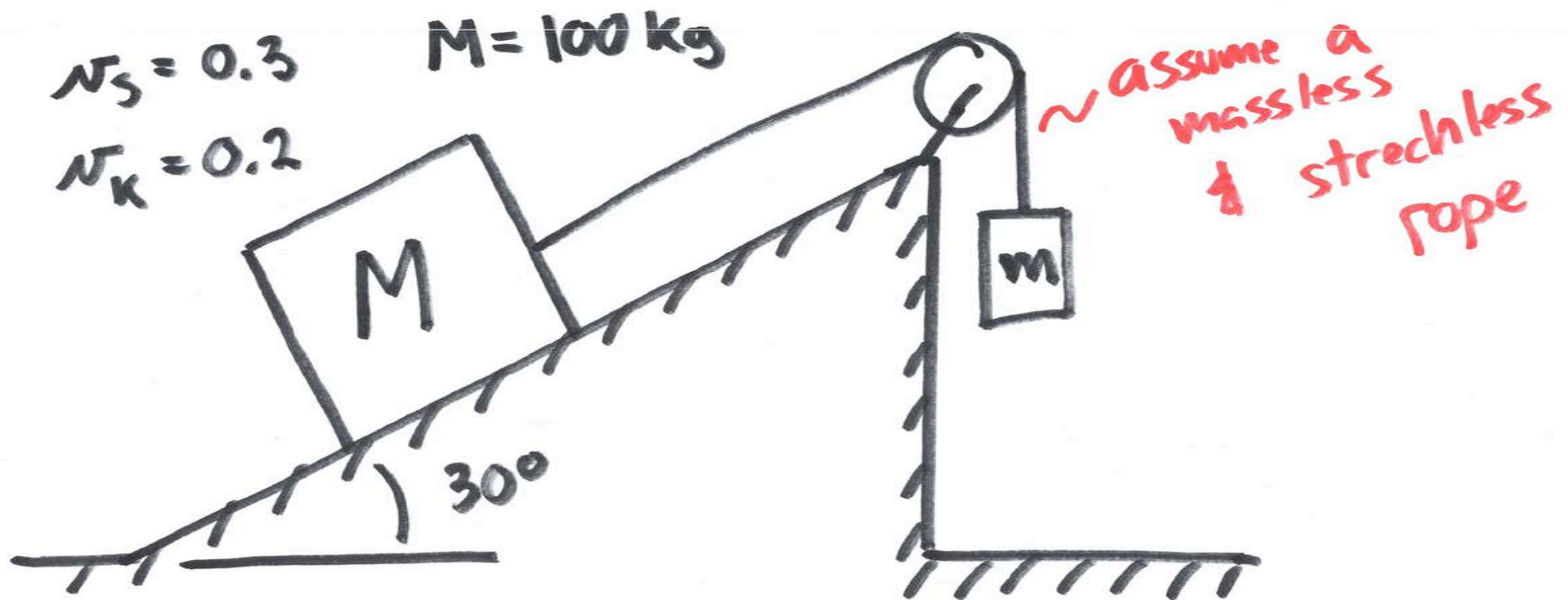
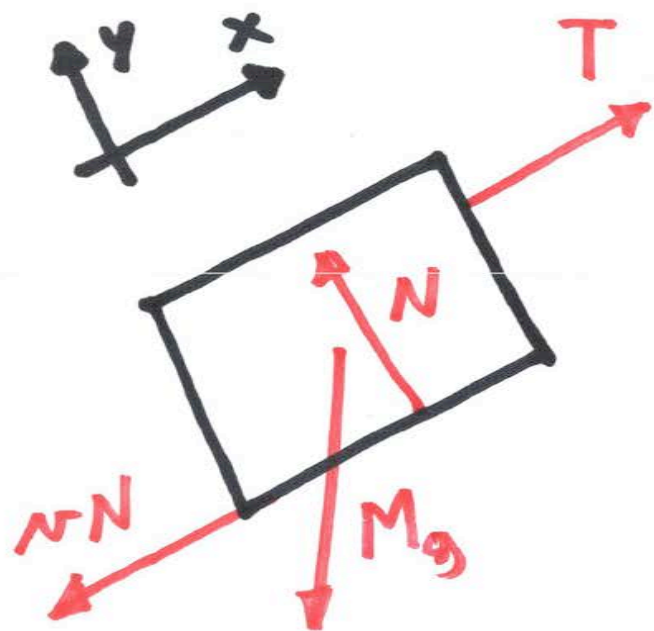


