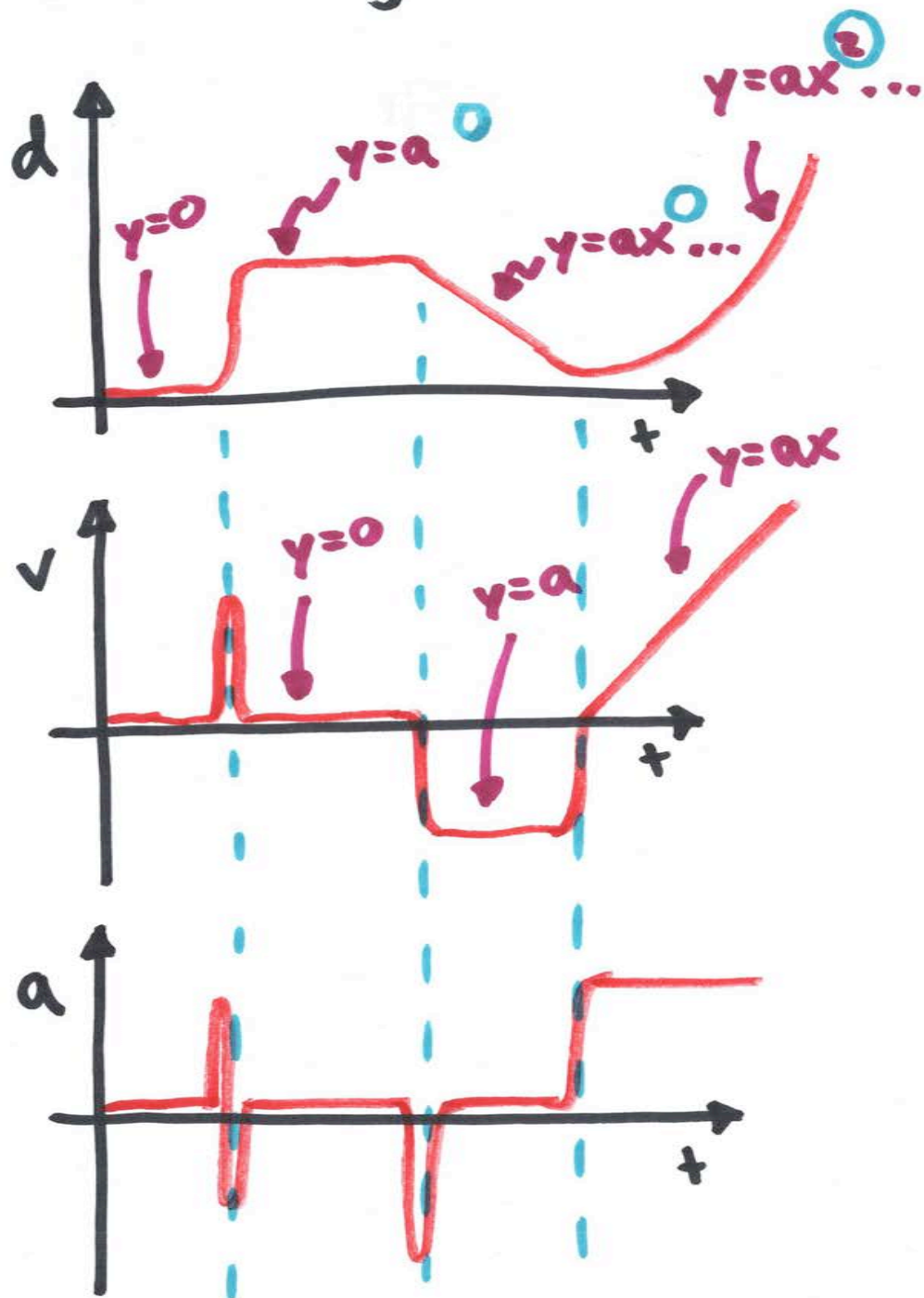


d-v-a graphs



$$y = a \quad \text{0th order poly.}$$

$$y = ax + b \quad \text{1st order poly.}$$

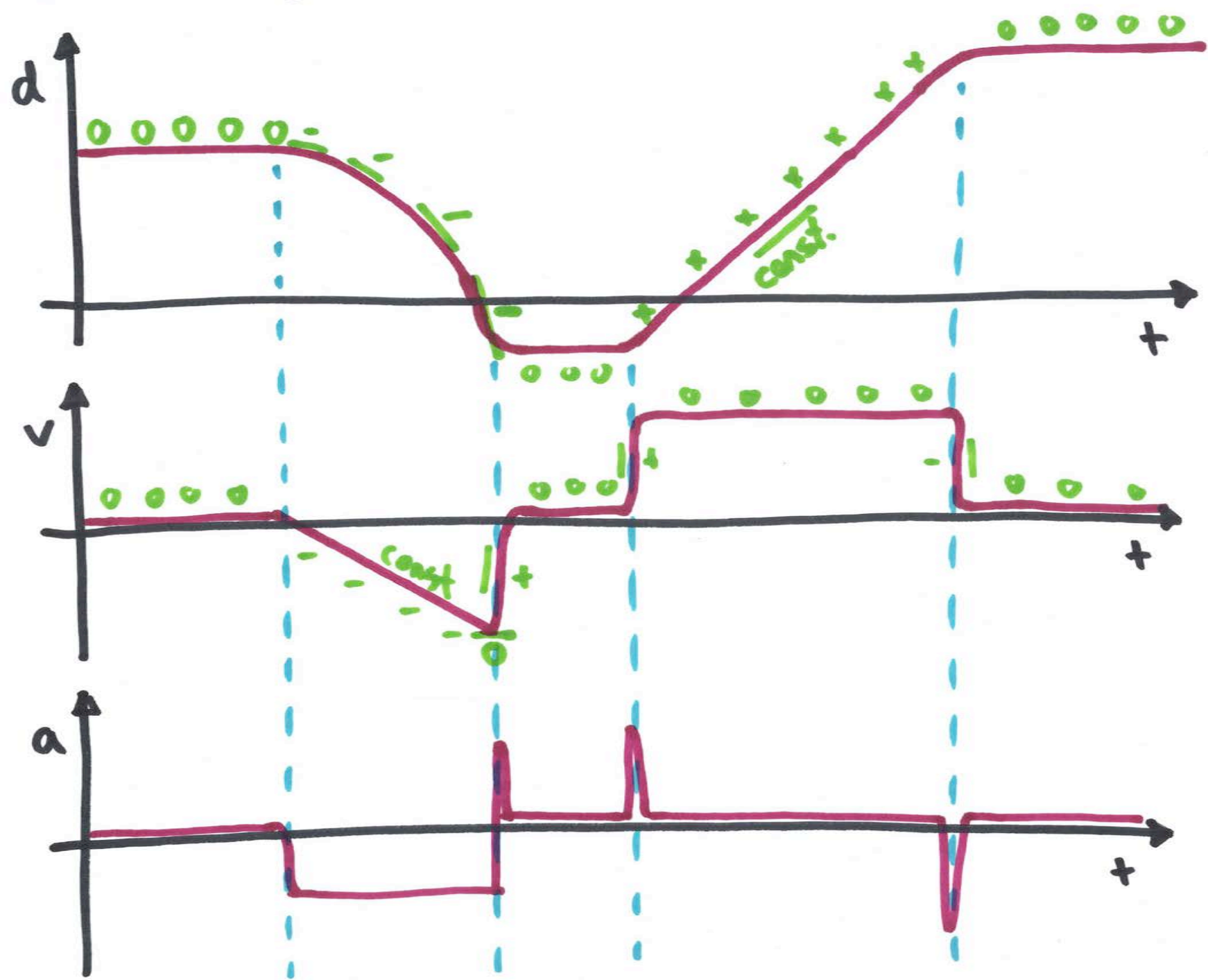
$$y = ax^2 + bx + c \quad \text{2nd order poly.}$$

★ 0th order $\rightarrow y = 0$

1st order $\rightarrow$ 0th order poly.

2nd order $\rightarrow$ 1st order poly.

d-v-a graph



Kinematics

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} \quad (1)$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad (2)$$

assume constant $\vec{a}$ we can solve (1) & (2):

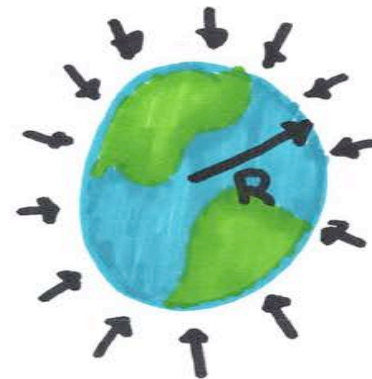
$$\begin{aligned} \vec{x}_f &= \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\ \vec{v}_f^2 &= \vec{v}_0^2 + 2 \vec{a} \Delta \vec{x} \\ \vec{v}_f &= \vec{v}_0 + \vec{a} t \end{aligned}$$

Annotations in the diagram:
- $(\vec{x}_f - \vec{x}_0)$ points to $\Delta \vec{x}$ in the second equation.
- "initial" points to $\vec{v}_0$ in the second equation.
- "final" points to $\vec{v}_f$ in the second equation.
- $(t_f - t_0)$ points to t in the third equation.
- $t_0 = 0$ points to t_0 in the third equation.

constant $\vec{a}$ kinematic equations

where can we use them?

