

Solving Systems of Equations

$$(1) \quad a + b = 2$$

$$(2) \quad a - b = 0$$

independent
equations

solve for a in (1)

$$a = 2 - b$$

sub into (2)

$$[2 - b] - b = 0$$

$$-2b = -2$$

$$b = 1$$

$$\therefore a = 1$$

$$\begin{cases} a + b = 2 & (1) \\ 2a + 2b = 4 & (2) \end{cases}$$

dependent
equations
(redundant)

solve for a in (1)

$$a = 2 - b$$

sub into (2)

$$2[2 - b] + 2b = 4$$

$$4 - \cancel{2b} + \cancel{2b} = 4$$

$$4 = 4$$

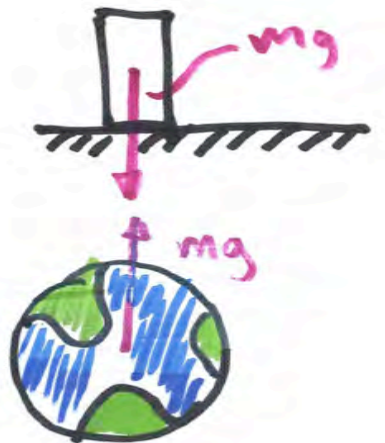
Newton's Three Laws of Motions

1. An object in motion stays in motion unless acted on by a force.



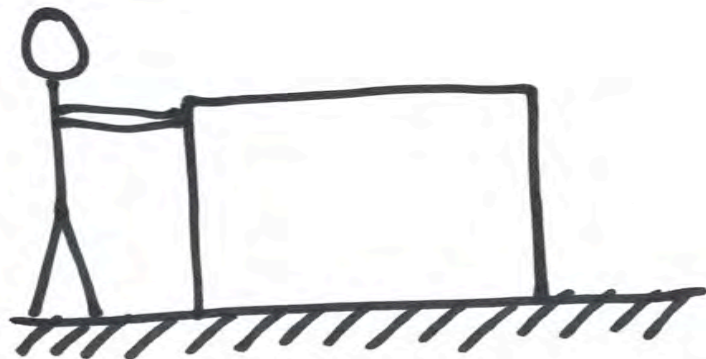
2. $\boxed{\sum \vec{F} = m\vec{a}}$ $\left\{ \begin{array}{l} \sum F_x = ma_x \\ \sum F_y = ma_y \end{array} \right.$

3. Every action force has an equal and opposite reaction force.

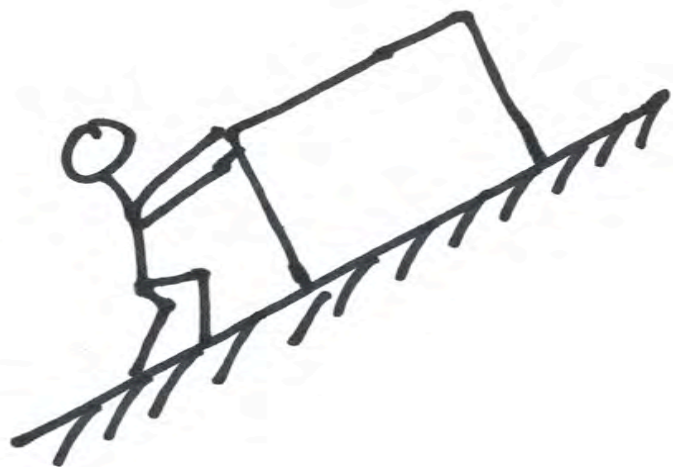
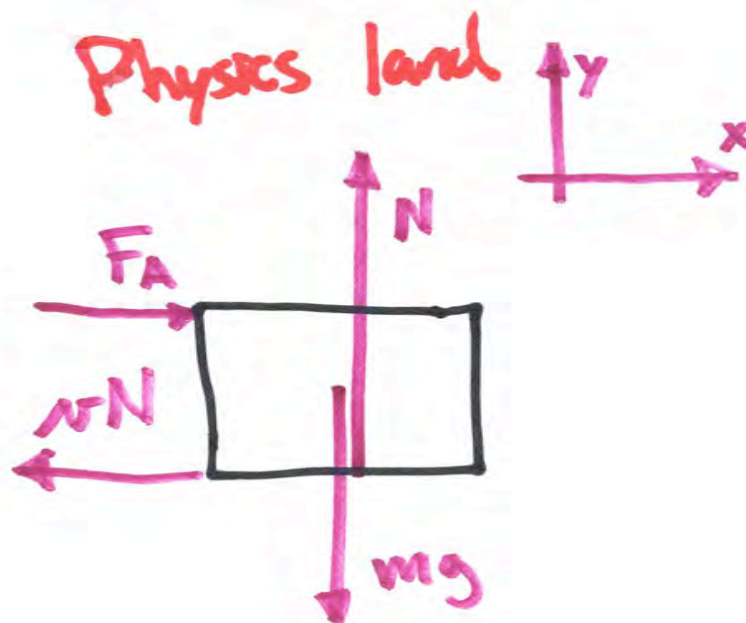


FBD:

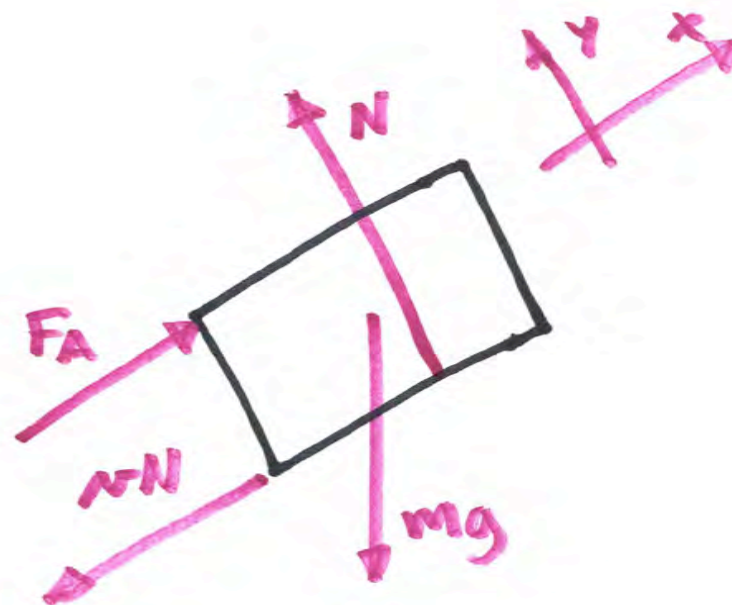
Real World

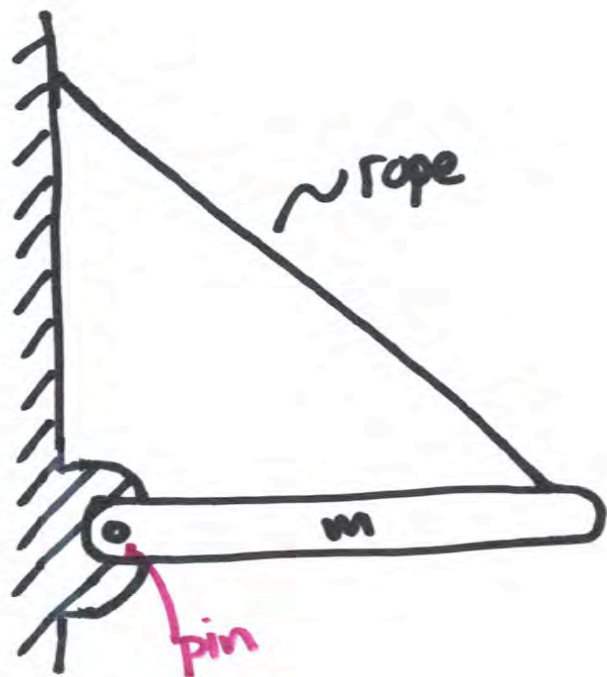


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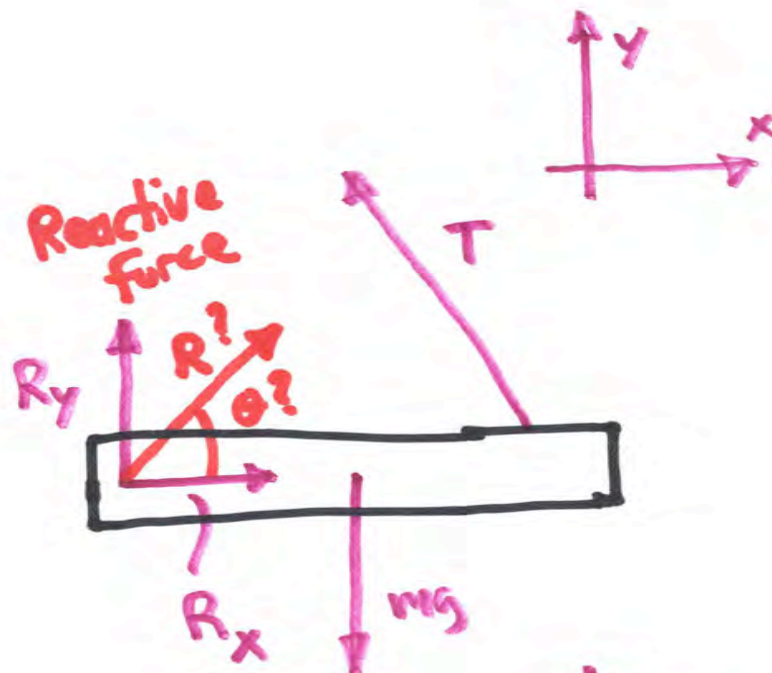


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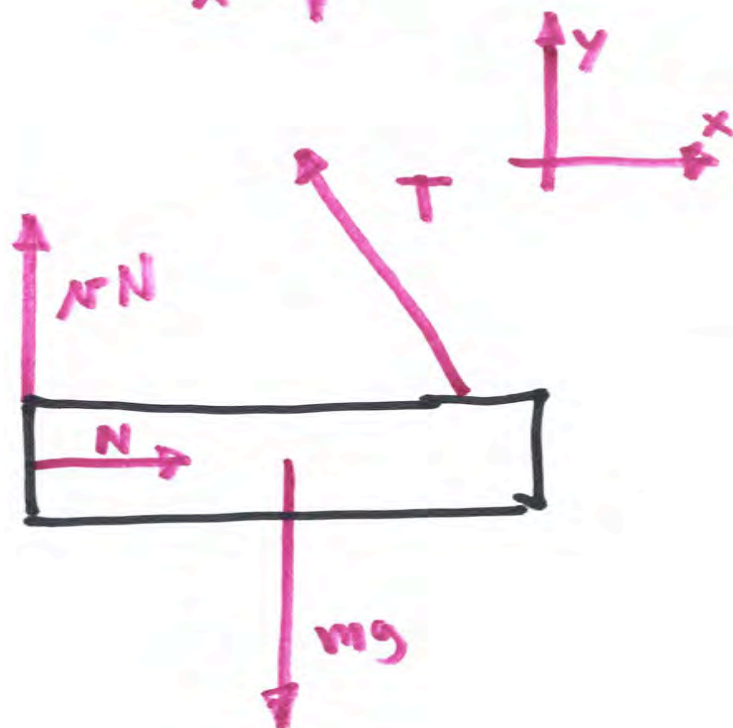
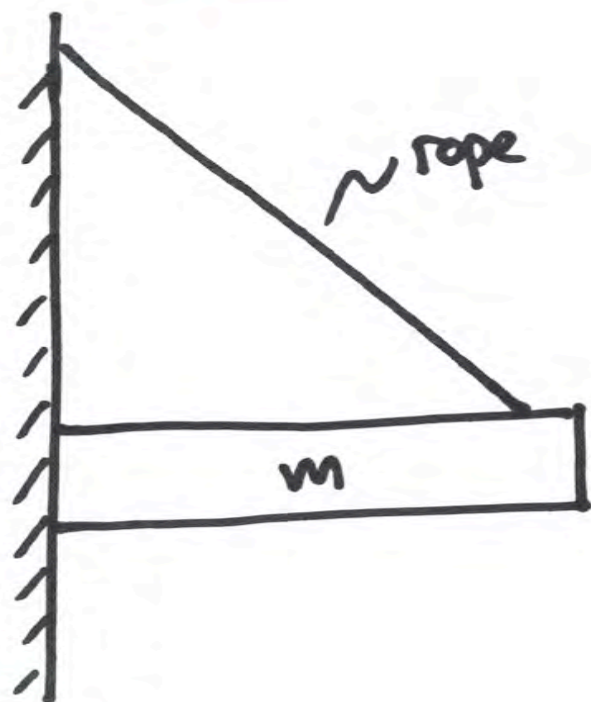