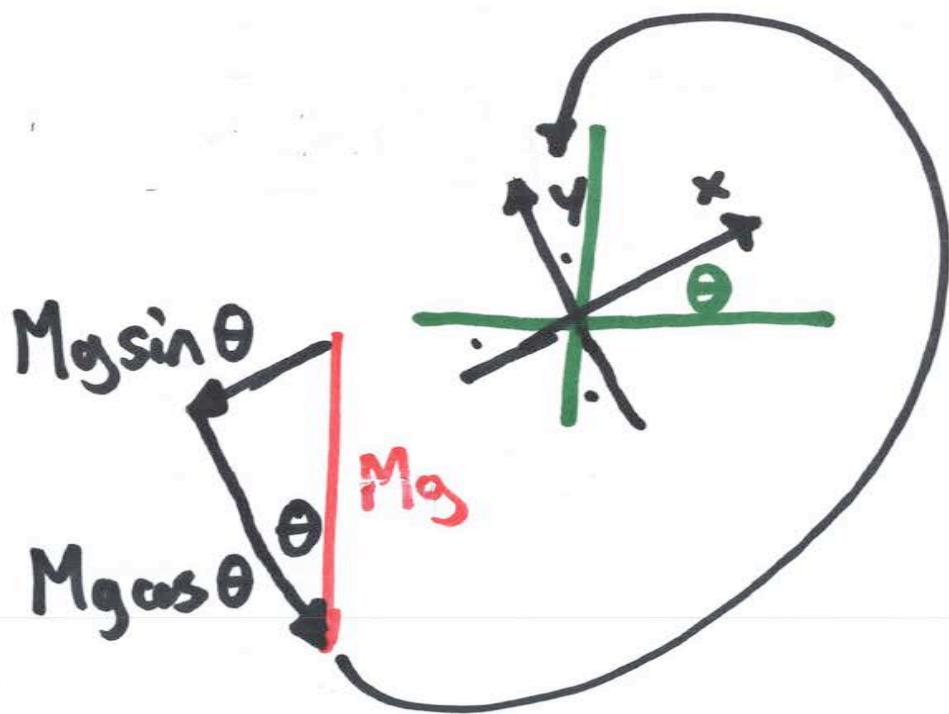
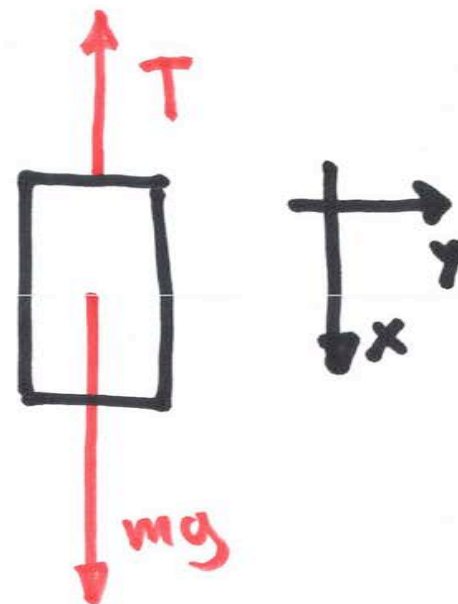
What is the min/max mass need to keep the larger mass from sliding?

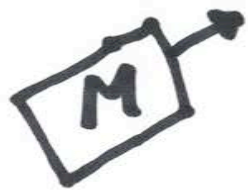




assume
massless
pulley

No
FBD





$$\Sigma F_x = Ma_x = T - \mu N - Mg \sin \theta \quad (1)$$

$$\Sigma F_y = Ma_y = N - Mg \cos \theta \quad (2)$$



$$\Sigma F_x = Ma_x = mg - T \quad (3)$$

$$\Sigma F_y = ma_y = 0$$

Because the rope is stretchless

$$a_{xM} = a_{xm} = a_x$$

sub (2) & (3) into (1)

$$Ma_x = \overbrace{[mg - ma_x]}^{(3)} - \mu \overbrace{[Mg \cos \theta]}^{(2)} - Mg \sin \theta$$

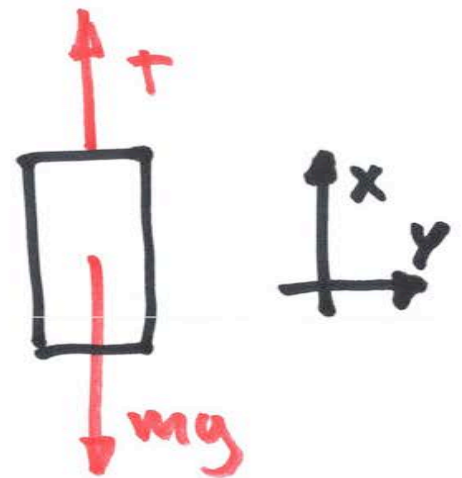
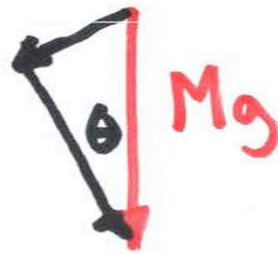
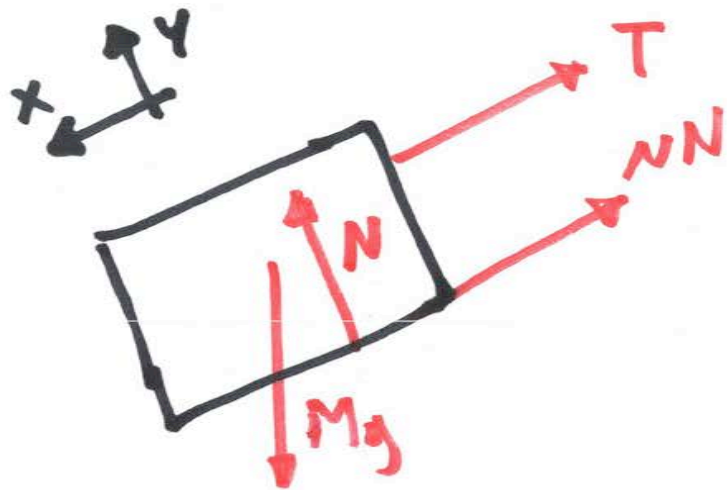
$$Ma_x = m(g - a_x) - \mu Mg \cos \theta - Mg \sin \theta$$

