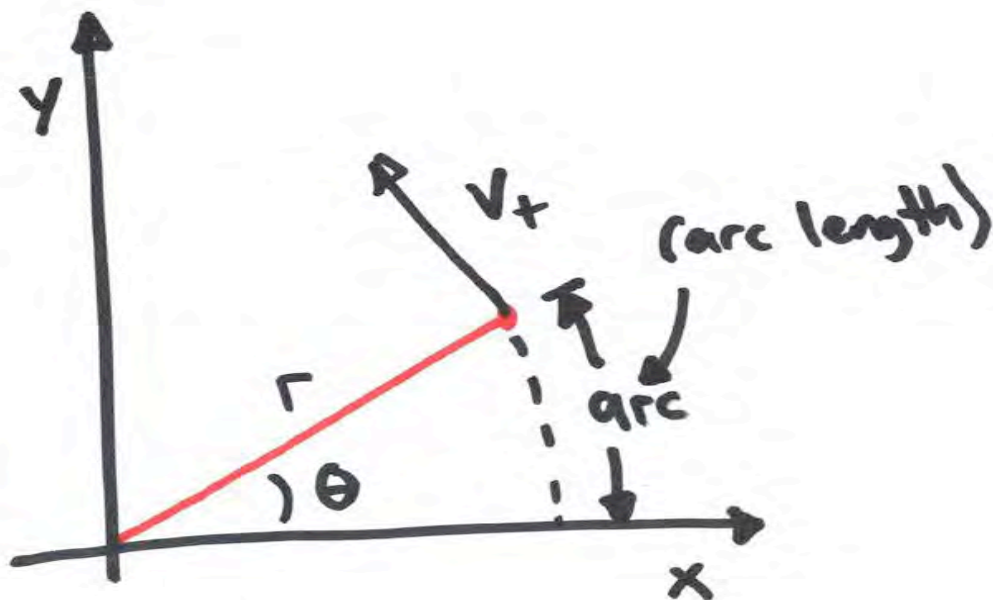


Kinematics (circular motion)



$$\theta = \frac{\text{arc}}{r}$$

$$\frac{d\theta}{dt} = \omega = \frac{v_t}{r}$$

angular velocity (pointing to ω)

$$\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \alpha = \frac{a}{r}$$

angular acceleration (pointing to α)

Kinematics

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_0^2 + 2 \vec{a} \Delta \vec{x}$$

$$\vec{v} = \vec{v}_0 + \vec{a} t$$

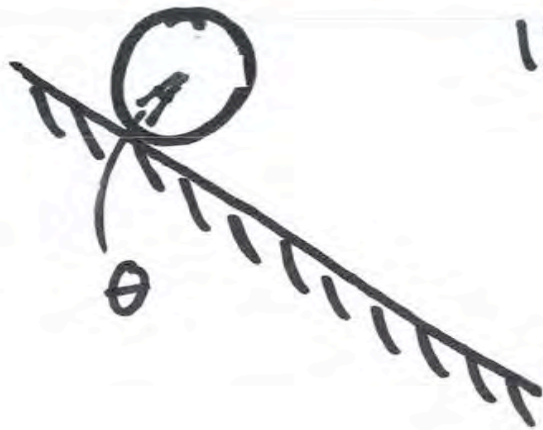
Kinematics

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2 \alpha \Delta \theta$$

$$\omega = \omega_0 + \alpha t$$

How many times does the ball rotate if it
it accelerates at 2 rads/sec^2 after
12 seconds starting from rest?