1. in free space ($\vec{a} = 0$)
2. on the earth surface

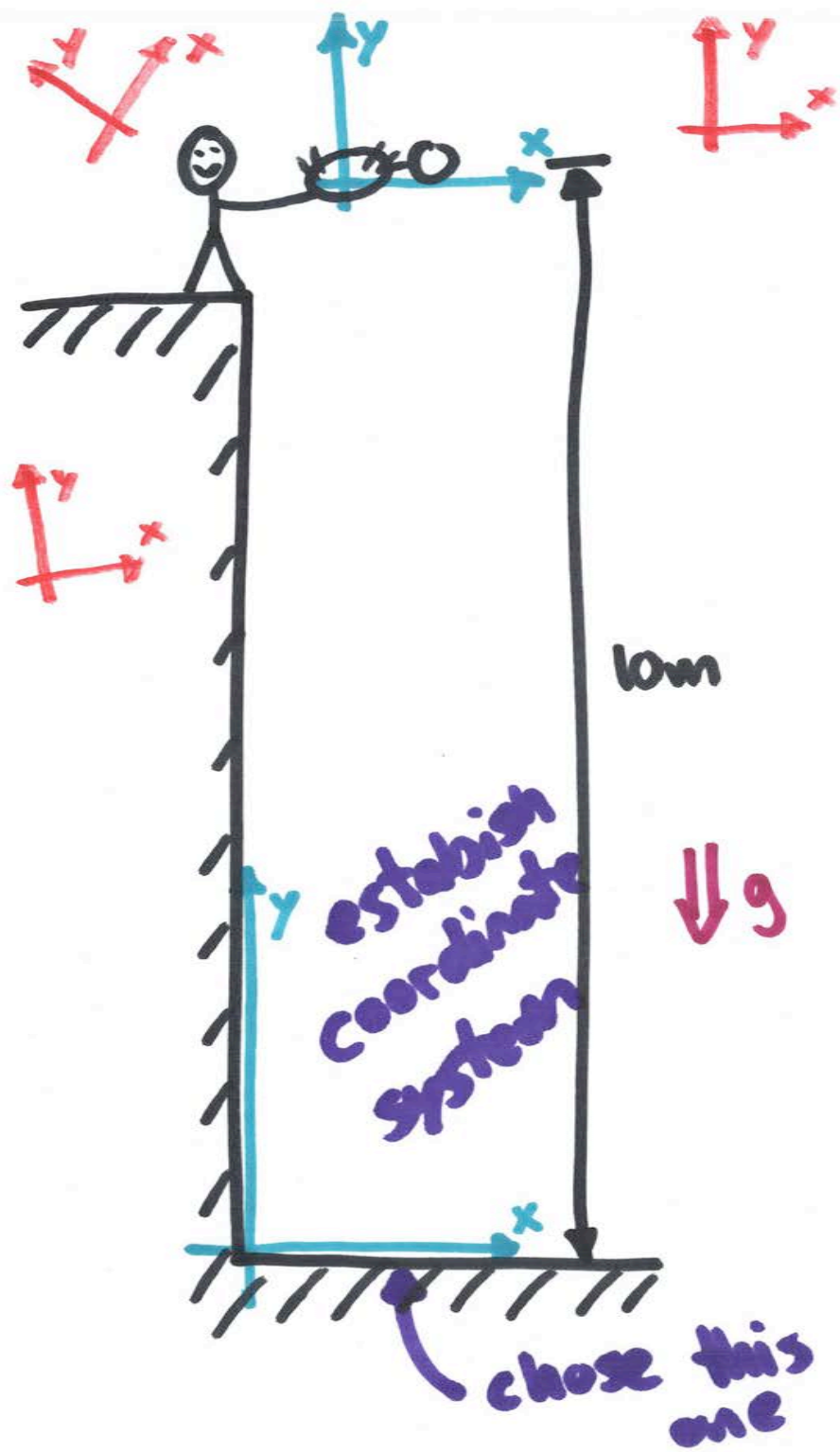


$\vec{a} = \vec{g} = 9.81 \text{ m/s}^2$
downward.

Kinematics equations expanded.

$$\begin{aligned}\vec{X}_f &= x_f \hat{x} + y_f \hat{y} + z_f \hat{z} \\ &= \left(x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \right) \hat{x} \\ &\quad + \left(y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \right) \hat{y} \\ &\quad + \left(z_0 + v_{0z}t + \frac{1}{2}a_z t^2 \right) \hat{z}\end{aligned}$$

$$\begin{aligned}\vec{V}_f^2 &= v_{xf}^2 \hat{x} + v_{yf}^2 \hat{y} + v_{zf}^2 \hat{z} \\ &= \left(v_{x0}^2 + 2a_x \Delta x \right) \hat{x} \\ &\quad + \left(v_{y0}^2 + 2a_y \Delta y \right) \hat{y} \\ &\quad + \left(v_{z0}^2 + 2a_z \Delta z \right) \hat{z}\end{aligned}$$



$$y_0 = 10 \text{ m}$$

$$y_f = 0 \text{ m}$$

$$v_{0y} = 0 \text{ m/s}$$

$$v_{fy} =$$

$$a_y = -9.81 \text{ m/s}^2$$

$$t =$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$t^2 = -\frac{2y_0}{a_y}, \quad t = \pm \sqrt{-\frac{2y_0}{a_y}}$$