$$m = \frac{Ma_x + \mu Mg \cos \theta + Mg \sin \theta}{g - a_x}$$

find max m (set $a_x = 0$)

$$m_{\max} = \frac{\cancel{(100)}(0) + (0.3)(100)(9.81) \cos(30) + (100)(9.81) \sin 30}{9.81 + 0}$$

$$m_{\max} = 76 \text{ kg}$$



Large Mass:

$$\Sigma F_x = Ma_x = -T - \mu N + Mg \sin \theta \quad (1)$$

$$\Sigma F_y = Ma_y = N - Mg \cos \theta \quad (2)$$

Small Mass:

$$\Sigma F_x = ma_x = T - mg \quad (3)$$

$$\Sigma F_y = ma_y = 0$$

stretchless pulley

$$a_{xM} = a_{xm} = a_x$$

sub (2) & (3) into (1):

$$Ma_x = -[ma_x + mg] - \mu[Mg \cos \theta] + Mg \sin \theta$$

$$Ma_x = -m(a_x + g) - \mu Mg \cos \theta + Mg \sin \theta$$

$$m = \frac{Mg \sin \theta - \mu Mg \cos \theta - Ma_x}{g + a_x}$$

find min m (set $a_x = 0$)

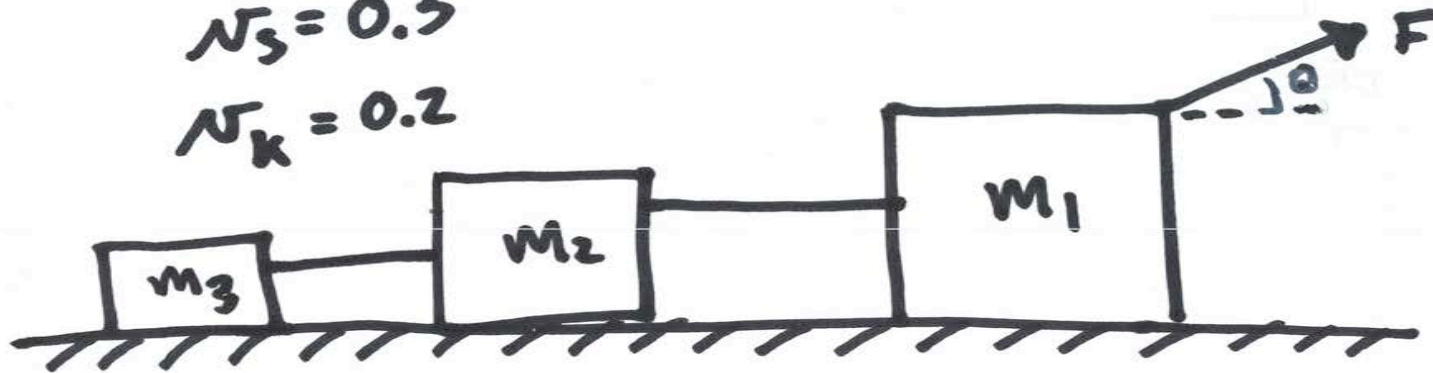
$$m = \frac{(100)(9.81) \sin(30) - (0.3)(100)(9.81) \cos(30) - (100)(0)}{9.81 + 0}$$

$$m = 24 \text{ kg}$$

Determine the tension in each rope.

$$\mu_s = 0.3$$

$$\mu_k = 0.2$$



$$m_1 = 100 \text{ kg}$$

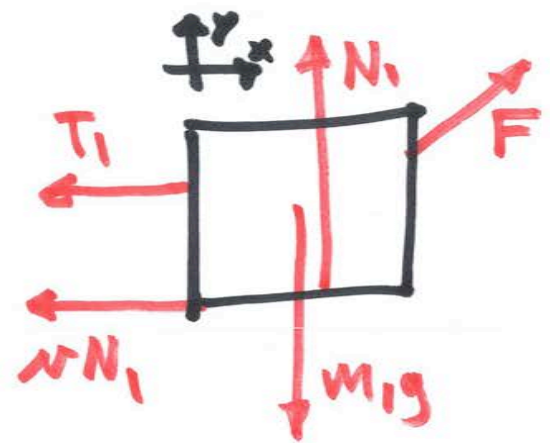
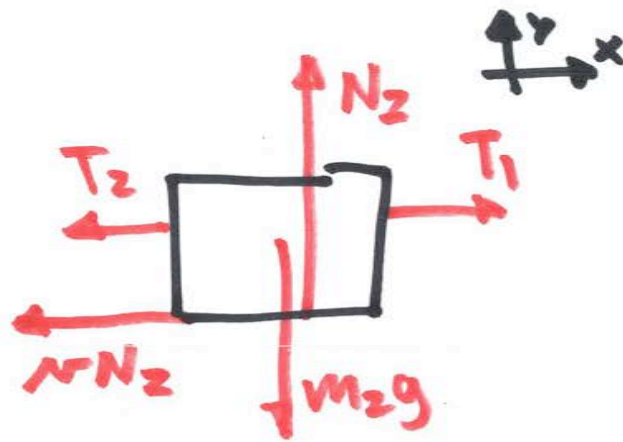
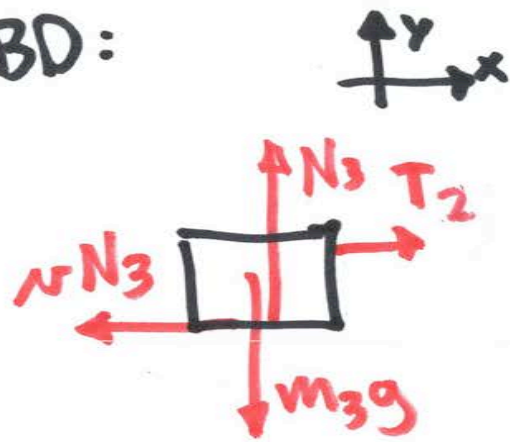
$$m_2 = 50 \text{ kg}$$