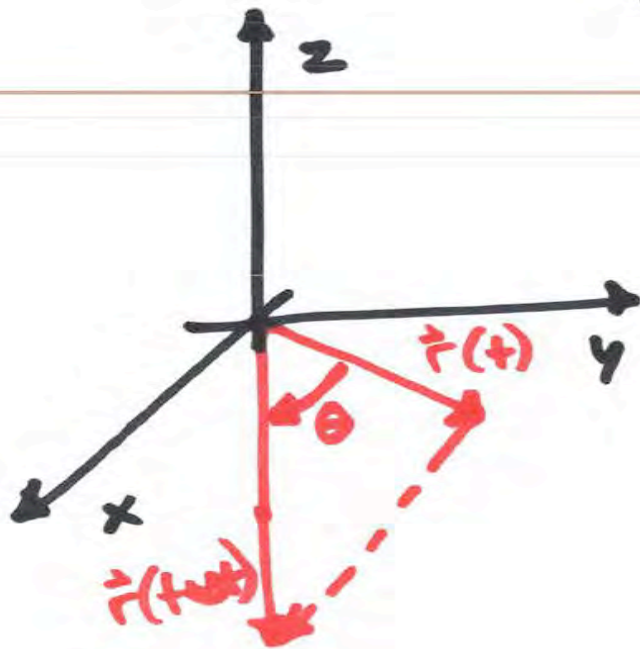


$$\theta = \cancel{\theta_0} + \cancel{\omega_0 t} + \frac{1}{2} \alpha t^2$$

$$\theta = \frac{1}{2} (2) (12)^2 = 144 \text{ rads}$$

$$\# \text{ of times} = \frac{144 \text{ rads}}{2\pi \frac{\text{rads}}{\text{cycle}}} = 22.9 \text{ cycles}$$

Equations of motion



$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{\Delta t}$$

$$\dot{r} = \frac{dr}{dt}, \quad \ddot{r} = \frac{d^2 r}{dt^2}$$

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

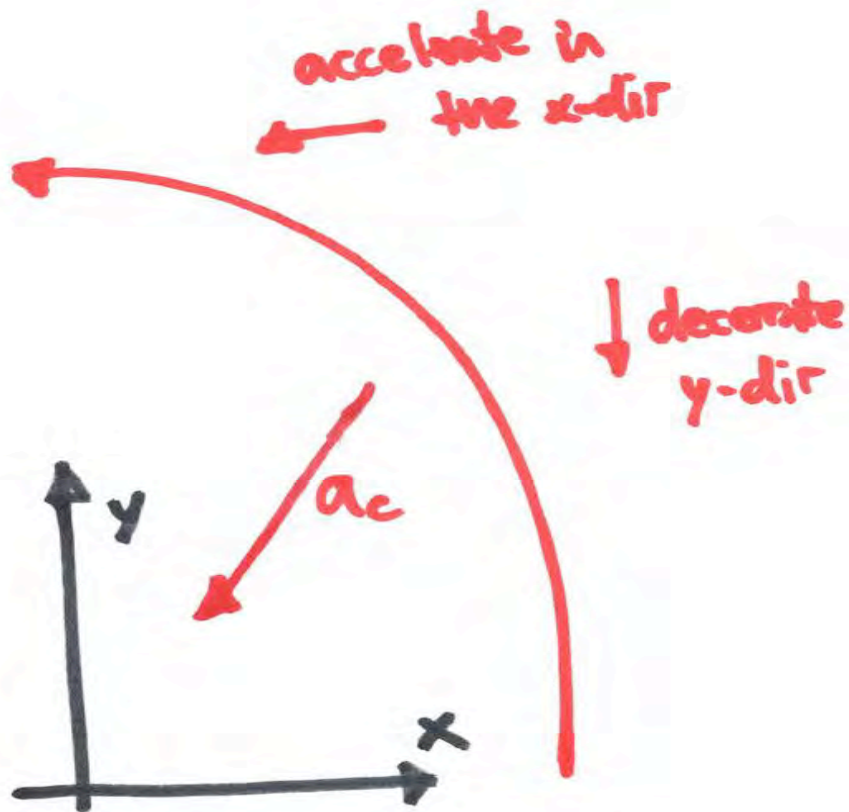
$$= \dot{\vec{r}} + \vec{\omega} \times \vec{r} = \vec{v}$$

$$\frac{d\vec{v}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{v}(t+\Delta t) - \vec{v}(t)}{\Delta t}$$