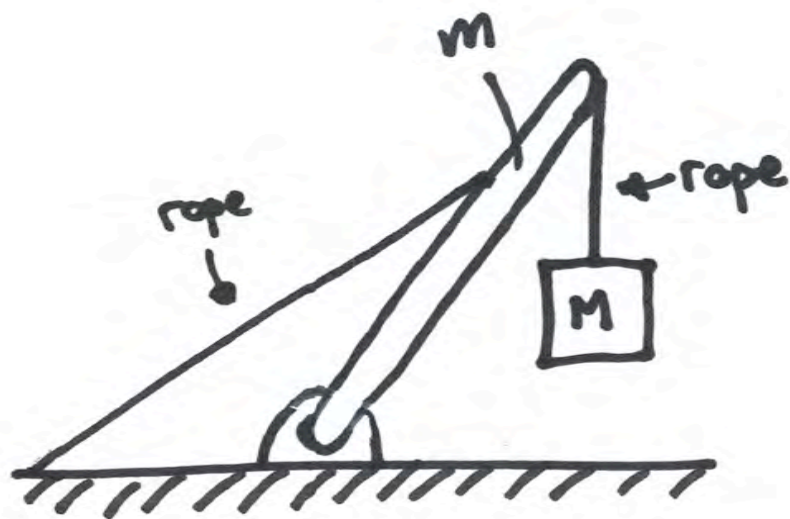


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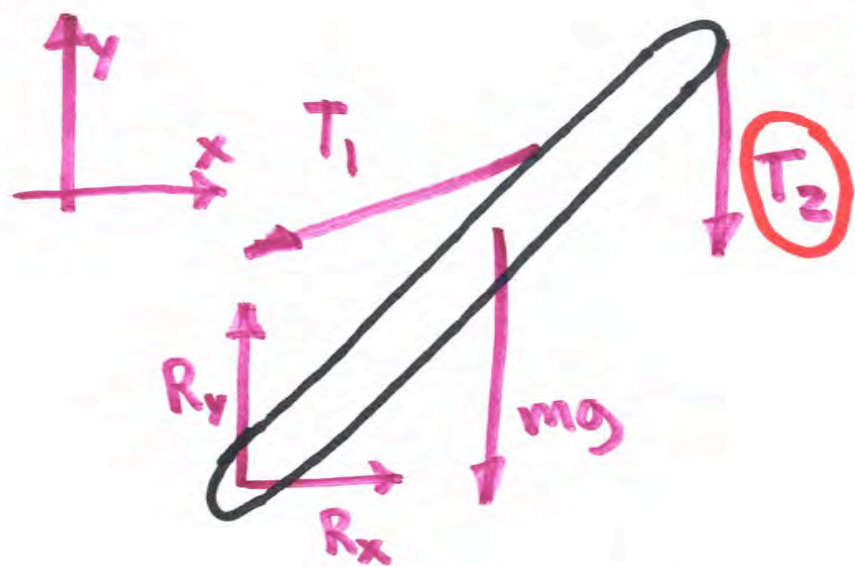


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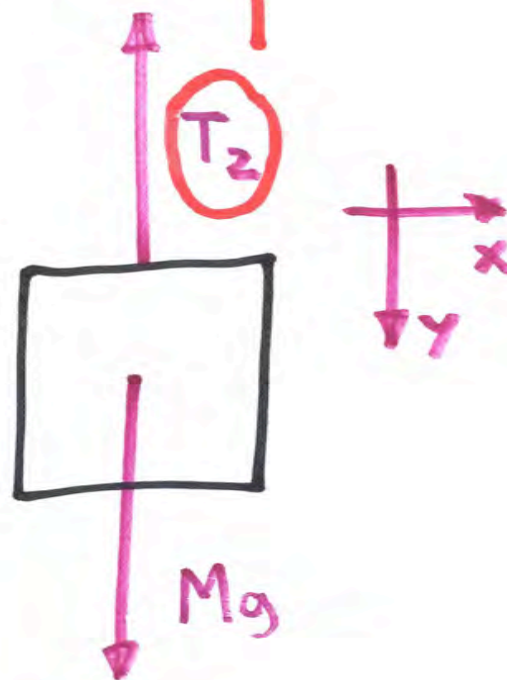