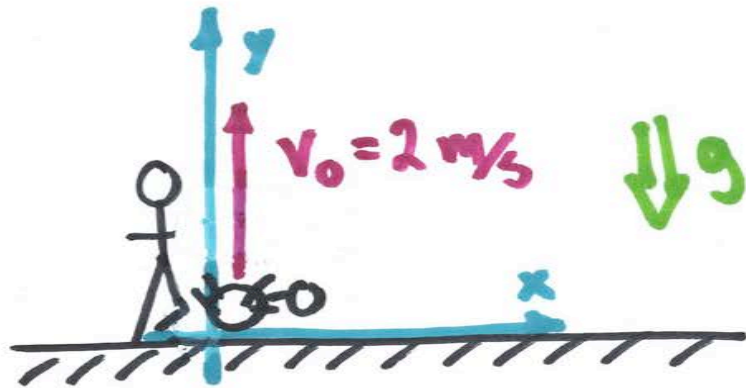
$$t = \sqrt{\frac{-2(10)}{-9.81}} = 1.4 \text{ s.}$$

$$t = 1.4 \text{ s}$$

write down variables.

find eqn. that will solve.

determine time to hit the ground



$$y_0 = 0 \text{ m}$$

$$y_f = 0 \text{ m}$$

$$v_{0y} = 2 \text{ m/s}$$

$$v_{fy} = -2 \text{ m/s}$$

$$a_y = -9.81 \text{ m/s}^2$$

$$t =$$

- y-dir

$$v_{fy}^2 = v_{0y}^2 + 2a_y \Delta y$$

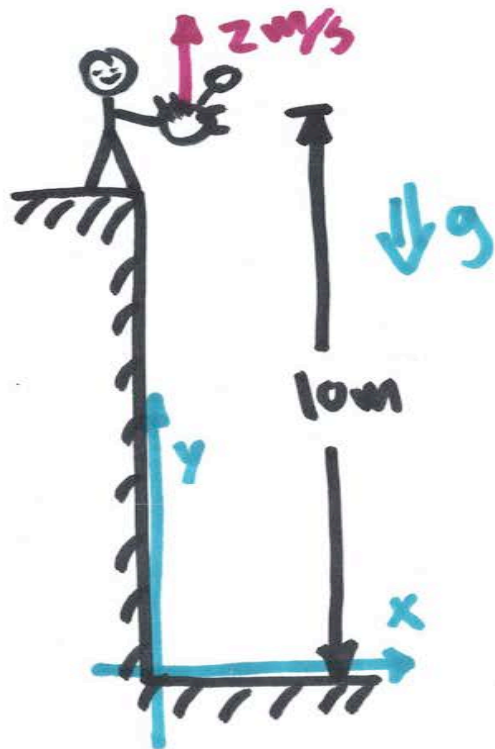
$$\begin{aligned} v_{fy} &= \pm \sqrt{v_{0y}^2 + 2a_y(y_f - y_0)} \\ &= \pm \sqrt{(2)^2 + 2(-9.81)(0-0)} \\ &= \pm 2 = -2 \text{ m/s} \end{aligned}$$

$$v_{fy} = v_{0y} + a_y t$$

$$t = \frac{v_{fy} - v_{0y}}{a_y} = \frac{(-2) - (2)}{-9.81}$$

$$t = 0.41 \text{ s}$$

determine time to hit ground and final velocity



$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}\Delta\vec{x}$$

$$v_{fy}^2 = v_{oy}^2 + 2a_y\Delta y$$

$$v_{fy} = \pm \sqrt{v_{oy}^2 + 2a_y(y_f - y_0)} \\ = \pm \sqrt{2^2 + 2(-9.81)(0 - 10)} = \pm 14.1 \text{ m/s} = -14.1 \text{ m/s}$$

$$v_{fy} = -14.1 \text{ m/s}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{fy} = v_{oy} + a_y t$$