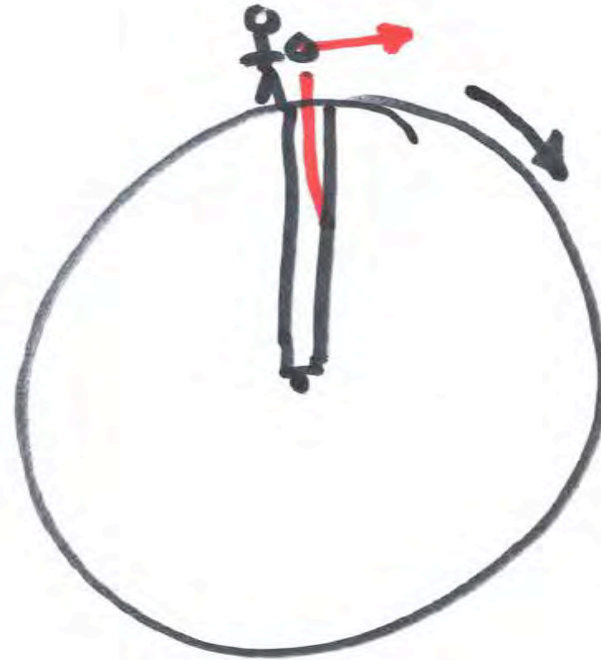
$$= \left[\ddot{\vec{r}} + \vec{\omega} \times \vec{\omega} \times \vec{r} \right] \hat{r} + \left[\vec{\alpha} \times \vec{r} + 2\vec{\omega} \times \vec{v} \right] \hat{\theta}$$

radial (pointing to $\ddot{\vec{r}}$)
this curve (pointing to $\vec{\omega} \times \vec{\omega} \times \vec{r}$)
centripetal (pointing to $\vec{\omega} \times \vec{\omega} \times \vec{r}$)
tangential (pointing to $\vec{\alpha} \times \vec{r}$)
coriolis (pointing to $2\vec{\omega} \times \vec{v}$)

Centripetal



Coriolis



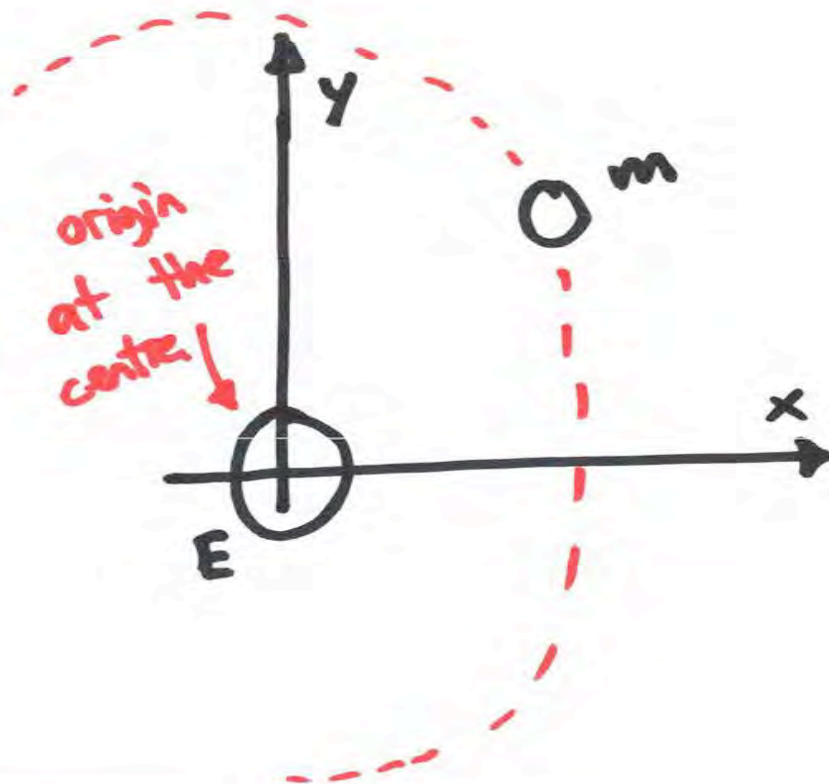
Orbits:

How far is the moon?