$$m_3 = 25 \text{ kg}$$

$$F = 500 \text{ N}$$

$$\theta = 20^\circ$$

FBD:



$$\sum F_x = m_3 a_{x_3} = T_2 - N_3 \quad (1) \quad \sum F_x = m_2 a_{x_2} = T_1 - T_2 - N_2 \quad (3)$$

$$\sum F_y = m_3 a_{y_3} = N_3 - m_3 g \quad (2) \quad \sum F_y = m_2 a_{y_2} = N_2 - m_2 g \quad (4)$$



$$\sum F_x = m_1 a_{x_1} = F \cos \theta - T_1 - N_1 \quad (5)$$

$$\sum F_y = m_1 a_{y_1} = F \sin \theta + N_1 - m_1 g \quad (6)$$

assume stretchless rope:

$$a_{x_3} = a_{x_2} = a_{x_1} = a_x \quad (7)$$

sub (2) & (7) into (1):

$$m_3 a_x = T_2 - \mu m_3 g \quad (1a)$$

sub (1a), (4) & (7) into (3)

$$m_2 a_x = T_1 - [m_3 a_x + \mu m_3 g] - \mu [m_2 g]$$

$$T_1 = m_2 a_x + m_3 a_x + \mu m_2 g + \mu m_3 g \quad (3a)$$

sub (3a), (6) & (7) into (5)

$$m_1 a_x = F \cos \theta - [m_2 a_x + m_3 a_x + \mu m_2 g + \mu m_3 g] \\ - \mu [m_1 g - F \sin \theta]$$

$$m_1 a_x + m_2 a_x + m_3 a_x = F \cos \theta + F \sin \theta - \mu g (m_1 + m_2 + m_3)$$

$$a_x = \frac{F \cos \theta + F \sin \theta - \mu g (m_1 + m_2 + m_3)}{m_1 + m_2 + m_3}$$