$$t = \frac{v_{fy} - v_{oy}}{a_y} = \frac{-14.1 - 2}{-9.81} = 1.65 \text{ s}$$

$$t = 1.65 \text{ s}$$

$$y_0 = 10 \text{ m}$$

$$y_f = 0 \text{ m}$$

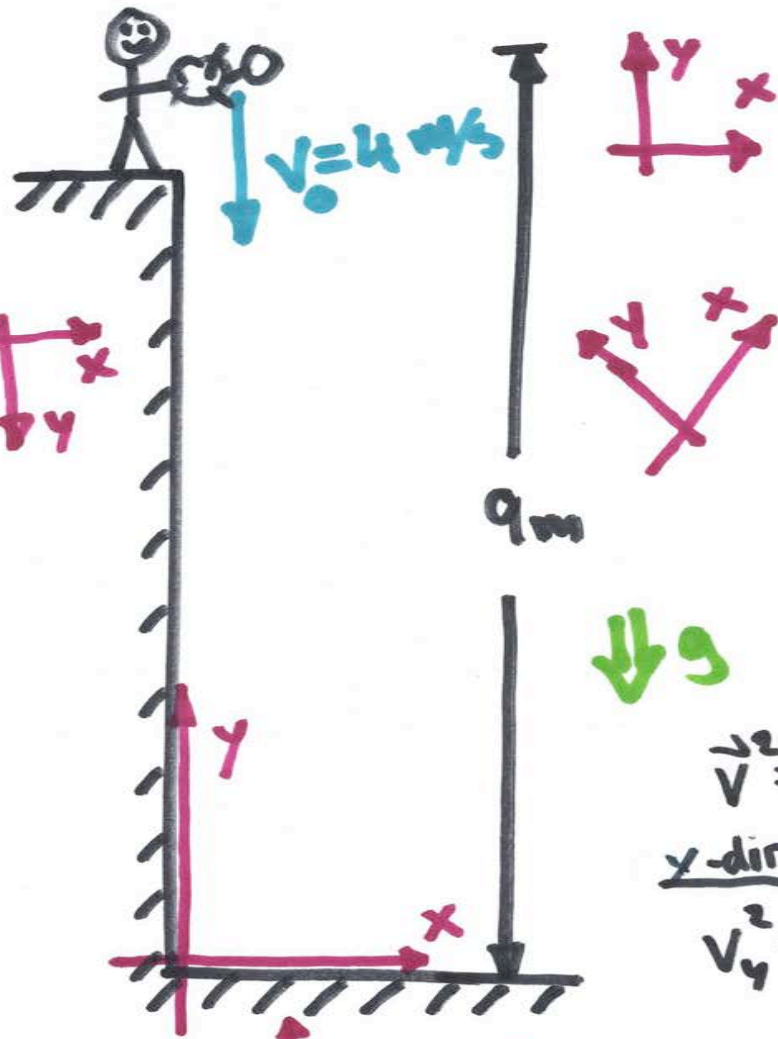
$$v_{oy} = 2 \text{ m/s}$$

$$v_{fy} = -14.1 \text{ m/s}$$

$$a_y = -9.81$$

$$t =$$

How long does it take to hit the ground?



establish
a coordinate
system.

$$y_0 = 9 \text{ m}$$

$$y_f = 0 \text{ m}$$

$$v_{0y} = -4 \text{ m/s}$$

$$v_{fy} =$$

$$a_y = -9.81 \text{ m/s}^2$$

$$t =$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}\Delta x$$

y-dir

$$v_y^2 = v_{0y}^2 + 2a_y\Delta y$$

$$v_y = \pm \sqrt{v_{0y}^2 + 2a_y(y_f - y_0)}$$

$$= \pm \sqrt{(-4)^2 + 2(-9.81)(0-9)}$$

$$= \pm 13.9 = -13.9 \text{ m/s}$$

set variables

find equations
to solve

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

y-dir

$$v_y = v_{0y} + a_y t$$

$$t = \frac{v_y - v_{0y}}{a_y}$$

$$t = \frac{-13.9 - (-4)}{-9.81}$$

$$t = 1.0 \text{ s}$$

Why $\pm$ on $\vec{v}^2 = \vec{v}_0^2 + 2\vec{a} \cdot \vec{\Delta x}$?

example:

$$x^2 = 4$$

solve for x :

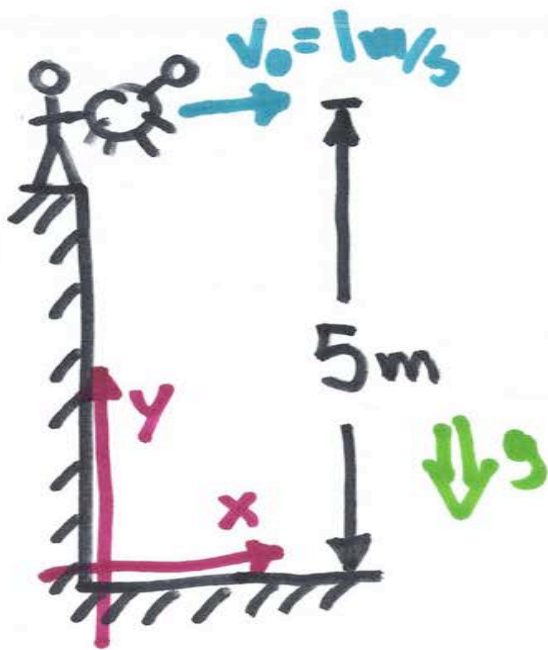
$$x = 2$$

but x can also

$$x = -2$$

$$\rightarrow \sqrt{x^2} = \pm \sqrt{4}$$

$$x = \pm 2$$



Determine time to hit ground?