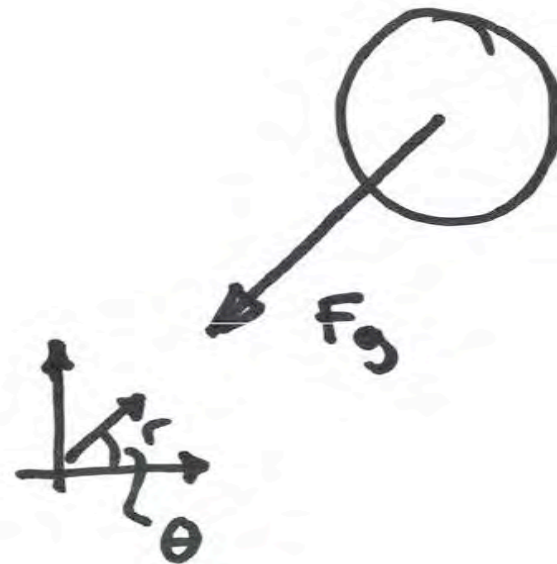
$$m_{\text{earth}} = 5.97 \times 10^{24} \text{ kg}$$

$$m_{\text{moon}} = 7.35 \times 10^{22} \text{ kg}$$

$$T_{\text{moon}} = 27.3 \text{ days}$$



FBD:



Newton's 2nd Law:

$$\sum F_r = m a_r = -F_g \quad (1)$$

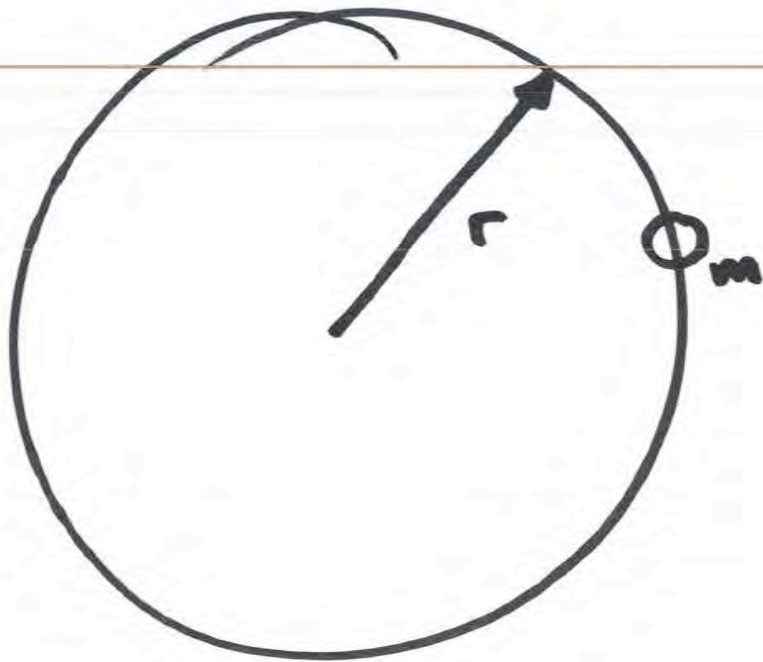
$$\sum F_\theta = m a_\theta = 0 \Rightarrow a_\theta = 0$$

sub F_g and a_r into (1)

$$m_e [\omega^2 r] = - \frac{G m_e m_m}{r^2}$$

$$r^3 = + \frac{G m_e}{\omega^2}$$

$$\omega = \frac{v}{r}$$



$$C = 2\pi r \quad T = 27.3 \text{ days}$$

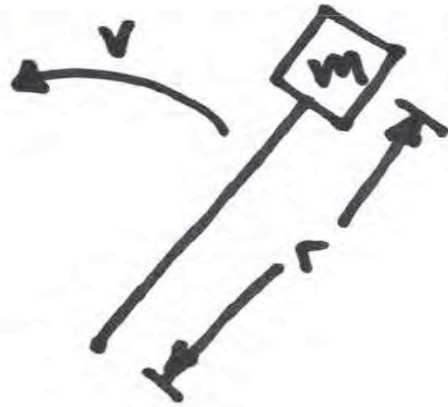
$$v = \frac{2\pi r}{T} = \omega r$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

$$r^3 = \frac{G m_e}{\left[\frac{2\pi}{T}\right]^2} = \frac{G m_e T^2}{(2\pi)^2} = \frac{(6.67 \times 10^{-11}) (5.97 \times 10^{24}) (2.36 \times 10^6)^2}{(2\pi)^2}$$

$$r = \sqrt[3]{5.615 \times 10^{25}} = 3.82 \times 10^8 \text{ m}$$

Determine tension at the top and bottom of the revolution:



$$v = 2 \text{ m/s}$$

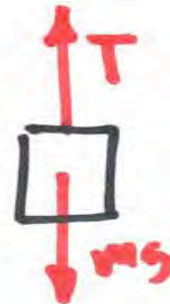
$$m = 1 \text{ kg}$$

$$r = 1.5 \text{ m}$$

FBD:
(top)