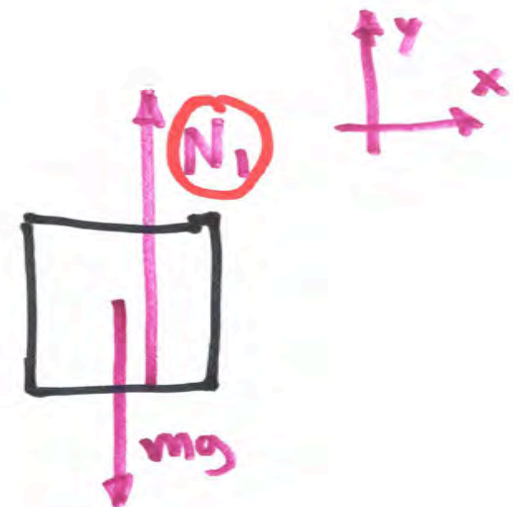
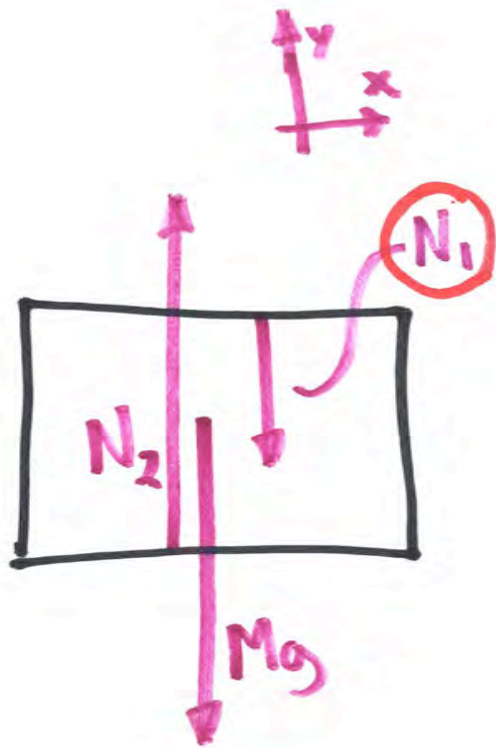
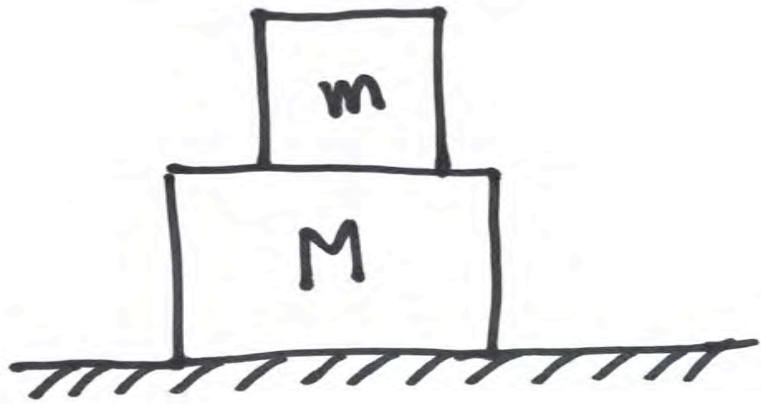


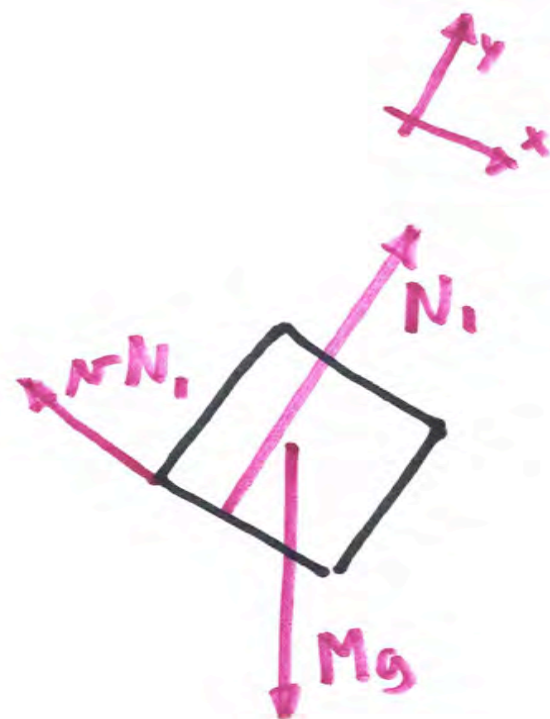
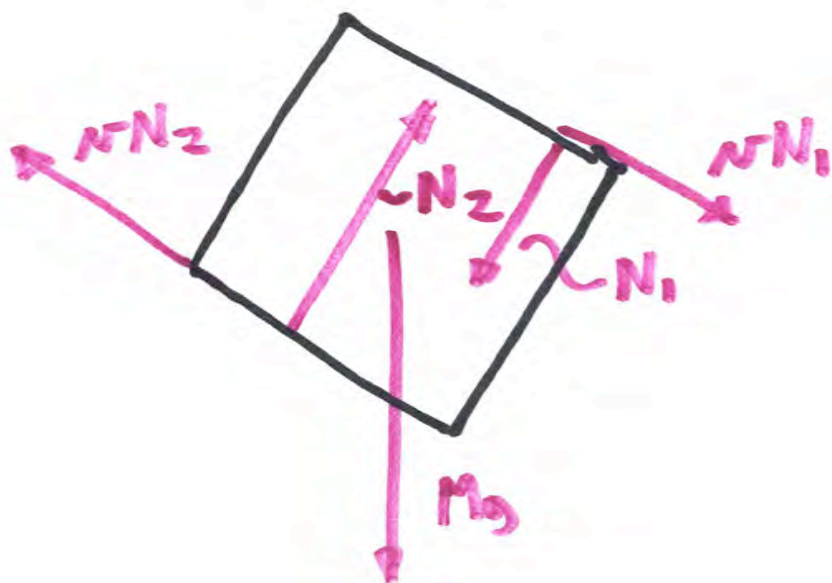
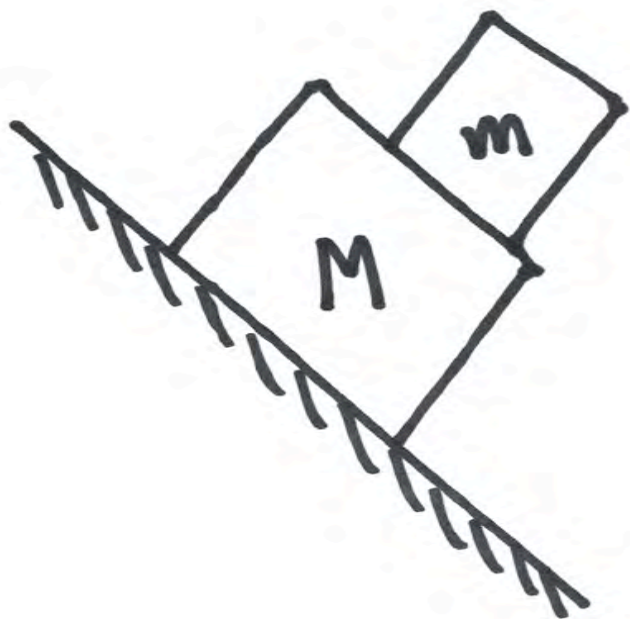
Newton's
3rd Law