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

-y-dir

$$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$t = \pm \sqrt{-\frac{2y_0}{a_y}}$$

$$= \pm \sqrt{-\frac{2(5)}{-9.81}}$$

$$t = \pm 1.0$$

$$t = 1.0 \text{ s}$$

$$\vec{v} = \vec{v}_0 + a t$$

-y-dir

$$v_y = v_{0y} + a_y t$$

$$= 0 + (-9.81)(1.0)$$

$$v_y = -9.81 \text{ m/s}$$

x-dir

$$x_0 = 0$$

$$x_f =$$

$$v_{0x} = 1$$

$$v_{fx} =$$

$$a_x = 0$$

-y-dir

$$y_0 = 5$$

$$y_f = 0$$

$$v_{0y} = 0$$

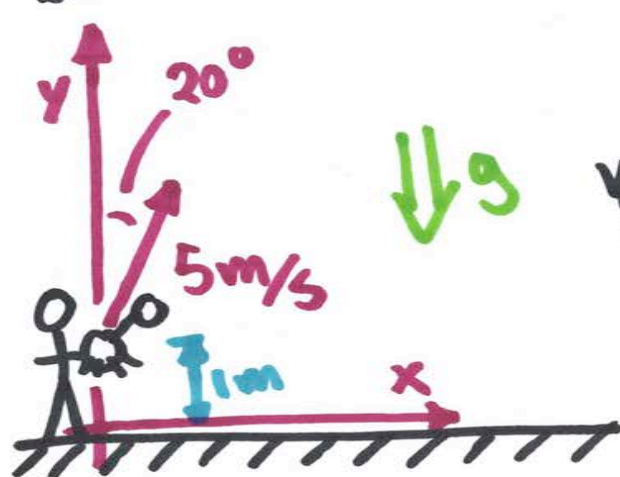
$$v_{fy} =$$

$$a_y = -9.81$$

time

$$t =$$

Determine time to hit ground?



$$v_{x0} = 5 \sin(20)$$

$$= 1.71$$

$$v_{y0} = 5 \cos(20)$$

$$= 4.70$$

$$x_0 = 0 \quad y_0 = 1$$

$$x_f = \quad y_f = 0$$

$$v_{0x} = 1.71 \quad v_{0y} = 4.70$$

$$v_{fx} = \quad v_{fy} =$$

$$a_x = 0 \quad a_y = -9.81$$

y-dir

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$v_y = \pm \sqrt{v_{0y}^2 + 2a_y(y_f - y_0)}$$

$$= \pm \sqrt{4.7^2 + 2(-9.81)(0-1)}$$

$$= \pm 6.46 = -6.46 \text{ m/s}$$

$$v_y = v_{0y} + a_y t$$

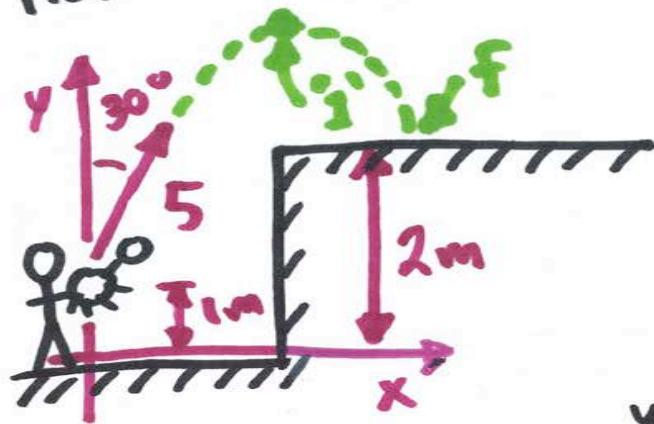
$$t = \frac{v_y - v_{0y}}{a_y}$$

$$= \frac{-6.46 - (4.7)}{-9.81}$$

$$t = 1.14 \text{ s}$$

$$t =$$

How far does the body go?



$$v_{0x} = 5 \sin(30)$$

$$= 2.5$$

$$v_{0y} = 5 \cos(30)$$

$$= 4.3$$

$$\begin{aligned} x_0 &= 0 & y_0 &= 1 \\ x_f &= & y_f &= 2 \\ v_{0x} &= 2.5 & v_{0y} &= 4.3 \\ v_{fx} &= & v_{fy} &= \\ a_x &= 0 & a_y &= -9.81 \\ t &= \end{aligned}$$

y-dir

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$v_y = \pm \sqrt{v_{0y}^2 + 2a_y(y_f - y_0)}$$

$$= \pm \sqrt{4.3^2 + 2(-9.81)(2-1)}$$

$$= \pm \sqrt{-1.13}$$

it doesn't
make it on the
ledge.

$$y_0 = 1$$

$$y_1 =$$

$$v_{0y} = 4.3$$

$$v_{1y} = 0$$

$$a_y = -9.81$$

y-dir

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$\Delta y = \frac{-v_{0y}^2}{2a_y}$$

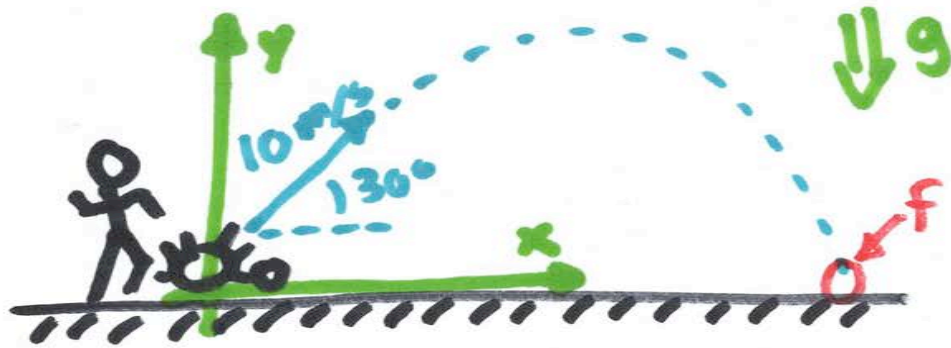
$$= y_1 - y_0$$

$$y_1 = -\frac{v_{0y}^2}{2a_y} + y_0$$

$$= -\frac{4.3^2}{2(-9.81)} + 1$$

$$y = 1.94 \text{ m}$$

How far does the baby go



$$x_0 = 0 \quad y_0 = 0$$

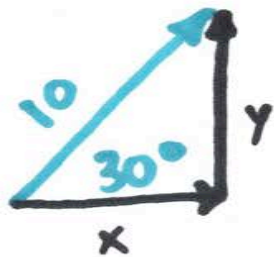
$$x_f = \quad y_f = 0$$

$$v_{0x} = 8.66 = v_0 \cos(\theta) \quad v_{0y} = 5 = v_0 \sin(\theta)$$

$$v_{fx} = \quad v_{fy} =$$

$$a_x = 0 \quad a_y = -9.81$$

$$t =$$



$$x = 10 \cos(30)$$

$$= 8.66$$

$$y = 10 \sin(30)$$

$$= 5$$

-y-dir

$$v_{fy}^2 = v_{0y}^2 + 2a_y \Delta y$$

$$v_{fy} = \pm \sqrt{v_{0y}^2 + 2a_y(y_f - y_0)}$$

$$= \pm \sqrt{5^2 + 2(-9.81)(0 - 0)}$$

$$= \pm 5 = -5 \text{ m/s}$$

-y-dir

$$v_{fy} = v_{0y} + a_y t$$

$$t = \frac{v_{fy} - v_{0y}}{a_y}$$

$$t = \frac{-5 - 5}{-9.81}$$

$$t = 1.02 \text{ s}$$

x-dir

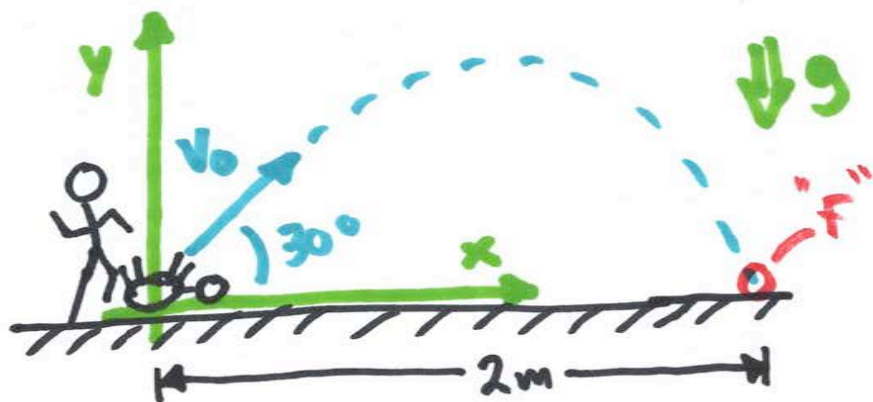
$$x_f = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$x_f = v_{0x} t$$

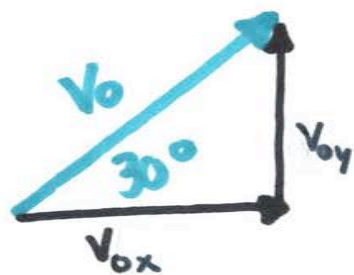
$$= (8.66)(1.02)$$

$$x_f = 8.83 \text{ m}$$

How fast did the baby leave (v_0)?



$$\begin{aligned} x_0 &= 0 & y_0 &= 0 \\ x_f &= 2 & y_f &= 0 \\ v_{0x} &= v_0 \cos(30) & v_{0y} &= v_0 \sin(30) \\ v_{fx} &= & v_{fy} &= -v_0 \sin(30) \\ a_x &= 0 & a_y &= -9.81 \\ t &= \end{aligned}$$



$$v_{0x} = v_0 \cos(30)$$

$$v_{0y} = v_0 \sin(30)$$

leave (v_0)?

x-dir

$$x_f = x_0 + v_{0x}t$$

$$2 = 0 + v_0 \cos(30) +$$

$$\boxed{2 = v_0 t \cos(30)} \quad (1)$$

y-dir

$$v_{fy}^2 = v_{0y}^2 + 2a_y(y_f - y_0)$$

$$v_{fy} = \pm v_{0y} = -v_{0y}$$

y-dir

$$v_{fy} = v_{0y} + a_y t$$

$$-v_0 \sin(30) = v_0 \sin(30) + (-9.81)(t)$$

$$\boxed{9.81t = 2v_0 \sin(30)} \quad (2)$$

Solve for v_0 in (1)

$$2 = v_0 t \cos(30)$$

$$v_0 = \frac{2}{t \cos(30)} \quad (1a)$$

sub (1a) into (2) and solve

$$9.81 t = 2 \left[\frac{2}{t \cos(30)} \right] \sin(30)$$

$$9.81 t^2 \cos(30) = 4 \sin(30)$$

$$t^2 = \frac{4 \sin(30)}{9.81 \cos(30)}$$

$$t = \pm \sqrt{\frac{4 \sin(30)}{9.81 \cos(30)}}$$

$$t = \pm 0.49 \text{ s}$$

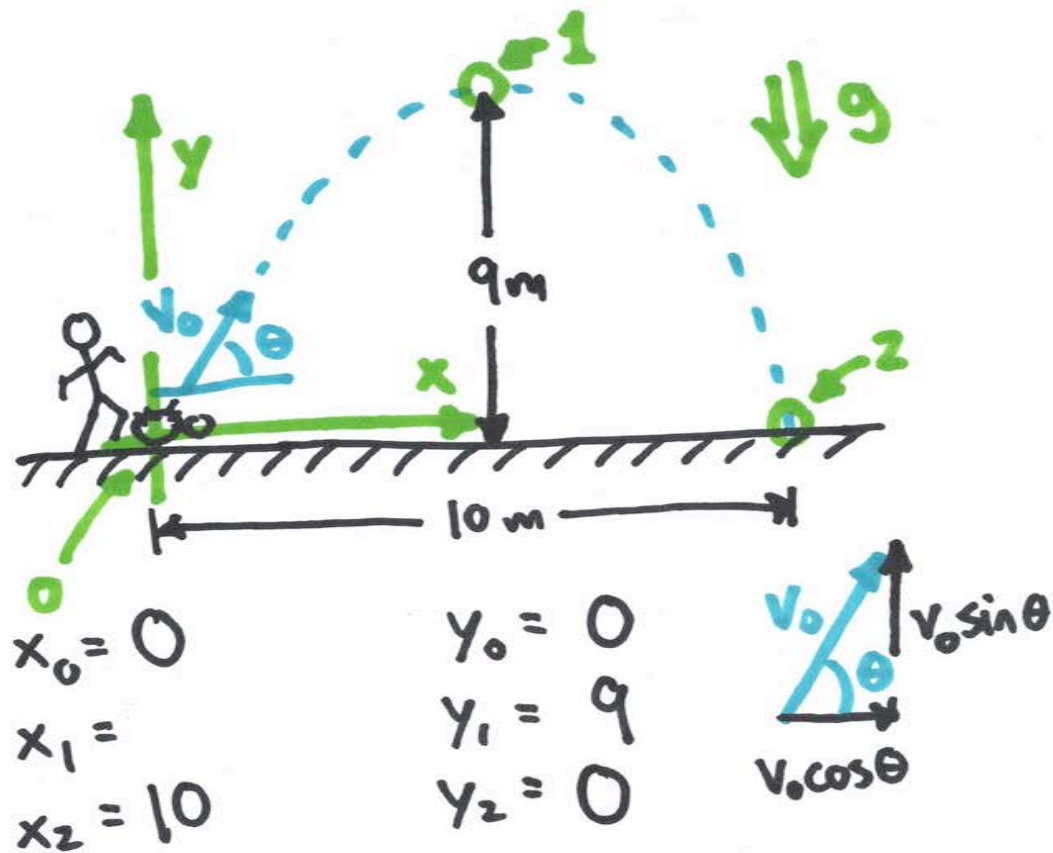
$$\boxed{t = 0.49 \text{ s}} \quad (2a)$$

sub (2a) back into (1a)

$$v_0 = \frac{2}{[0.49] \cos(30)}$$

$$\boxed{v_0 = 4.76 \text{ m/s}}$$

Determine the initial velocity and angle of the kick?



$$v_{0x} = v_0 \cos \theta \quad v_{0y} = v_0 \sin \theta$$

$$v_{1x} = v_0 \cos \theta \quad v_{1y} = 0$$

$$v_{2x} = v_0 \cos \theta \quad v_{2y} =$$

$$a_x = 0$$

$$a_y = -9.81$$

$$+ =$$