FBD:
(bottom)



Newton's 2nd Law

Top

$$\Sigma F_r = ma_r = -mg - T$$

$$m\left(-\frac{v^2}{r}\right) = -mg - T$$

$$T = m\frac{v^2}{r} - mg$$

$$= (1)\frac{2^2}{1.5} - (1)(9.81)$$

$$= -7.14 \text{ N}$$

 is not
tight at top of
circle.

Bottom

$$\Sigma F_r = ma_r = mg - T$$

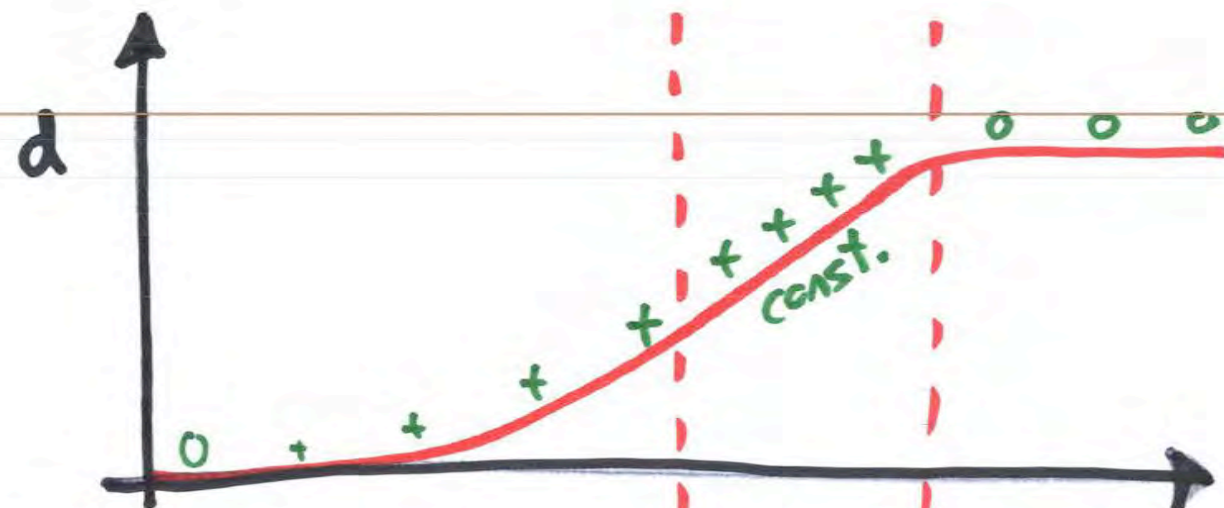
$$m\left(-\frac{v^2}{r}\right) = mg - T$$