Reaction force

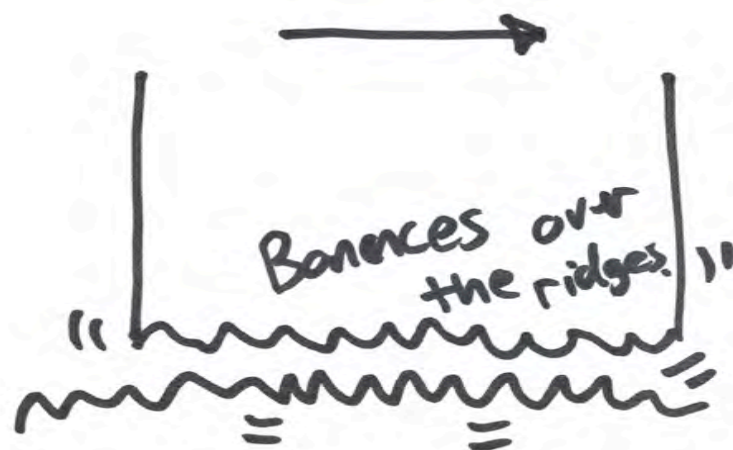






N_s

vs.

 N_k 

$$N_s > N_k$$

Drag. Force

$$R = F(v^2)$$

function of
velocity.

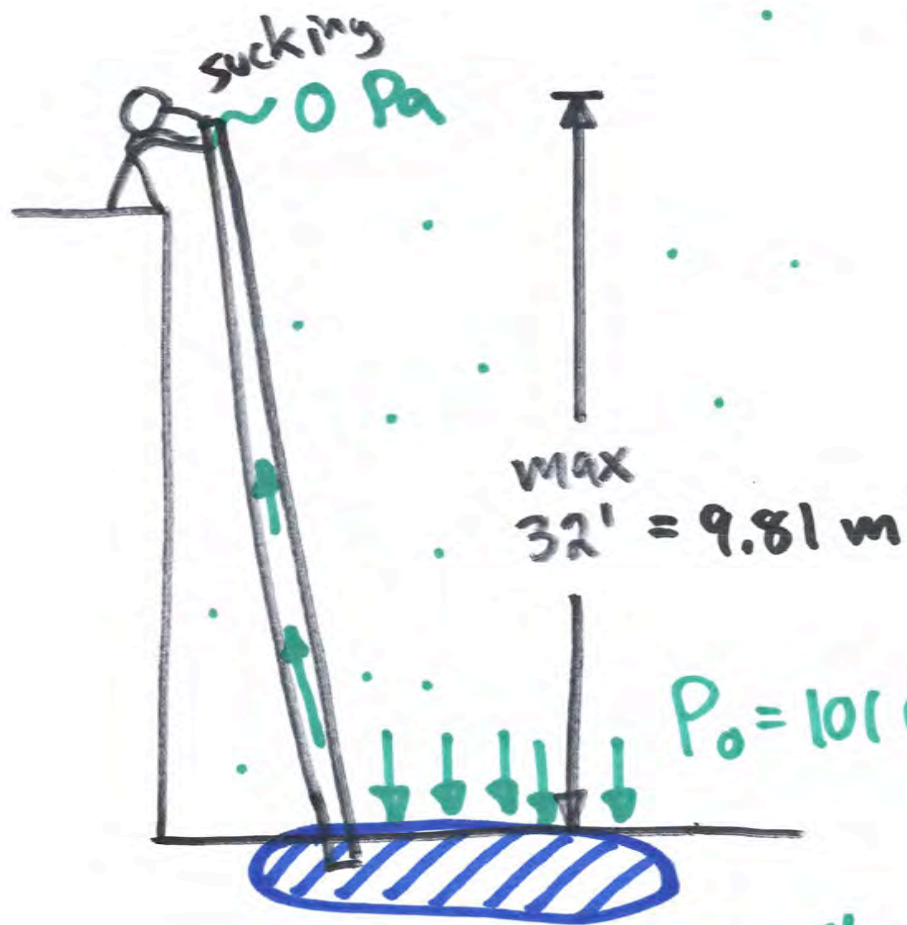
Deal with this

next year ... it

become iterative ...

Pressure force

(Fluid Mechanics)



max
32' = 9.81 m

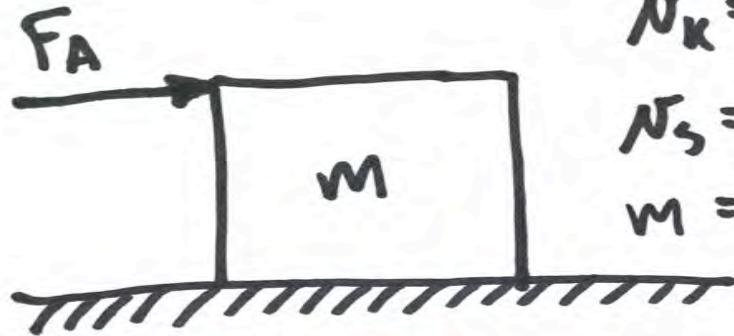
$$P_0 = 101000 \text{ Pa} = 101000 \text{ N/m}^2$$

$$P = \rho g h$$

gravity
height
density

$$h = \frac{P}{\rho g} = \frac{101000}{(1000)(9.81)} = 10 \text{ m}$$

Does it move?



$$F_A = 575 \text{ N}$$

$$\mu_k = 0.2$$

$$\mu_s = 0.3$$

$$m = 200 \text{ kg}$$

Newton's 2nd Law:

$$\sum F_x = m \underline{a_x} = \underline{\sum F_x} \\ = F_A - \mu \underline{N} \quad (1)$$

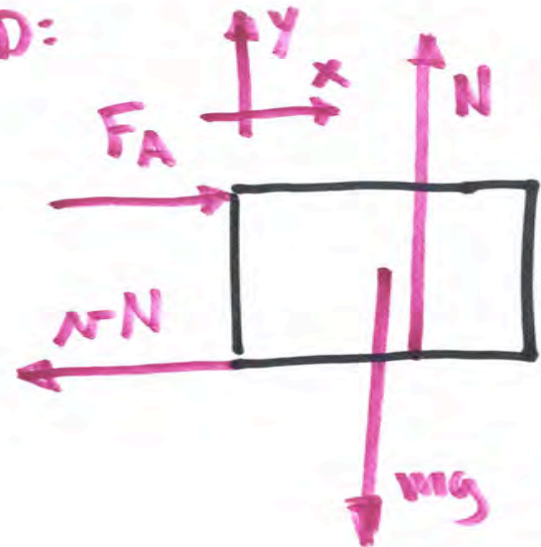
$$\sum F_y = \cancel{m \underline{a_y}} = \underline{N} - mg \quad (2)$$

Solve (2) for N

$$0 = N - mg$$

$$N = mg \quad (2a)$$

FBD:



Sub (2a) into (1)

$$ma_x = F_A - \mu [mg]$$

$$a_x = \frac{1}{m} (F_A - \mu mg)$$

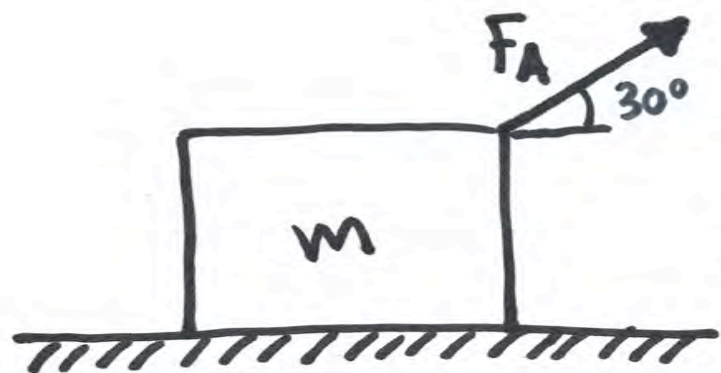
check μ_s :

$$a_x = \frac{1}{200} (575 - (0.3)(200)(9.81))$$

$$= -0.07 \text{ m/s}^2$$

does not move.

Does it move?



$$\begin{aligned}F_A &= 575 \\ \mu_k &= 0.2 \\ \mu_s &= 0.3 \\ m &= 200 \text{ kg}\end{aligned}$$

Newton's 2nd Laws:

$$\sum F_x = ma_x = F_A \cos \theta - \mu N \quad (1)$$

$$\sum F_y = ma_y = N + F_A \sin \theta - mg \quad (2)$$

solve (2) for N:

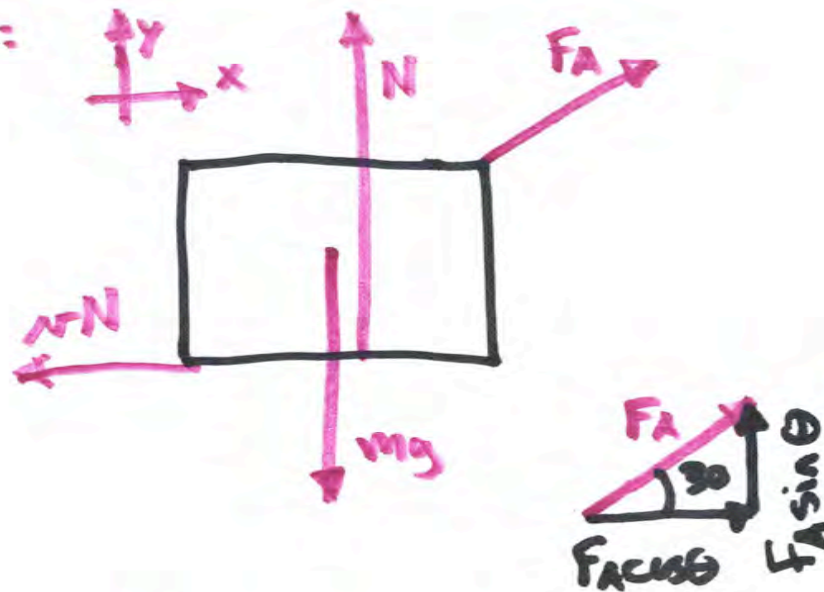
$$N = mg - F_A \sin \theta \quad (2a)$$

sub (2a) into (1)

$$ma_x = F_A \cos \theta - \mu [mg - F_A \sin \theta]$$

$$ma_x = F_A \cos \theta - \mu mg + \mu F_A \sin \theta$$

FBD:



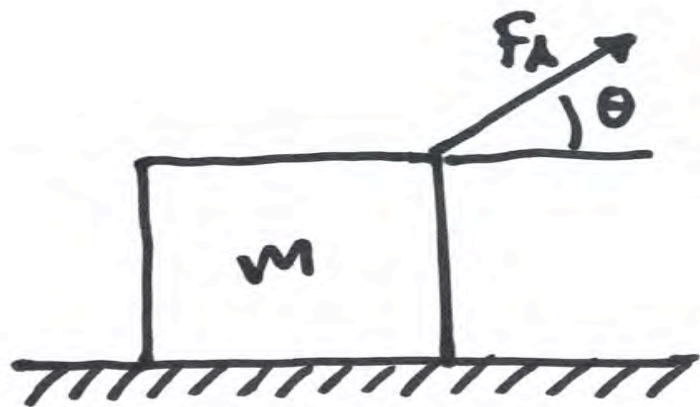
check μ_s :

$$\begin{aligned}ma_x &= (575) \cos(30) - \\ &\quad (0.3)(200)(9.81) + \\ &\quad (0.3)(575) \sin(30) \\ &= -4.36 \text{ N}\end{aligned}$$

$$a_x = -0.02 \text{ m/s}^2$$

does not move.

Can it Move?



determine max/min:

$$m \frac{da_x}{d\theta} = -F_A \sin\theta + \mu F_A \cos\theta = 0 \quad \text{max/min}$$

$$\mu F_A \cos\theta = F_A \sin\theta \quad \text{max}$$

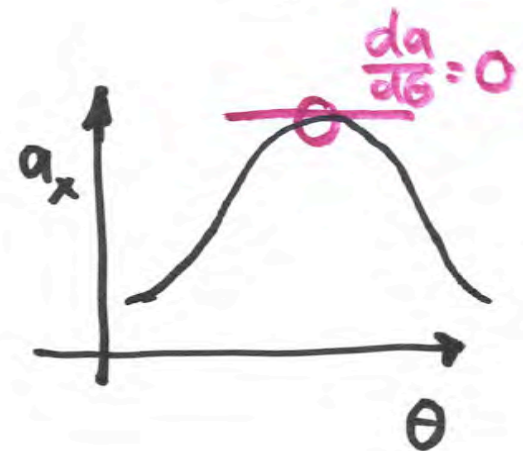
$$\mu = \frac{\sin\theta_{\text{max}}}{\cos\theta_{\text{max}}}$$

$$\theta_{\text{max}} = \tan^{-1}(\mu)$$

$$ma_x = F_A \cos\theta - \mu mg + \mu F_A \sin\theta$$

put into computer:

θ	a_x
0°	$\vdots$
1°	$\vdots$
2°	
3°	
$\vdots$	



for $\mu_s = 0.3$

$$\theta_{\text{max}} = 17^\circ$$

Will it move with $\theta = 17^\circ$

$$m_{ax} = F_A \cos \theta - \mu mg + \mu F_A \sin \theta$$

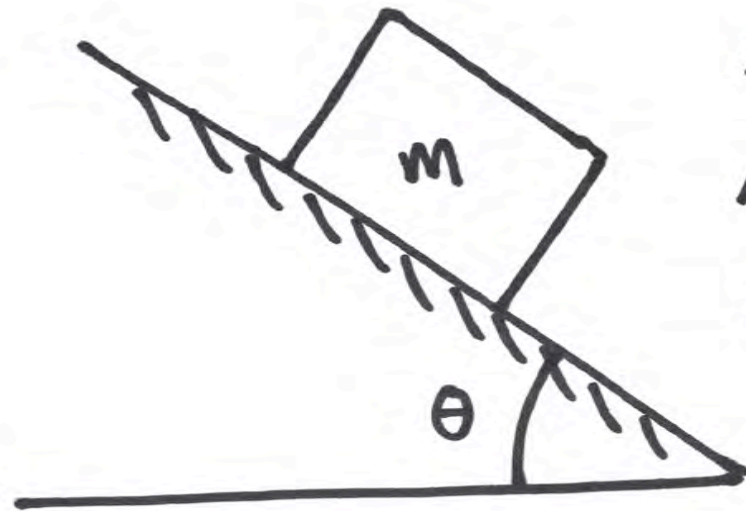
$$a_x = \frac{1}{200} (575 \cos 17 - (0.3)(200)(9.81) + (0.3)(575) \sin 17)$$
$$= 0.059 \text{ m/s}^2 \quad \text{IT MOVES!!}$$

What is the actual acceleration?

$$a_x = \frac{1}{200} (575 \cos 17 - (0.2)(200)(9.81) + (0.2)(575) \sin 17)$$

$$a_x = 0.96 \text{ m/s}^2$$

Does it slide?



$$m = 100 \text{ kg}$$

$$\mu_k = 0.2$$

$$\mu_s = 0.3$$

$$\theta = 30^\circ$$

$$\Sigma F_x = ma_x = mg \sin \theta - \mu N \quad (1)$$

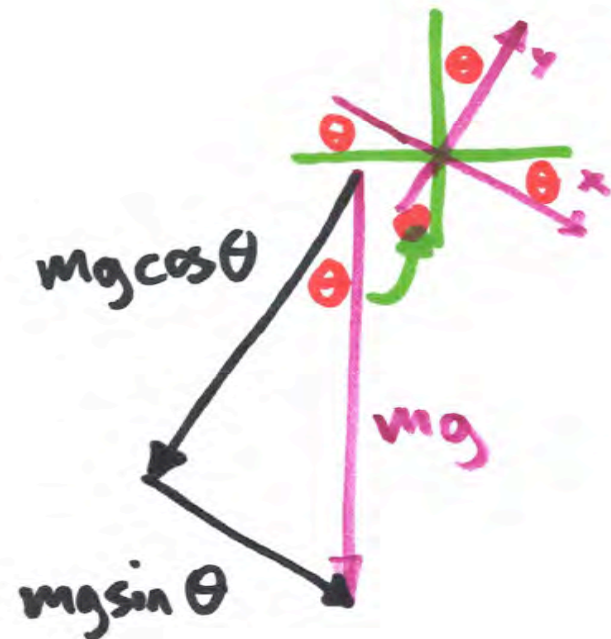
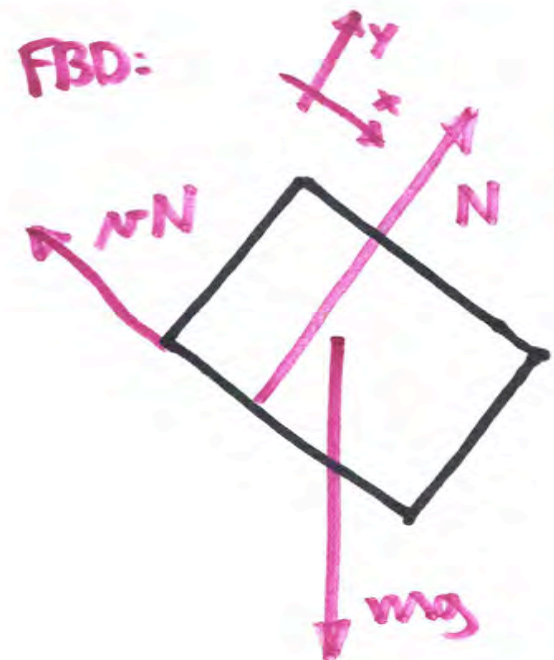
$$\Sigma F_y = ma_y = N - mg \cos \theta \quad (2)$$

Solve (2) for N :

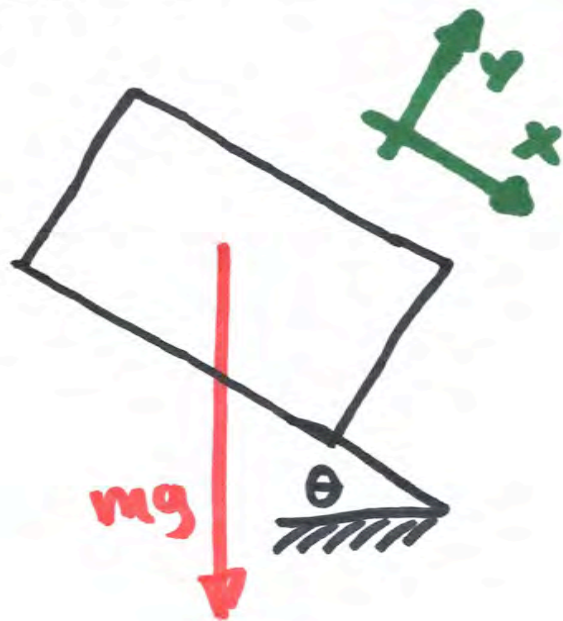
$$N = mg \cos \theta \quad (2a)$$

Sub (2a) into (1)

$$ma_x = mg \sin \theta - \mu [mg \cos \theta]$$



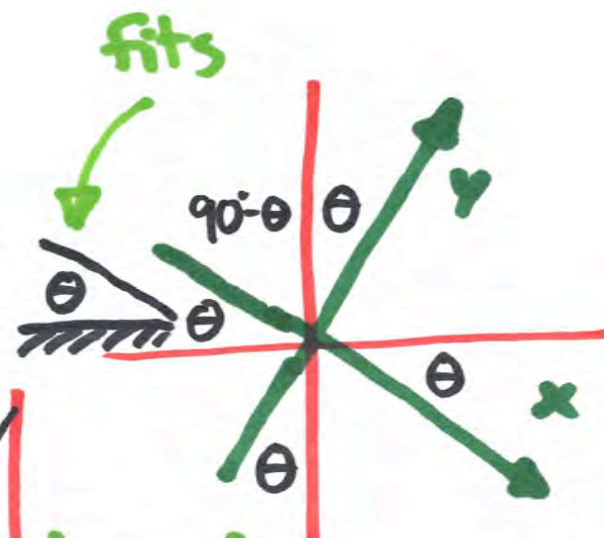
Where does the angle go?



y component $\rightarrow$



x component



check N_s :

$$ma_x = (100)(9.81)\sin 30 - (0.3)(100)(9.81)\cos 30$$
$$= 236 \text{ N}$$

$$a_x = 2.36 \text{ m/s}^2$$

actual acceleration:

$$a_x = \frac{1}{100} \left((100)(9.81)\sin(30) - (0.2)(100)(9.81)\cos(30) \right)$$

$$a_x = 3.21 \text{ m/s}^2$$