$$a_x = \frac{500 \cos(20) + 500 \sin(20) - (0.3)(9.81)(175)}{175}$$

$$a_x = 0.719 \text{ m/s}^2 \quad \text{it moves!}$$

$$a_x = 1.7 \text{ m/s}^2$$

$$\mu_k = 0.2.$$

$$T_1 = (1.7)(50+25) + (0.2)(9.81)(50+25)$$

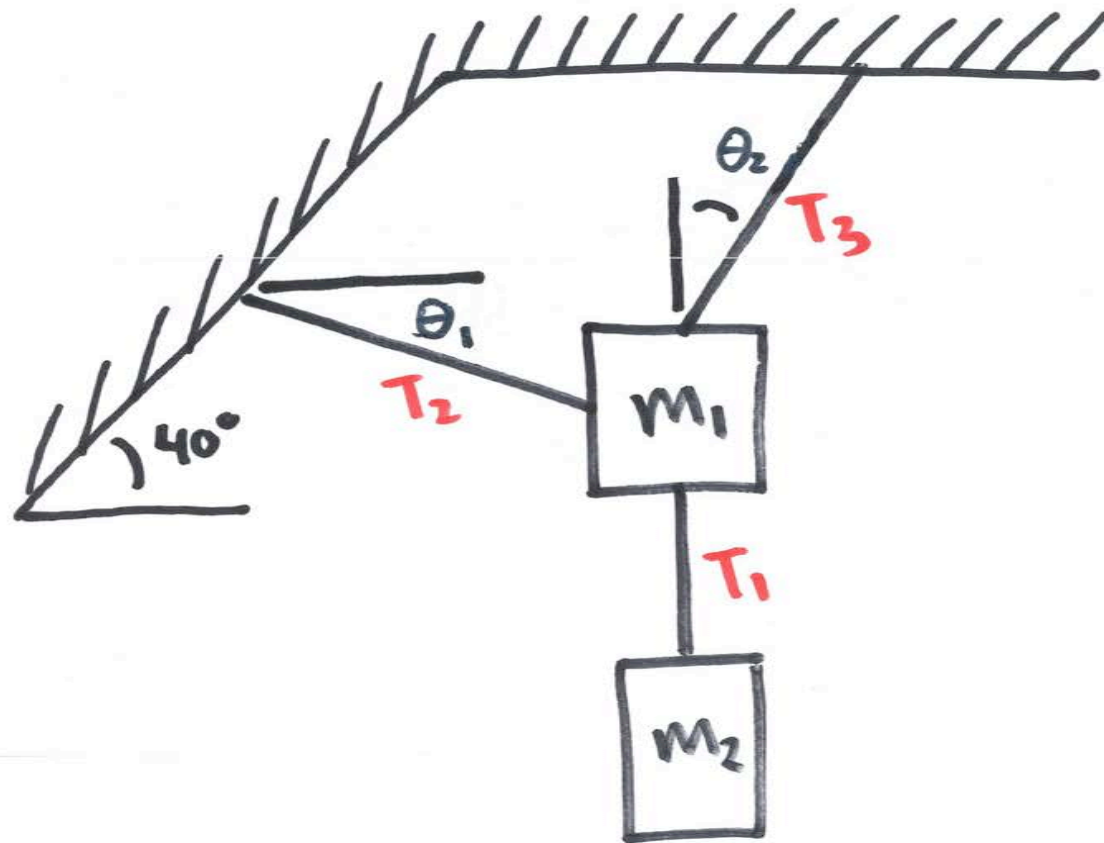
$$T_1 = 275 \text{ N}$$

$$= a_x(m_2+m_3) + N_k g(m_2+m_3)$$

$$T_2 = m_3 a_x + N m_3 g$$

$$T_2 = (25)(1.7) + (0.2)(25)(9.81)$$

$$T_2 = 96 \text{ N}$$



Find the tension
in each rope.

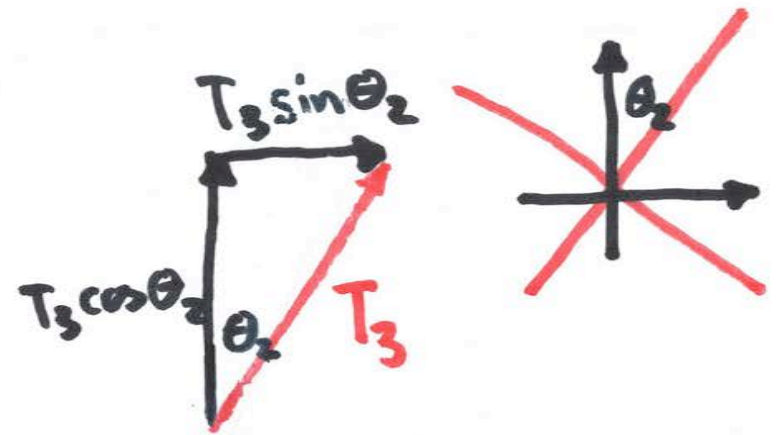
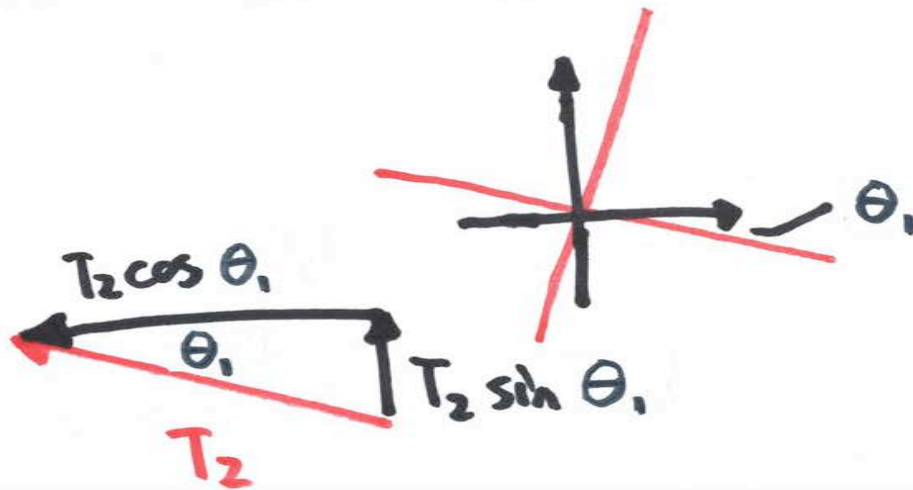
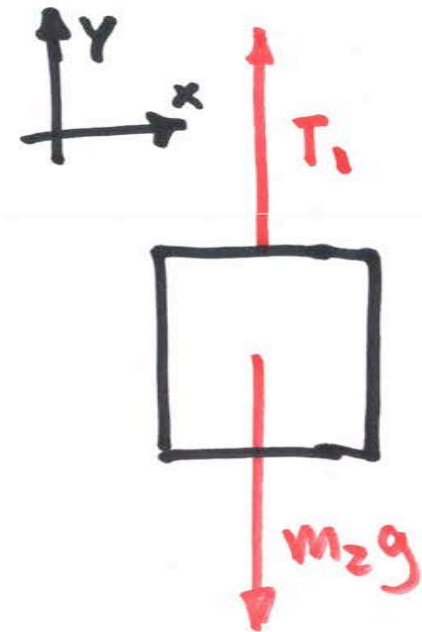
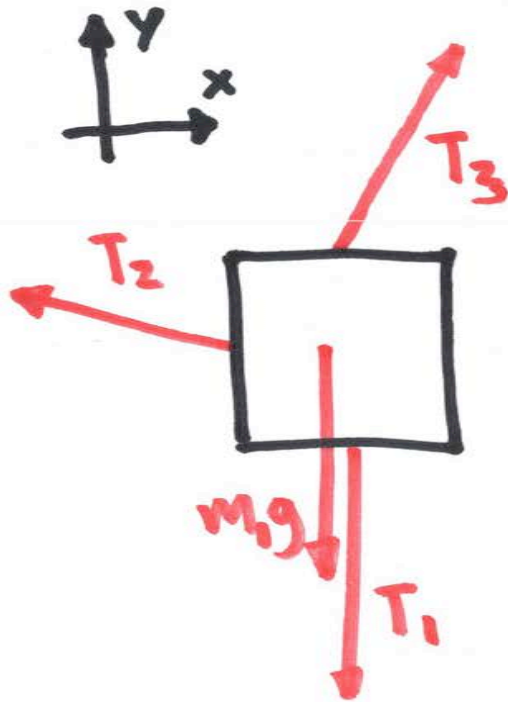
$$\theta_1 = 20^\circ$$

$$\theta_2 = 30^\circ$$

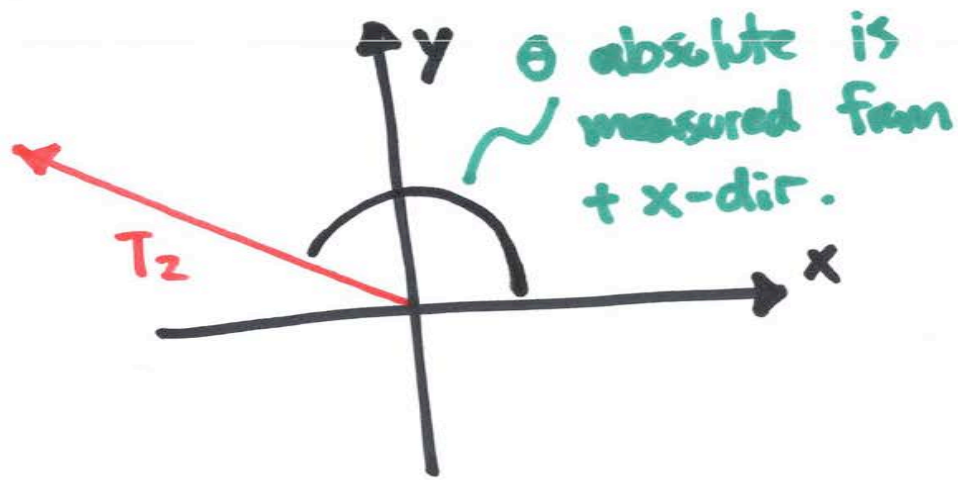
$$m_1 = 100 \text{ kg}$$

$$m_2 = 50 \text{ kg}$$

FBD:



Relative vs. Absolute Angle.



$$+ T_2 \cos \theta \quad (x\text{-dir})$$

$$+ T_2 \sin \theta \quad (y\text{-dir.})$$

For example $\theta = 160^\circ$

$$\cos(160) = -0.94$$

$$\sin(160) = 0.34$$

Don't use absolute
use relative.

Newton's 2nd Law: (m_1)

$$\sum F_x = m_1 a_x = T_3 \sin \theta_2 - T_2 \cos \theta_1 \quad (1)$$

$$\sum F_y = m_1 a_y = T_3 \cos \theta_2 + T_2 \sin \theta_1 - m_1 g - T_1 \quad (2)$$

Newton's 2nd Law: (m_2)

$$\sum F_x = m_2 a_x = 0 \quad (3)$$

$$\sum F_y = m_2 a_y = T_1 - m_2 g \quad (4)$$

Solve for T_1 in (4):

$$\boxed{T_1 = m_2 g} \quad (4a)$$

Solve for T_2 in (1):

$$\boxed{T_2 = T_3 \frac{\sin \theta_2}{\cos \theta_1}} \quad (1a)$$

sub (1a), (4a) into (2):

$$0 = T_3 \cos \theta_2 + \left[T_3 \frac{\sin \theta_2}{\cos \theta_1} \right] \sin \theta_1 - m_1 g - [m_2 g]$$

$$T_3 \left(\cos \theta_2 + \sin \theta_2 \frac{\sin \theta_1}{\cos \theta_1} \right) = m_1 g + m_2 g$$

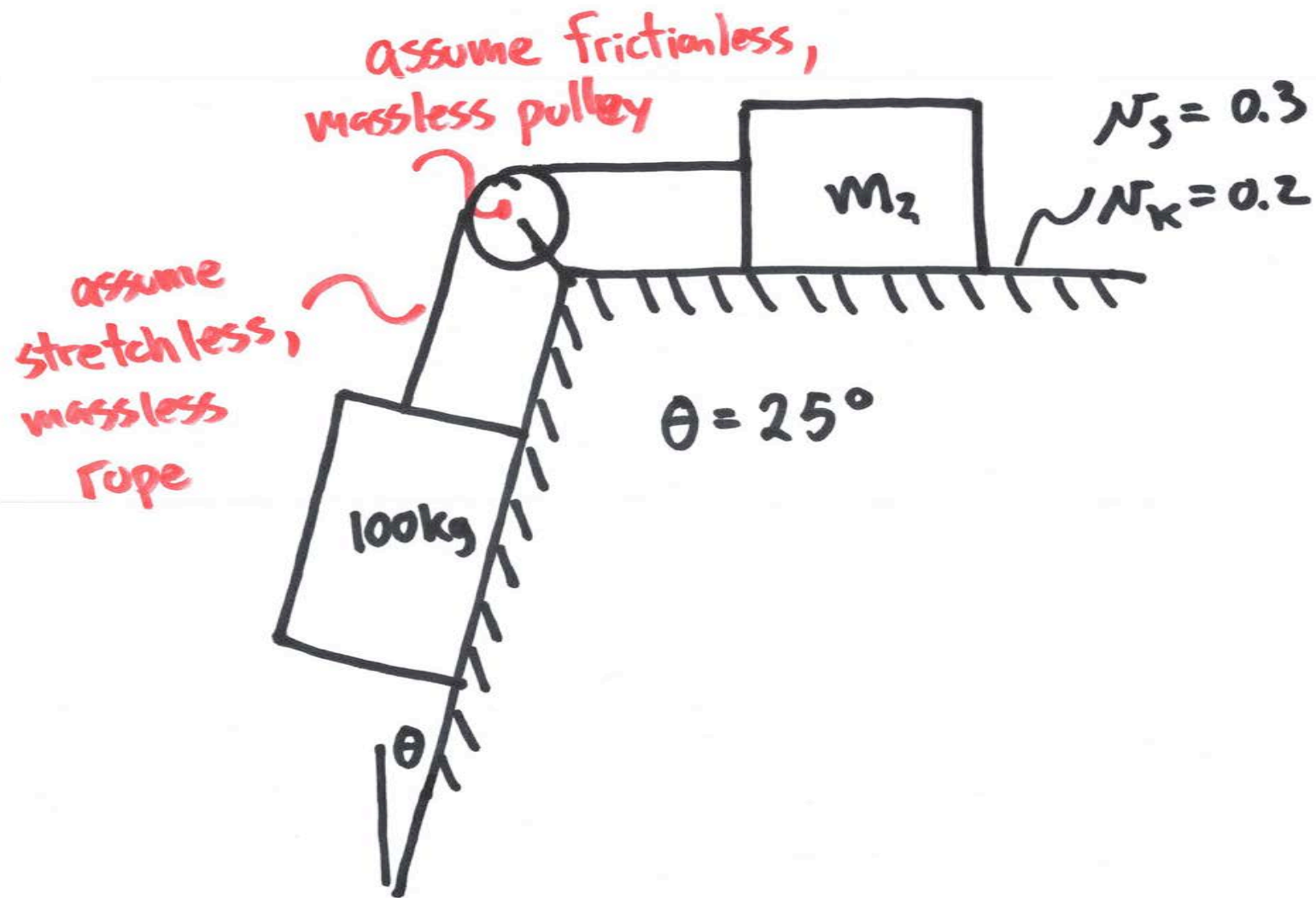
$$T_3 = g \frac{m_1 + m_2}{\cos \theta_2 + \sin \theta_2 \tan \theta_1}$$

$$T_1 = (50)(9.81) = 490.5 \text{ N}$$

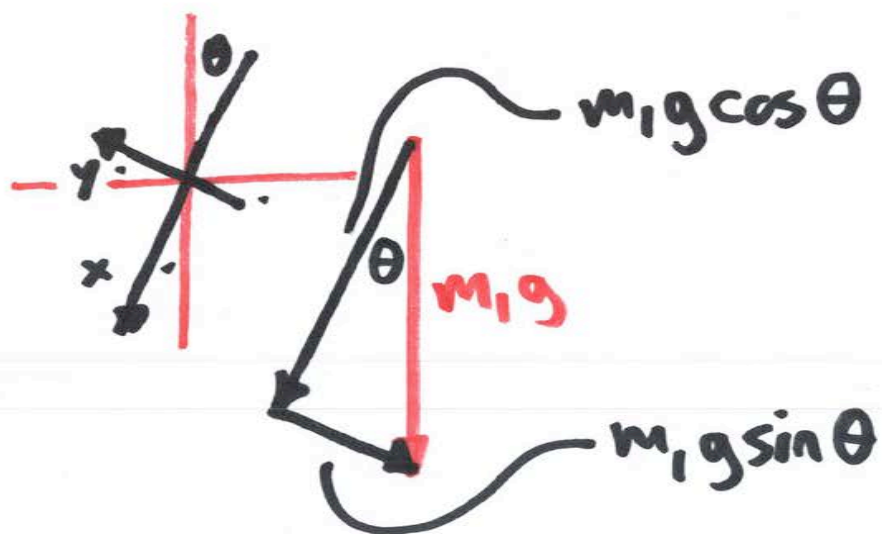
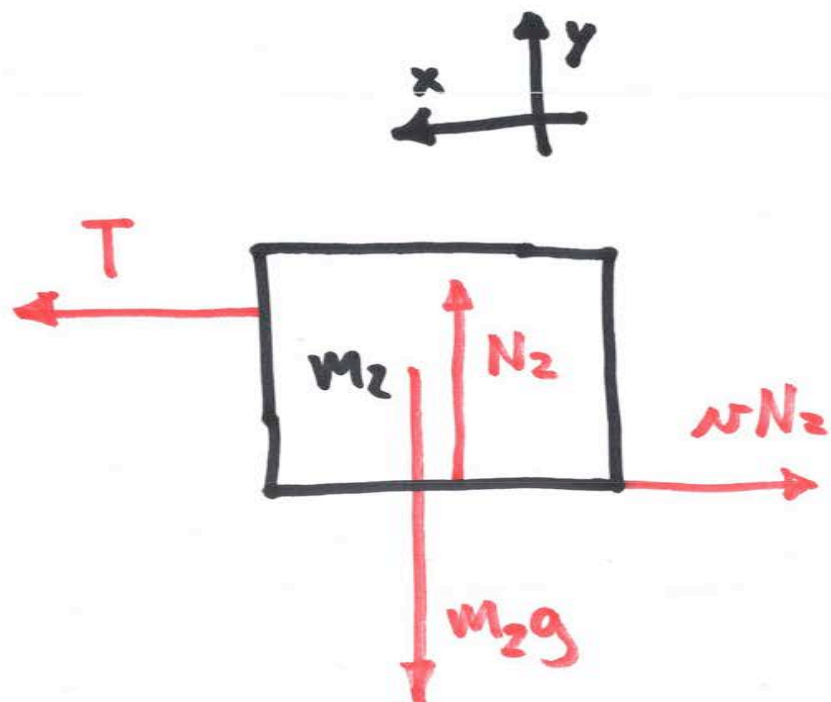
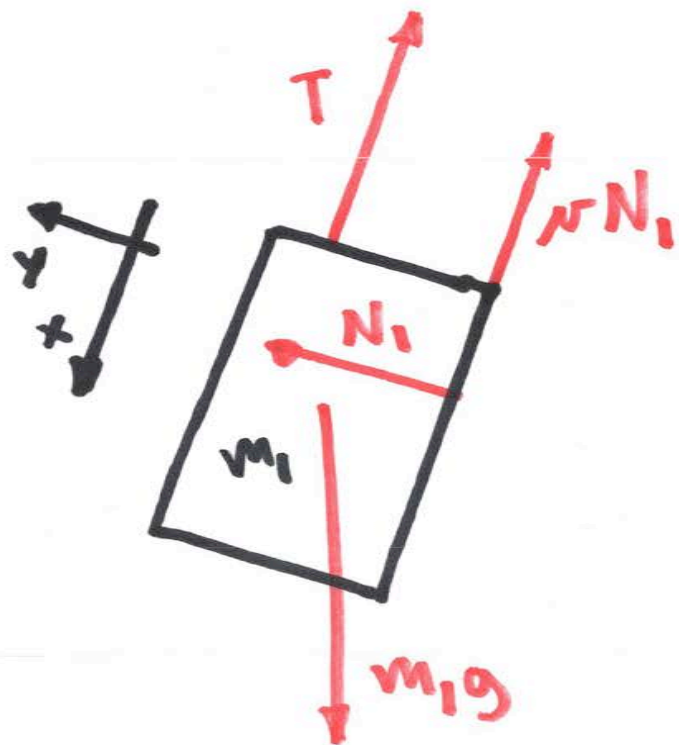
$$T_3 = (9.81) \frac{100 + 50}{\cos(30) + \sin(30)\tan(20)} = 1404 \text{ N}$$

$$T_2 = (1404) \frac{\sin 30}{\cos 20} = 747 \text{ N}$$

Determine minimum mass so the objects don't slide.



FBD:



Newton's 2nd Law: (m_1)

$$\Sigma F_x = m_1 a_{x_1} = m_1 g \cos \theta - T - \mu N_1 \quad (1)$$

$$\Sigma F_y = m_1 a_{y_1} = N_1 - m_1 g \sin \theta \quad (2)$$

Newton's 2nd Law: (m_2)

$$\Sigma F_x = m_2 a_{x_2} = T - \mu N_2 \quad (3)$$

$$\Sigma F_y = m_2 a_{y_2} = N_2 - m_2 g \quad (4)$$

Assumed stretchless rope:

$$a_{x_1} = a_{x_2} = a_x \quad (5)$$

Solve (4) for N_2 :

$$\boxed{N_2 = m_2 g}$$

(4a)

Solve (2) for N_1 :

$$\boxed{N_1 = m_1 g \sin \theta}$$

(2a)

Sub (4a), (5) into (3) solve T :

$$m_2 a_x = T - \mu [m_2 g]$$

$$\boxed{T = m_2 a_x + \mu m_2 g}$$

(3a)

Sub (2a), (3a), (5) into (1):

$$m_1 a_x = m_1 g \cos \theta - [m_2 a_x + \mu m_2 g] - \mu [m_1 g \sin \theta]$$

$$m_1 a_x = m_1 g \cos \theta - m_2 a_x - \mu m_2 g - \mu m_1 g \sin \theta$$

$$m_2 (a_x + \mu g) = m_1 g \cos \theta - m_1 a_x - \mu m_1 g \sin \theta$$

$$m_2 = m_1 \frac{g (\cos \theta - \mu \sin \theta) - a_x}{g \mu + a_x}$$

solve for $m_{2\min}$. set $a_x = 0$

$$m_{2\min} = (100) \frac{(9.81)(\cos 25 - (0.3)\sin 25)}{(9.81)(0.3)}$$

$$m_{2\min} = 260 \text{ kg}$$