$$T = m\frac{v^2}{r} + mg$$

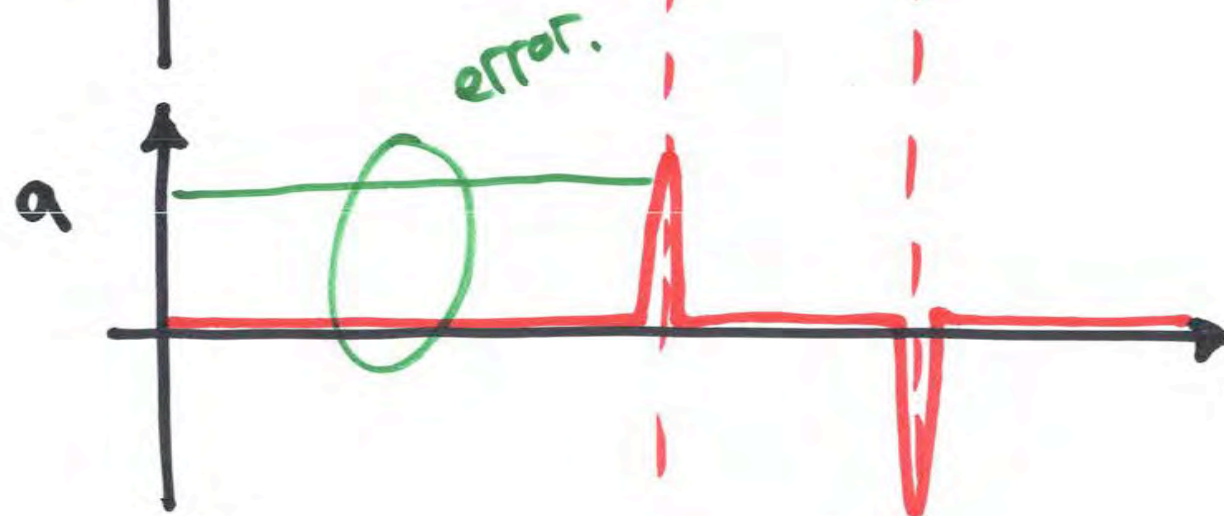
$$= (1)\frac{2^2}{1.5} + (1)(9.81)$$

$$= 12.5 \text{ N}$$

Test review.

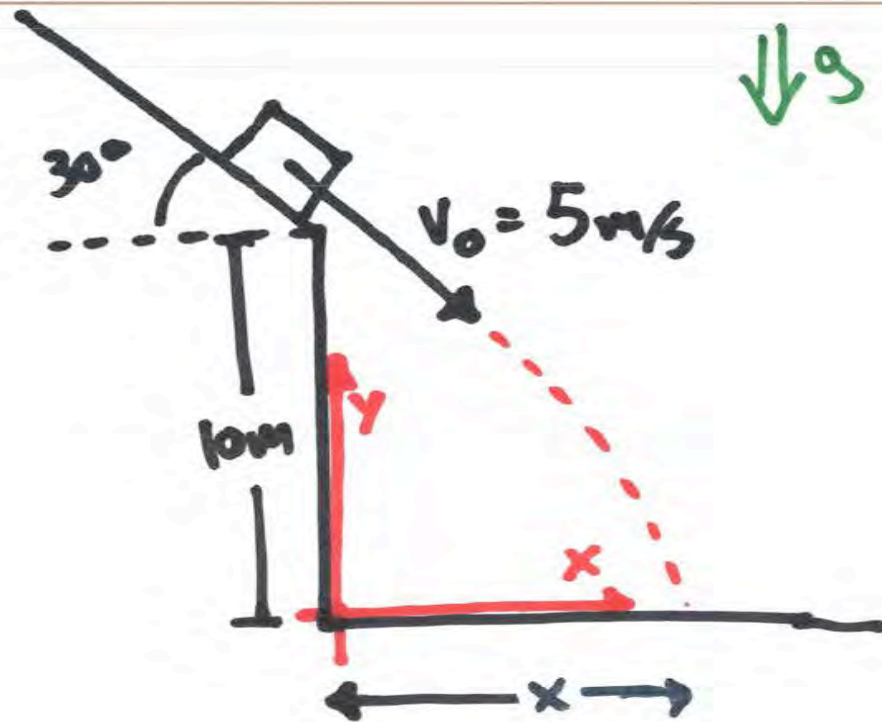


Find the error.



$$v = \frac{dx}{dt}$$
$$a = \frac{dv}{dt}$$

Determine x ?



$$x_0 = 0 \quad y_0 = 10$$

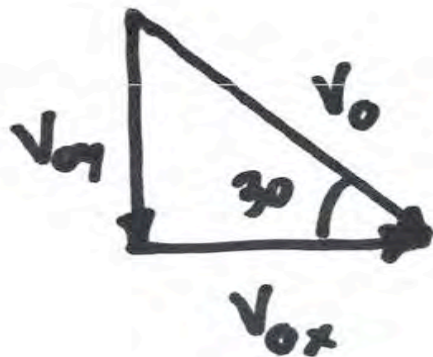
$$x_f = \quad y_f = 0$$

$$v_{0x} = 4.33 \quad v_{0y} = -2.5$$

$$v_{fx} = 4.33 \quad v_{fy} =$$

$$a_x = 0 \quad a_y = -9.81$$

$$t =$$



$$v_{0x} = v_0 \cos(30) = 4.33$$

$$v_{0y} = v_0 \sin(30) = 2.5$$

Kinematics:

$$V_{yF}^2 = V_{oy}^2 + 2a_y \Delta y$$

$$V_{Fy} = \pm \sqrt{(-2.5)^2 + 2(-9.81)(0-10)} = -14.2 \text{ m/s}$$

$$V_{Fy} = V_{oy} + at$$

$$t = \frac{V_{Fy} - V_{oy}}{a} = \frac{-14.2 + 2.5}{-9.81} = 1.2 \text{ s.}$$

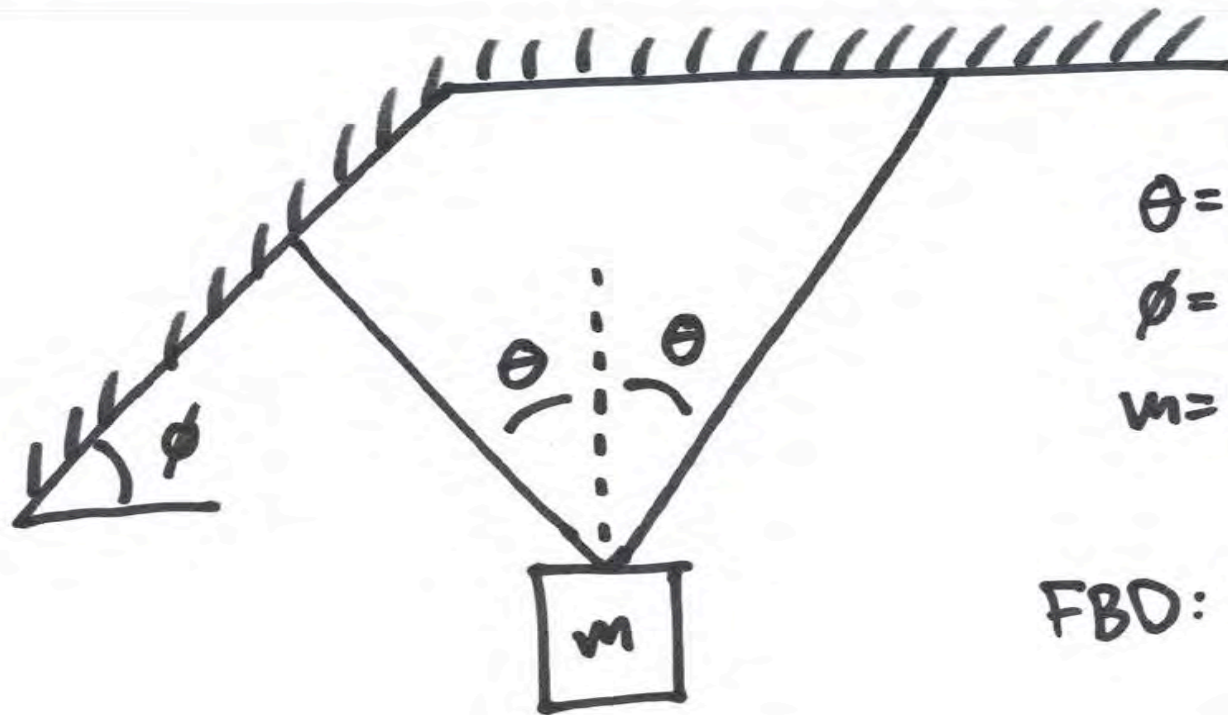
$$x = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$$



$$= 0 + (4.33)(1.2)$$

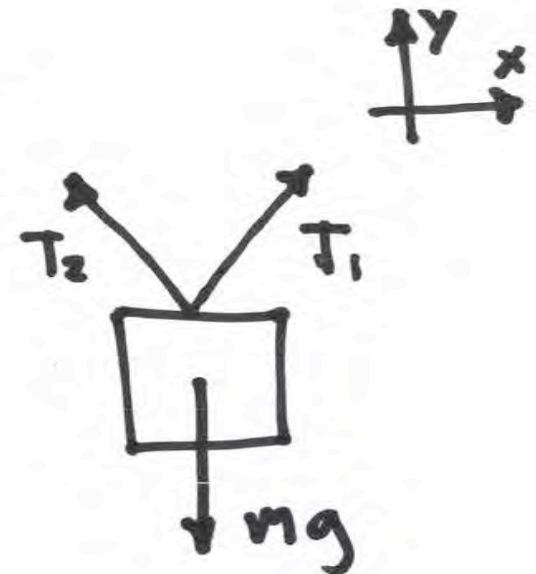
$x = 5.16 \text{ m}$

determine the tension in each rope.



$$\theta = 40^\circ$$
$$\phi = 50^\circ$$
$$m = 100 \text{ kg}$$

FBD:



$$x = T \cos \theta$$
$$y = T \sin \theta$$

Newton's 2nd Law:

$$\boxed{\sum F_x = \cancel{m\vec{a}_x}} = T_1 \cos \theta - T_2 \cos \theta \quad (1)$$

$$\sum F_y = \cancel{m\vec{a}_y} = T_1 \sin \theta + T_2 \sin \theta - mg \quad (2)$$

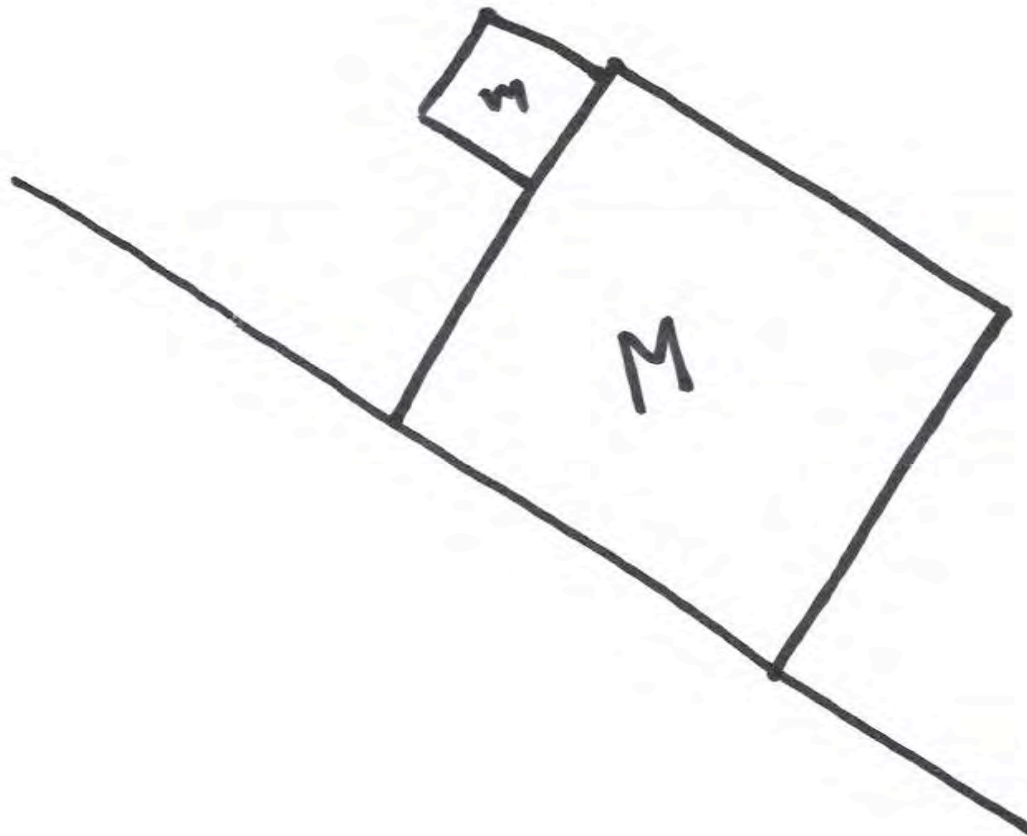
sub (1) into (2)

$$0 = [T_2] \sin \theta + T_2 \sin \theta - mg$$

$$\boxed{T_2 = \frac{mg}{2 \sin \theta} = \frac{(100)(9.81)}{2(\sin 40)} = 763 \text{ N}}$$

$$\boxed{T_1 = 763 \text{ N}}$$

Drawing FBD on:



FBD:

