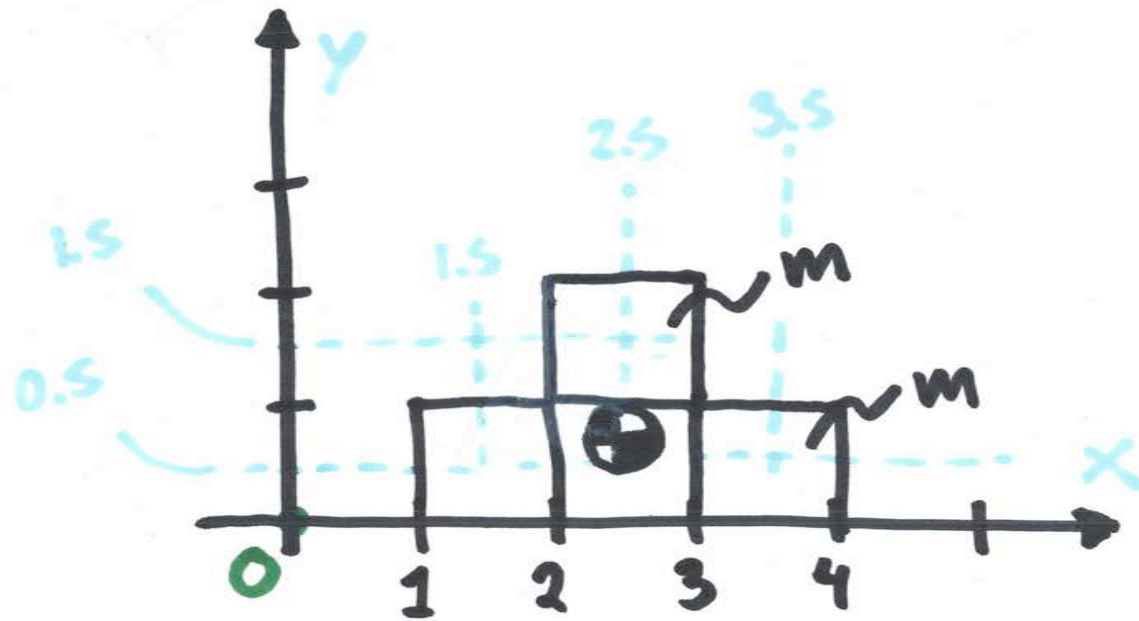


Centre of Mass



$$\sum M_o = I_o \alpha_o = m_1 x_1 + m_2 x_2 + \dots$$

$$= M x_{cg}$$

$$\text{where } M = \sum m_i$$

Centre of Mass equation

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$m_1 x_1 + m_2 x_2 + \dots = M x_{cg}$$

$$\frac{m_1 x_1 + m_2 x_2 + \dots}{\sum m_i} = x_{cg}$$

for example:

$$x_{cg} = \frac{\sum m x_i}{\sum m} = \frac{m(1.5) + m(2.5) + m(3.5) + m(2.5)}{4m}$$

$$= \frac{m(1.5 + 2.5 + 3.5 + 2.5)}{4m}$$

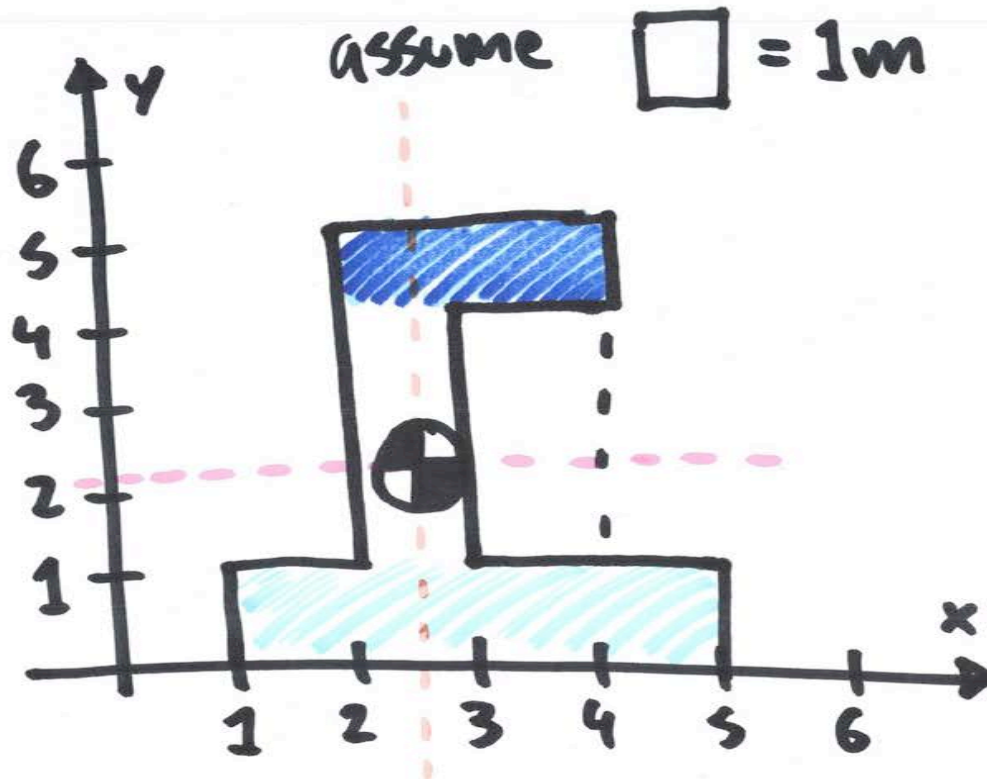
$$= 2.5$$

$$\begin{aligned}
 y_{cg} &= \frac{\sum m_i y_i}{\sum m_i} = \frac{m(0.5) + m(0.5) + m(0.5) + m(1.5)}{4m} \\
 &= \frac{m(0.5 \cdot 3 + 1.5)}{4m} \\
 &= \frac{3}{4} = 0.75
 \end{aligned}$$

$$\vec{x}_{cg} = (2.5, 0.75)$$

find centre of mass

$$\vec{x}_{cg} = \frac{\sum m \vec{x}_i}{\sum m}$$

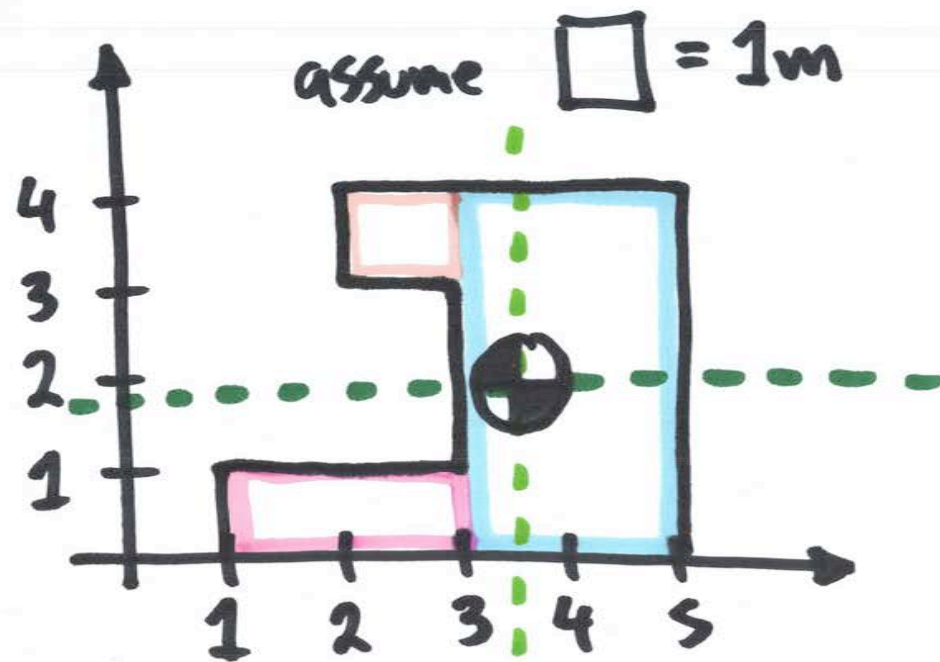


$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(4m)(0.5) + (3m)(2.5) + (2m)(4.5)}{9m}$$

$$= \frac{2 + 7.5 + 9}{9} = \underline{2.06}$$

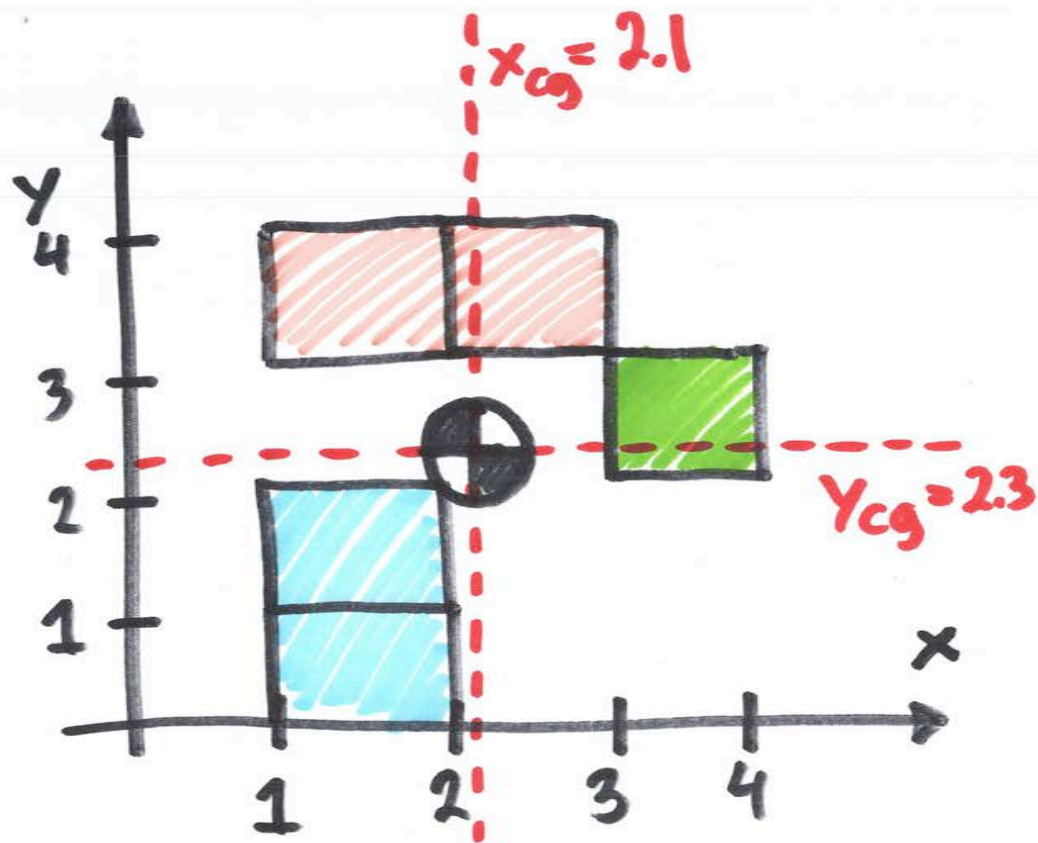
$$\begin{aligned} x_{cg} &= \frac{\sum m x_i}{\sum m} \\ &= \frac{(4m)(3) + (3m)(2.5) + (2m)(3)}{9m} \\ &= \frac{(12) + (7.5) + (6)}{9} \\ &= \underline{2.56} \end{aligned}$$

find centre of mass



$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(2\text{m})(2) + (8\text{m})(4) + (1\text{m})(2.5)}{11\text{m}} = \underline{3.5}$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2\text{m})(0.5) + (8\text{m})(2) + (1\text{m})(3.5)}{11\text{m}} = \underline{1.86}$$



$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{(2m)(1.5) + (2m)(2) + (1m)(3.5)}{5m}$$

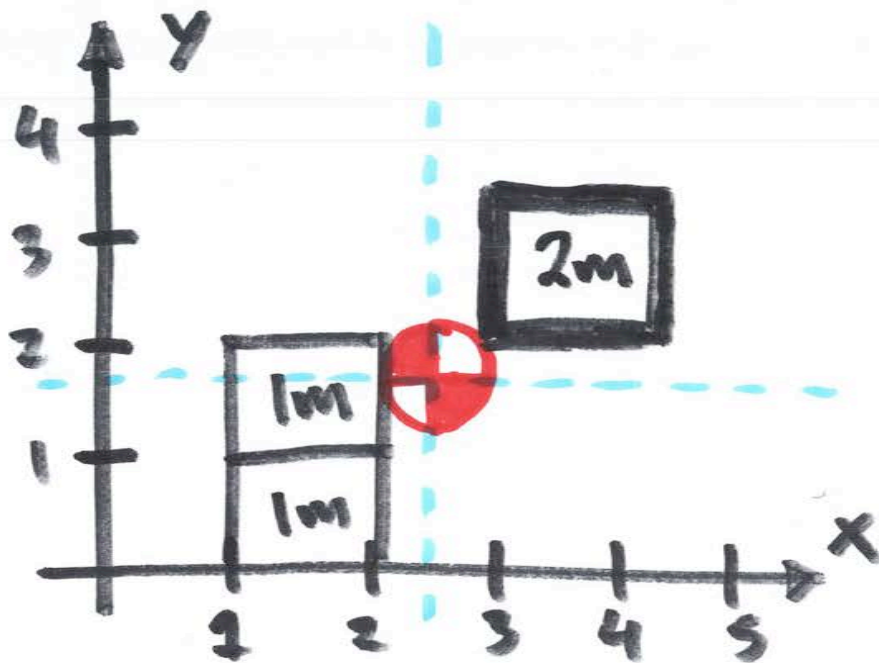
$$= \frac{(2)(1.5) + (2)(2) + 3.5}{5}$$

$$= 2.1$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i}$$

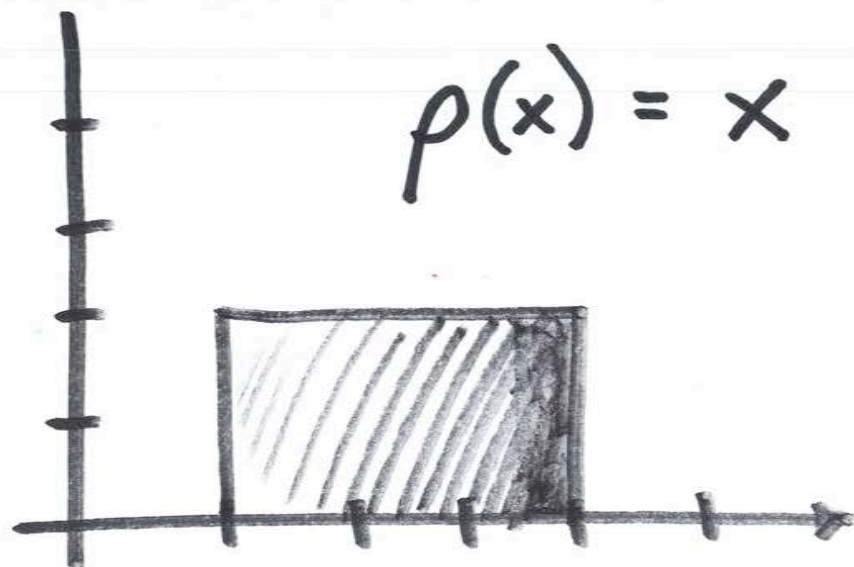
$$= \frac{(2m)(1) + (2m)(3.5) + (1m)(2.5)}{5m}$$

$$= 2.3$$



$$x_{cg} = \frac{(2m)(1.5) + (2m)(3.5)}{4m} = 2.5$$

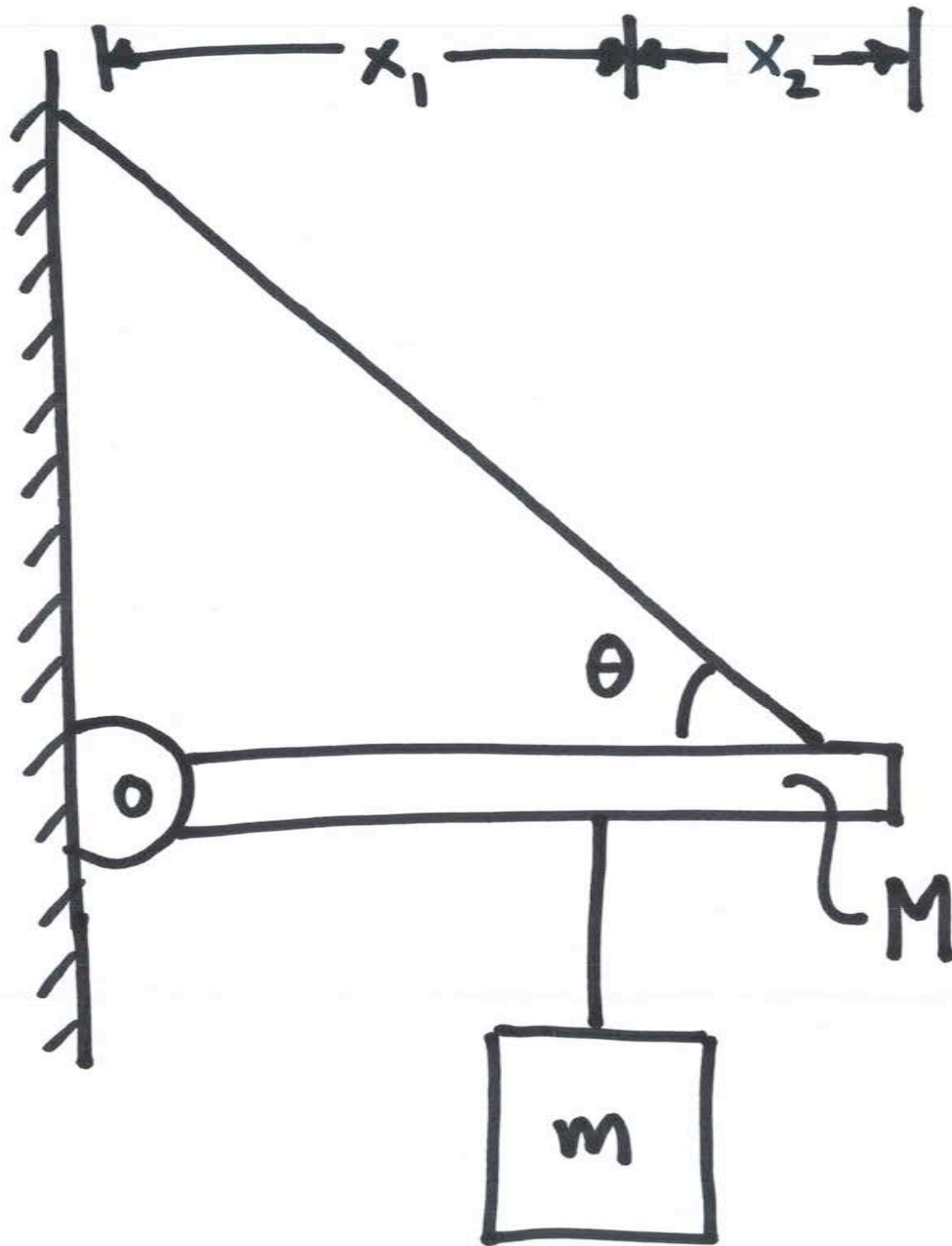
$$y_{cg} = \frac{(2m)(1) + (2m)(2.5)}{4m} = 1.75$$



$$x_{\text{cg}} = \frac{\int m x \, dx}{\int m \, dx}$$

math 20
integral

Solve for reaction force



$$x_1 = 2 \text{ m}$$

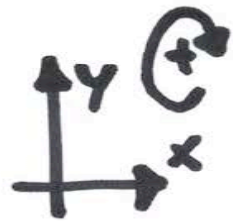
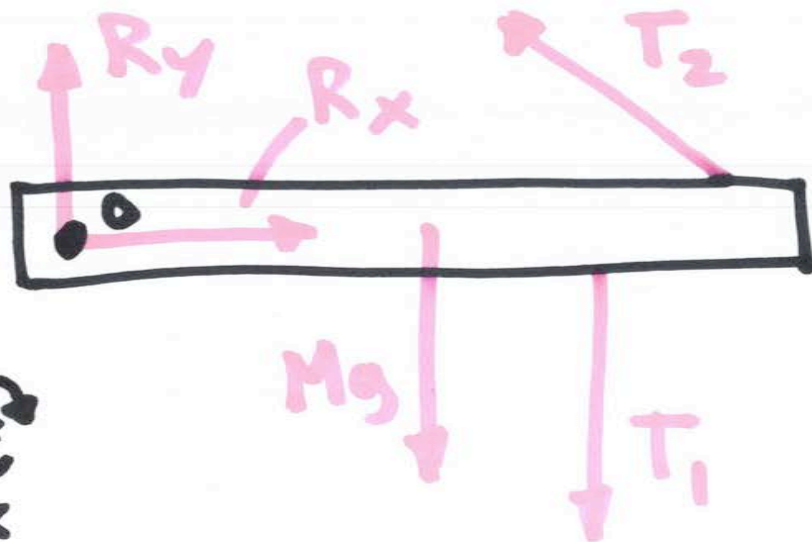
$$x_2 = 1 \text{ m}$$

$$\theta = 40^\circ$$

$$M = 10 \text{ kg}$$

$$m = 20 \text{ kg}$$

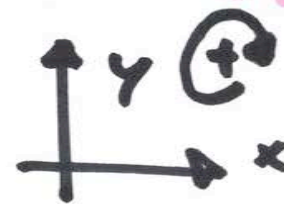
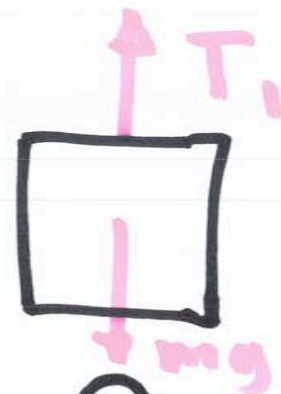
FBD:



$$\sum F_x = \cancel{m a_x} = R_x - T_2 \cos \theta$$

$$\sum F_y = \cancel{m a_y} = R_y - Mg - T_1 + T_2 \sin \theta$$

$$\sum M_o = \cancel{I \alpha_o} = (Mg)(1.5) + (T_1)(2) - (T_2 \sin \theta)(3)$$



$$\sum F_y = \cancel{m a_y} = T_1 - mg$$

$$T_1 = mg$$

solve (3) for T_2 :

$$T_2 = \frac{(1.5)Mg + 2[mg]}{3 \sin \theta} = \frac{(1.5)(10)(9.81) + (2)(20)(9.81)}{3 \sin(40)^\circ}$$

$$= 279.8 \text{ N}$$

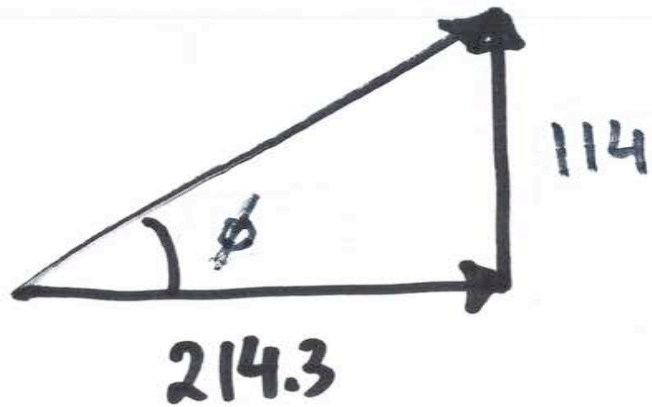
solve (1) & (2) for R_x & R_y :

$$R_x = T_2 \cos \theta = (279.8) \cos(40) = 214.3 \text{ N}$$

$$R_y = Mg + [mg] = (10)(9.81) + \overset{-T_2 \sin \theta}{(20)(9.81)} = 114 \text{ N}$$

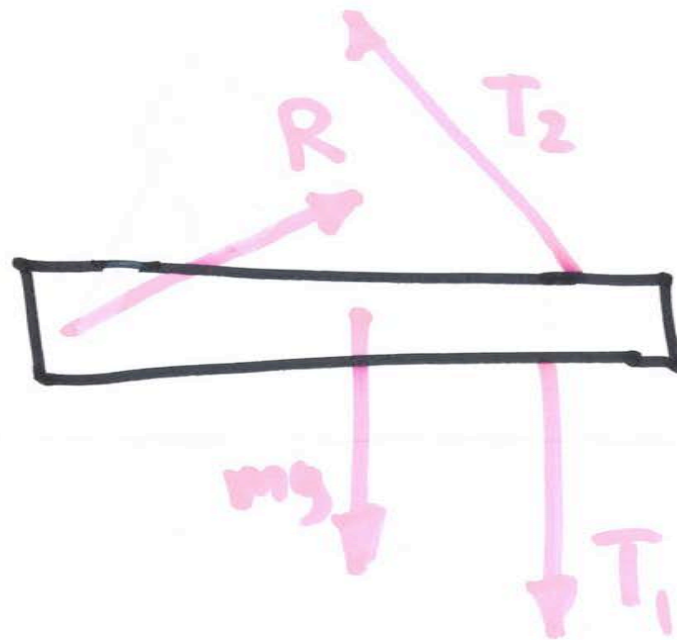
$$R = \sqrt{R_x^2 + R_y^2} = 243 \text{ N}$$

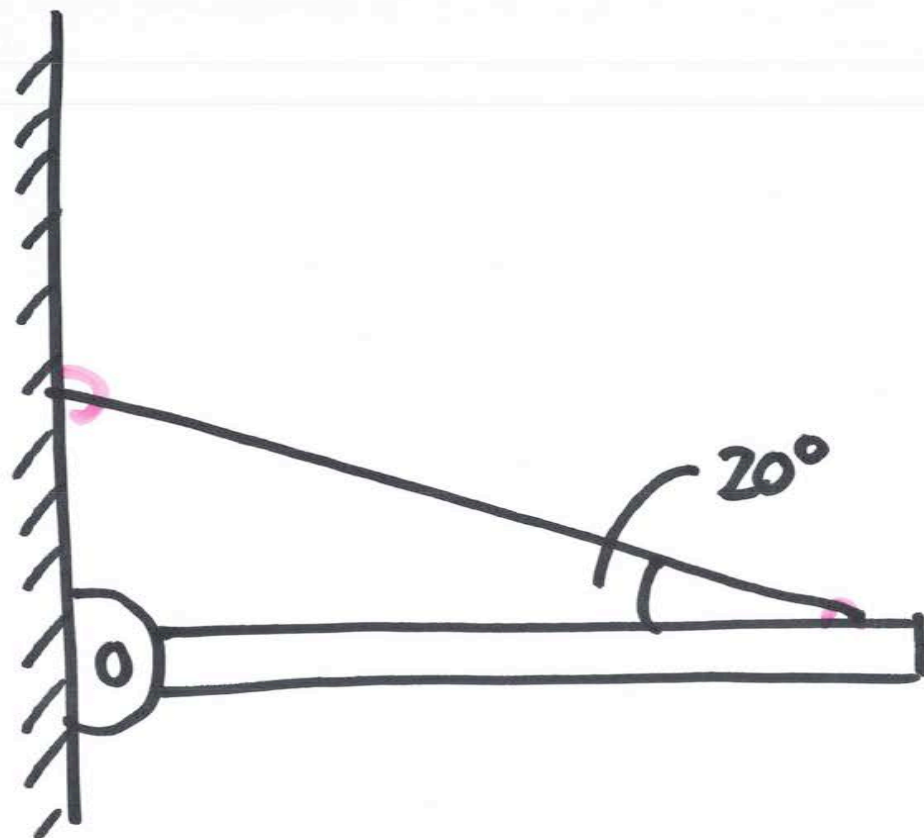
Direction:



$$\phi = \arctan\left(\frac{114}{214.3}\right)$$

$$\phi = 28^\circ$$

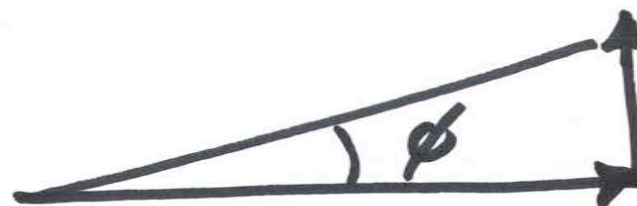




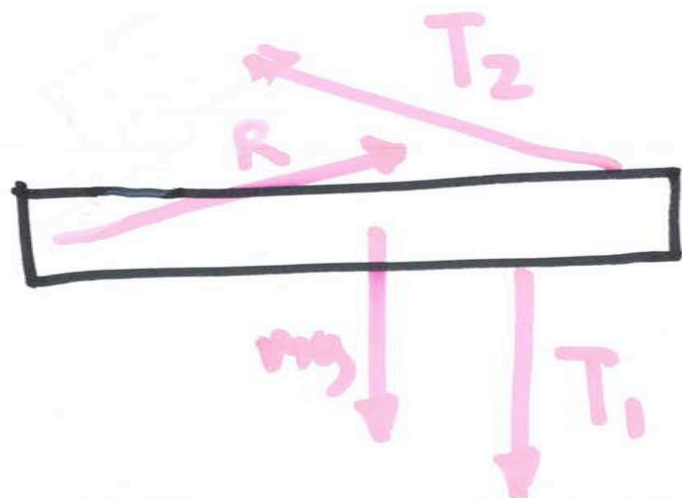
$$T_2 = \frac{(1.5Mg) + 2mg}{3\sin\theta} = 526 \text{ N}$$

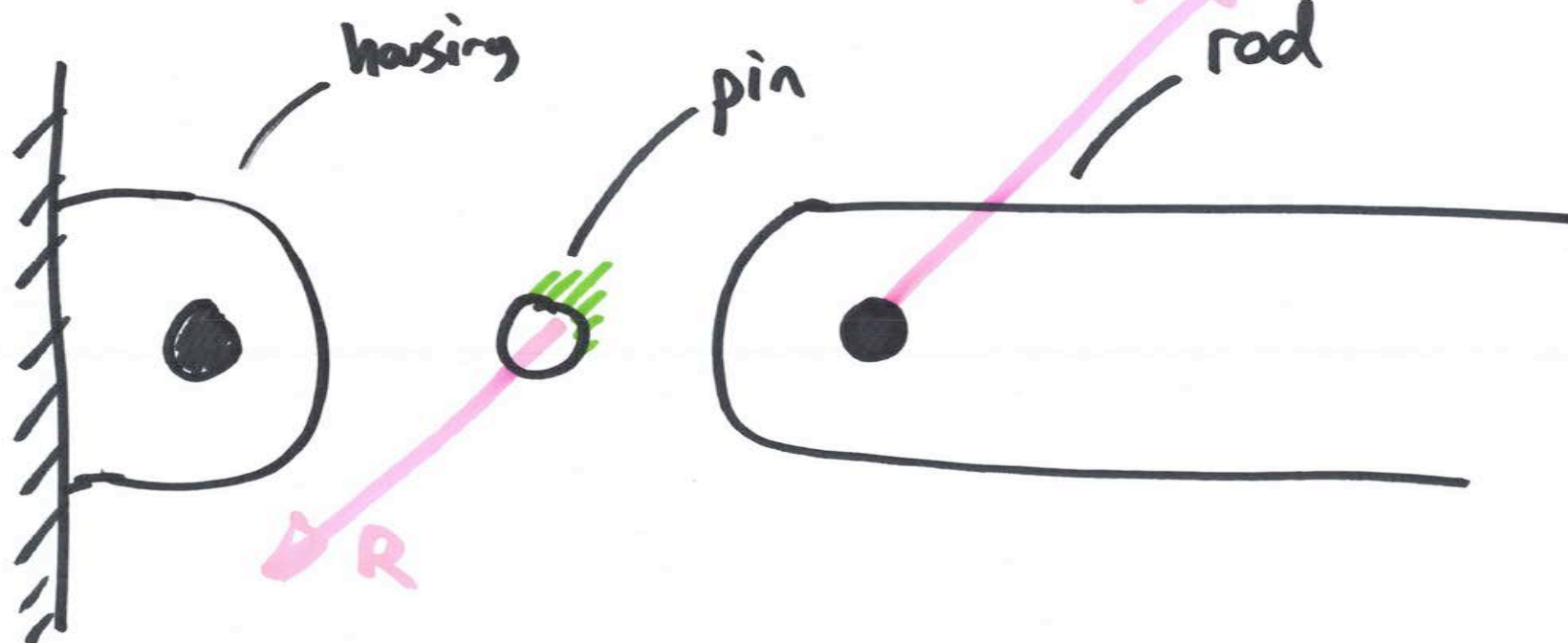
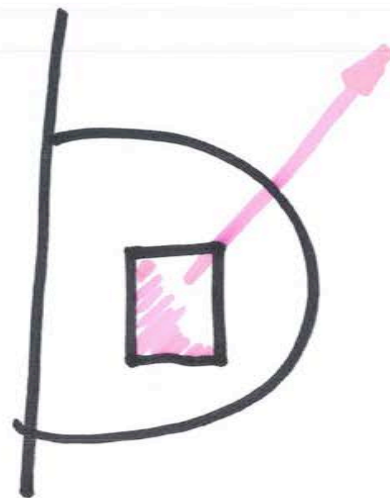
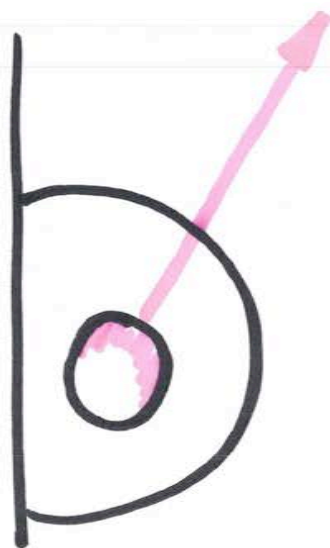
$$R_x = 494 \text{ N}$$

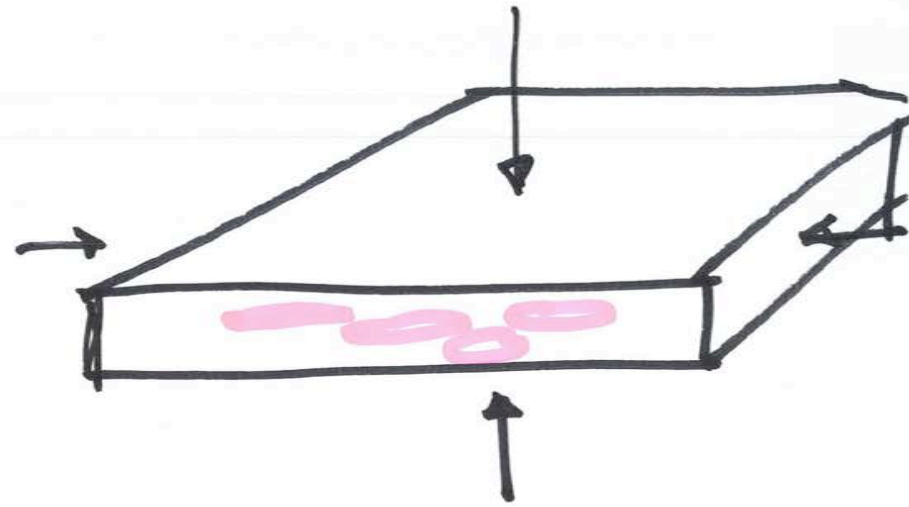
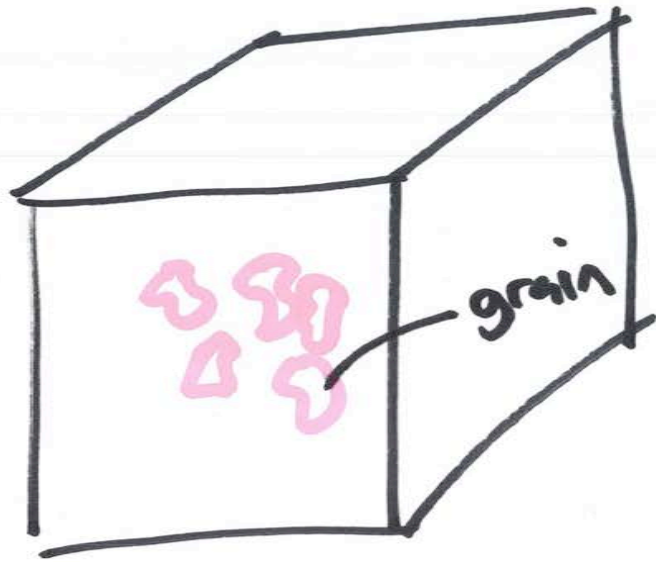
$$R_y = 114 \text{ N}$$



$$\phi = \arctan\left(\frac{R_y}{R_x}\right) = 13^\circ$$

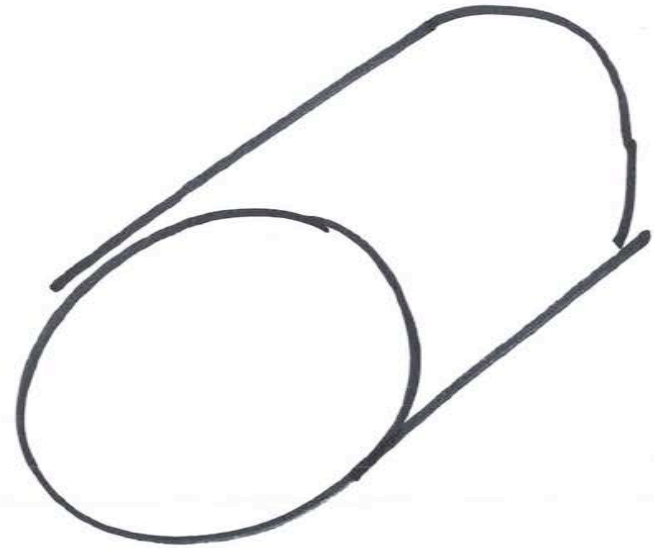




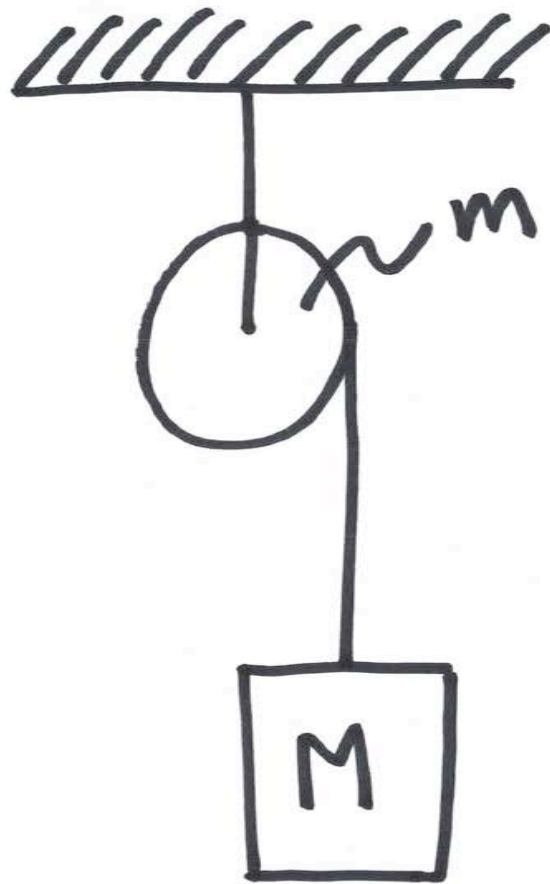


the science of

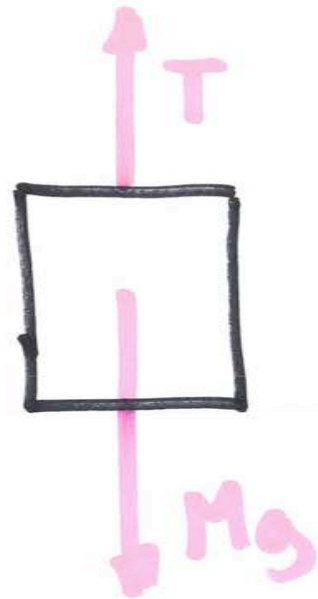
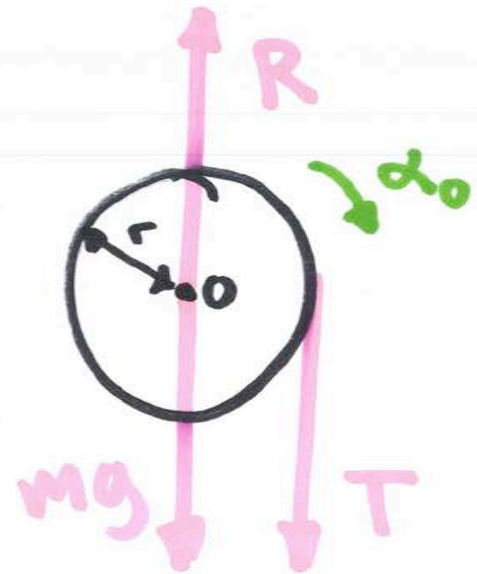
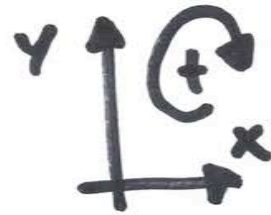
Metallurgy



Determine the acceleration



FBD:



$$\odot \quad \Sigma F_x = ma_x = 0$$

$$\Sigma F_y = ma_y = R - mg - T \quad (1)$$

$$\Sigma M_o = I_o \alpha_o = Tr \quad (2)$$

$$\square \quad \Sigma F_x = ma_x = 0$$

$$\Sigma F_y = ma_{y_H} = T - Mg \quad (3)$$

convert for α to a_y : (assume straightless rope)
massless rope

$$r \alpha_o = -a_{y_H} \quad (4)$$

sub (4) into (3):

$$M(-r\alpha_0) = T - Mg \quad (3a)$$

sub (3a) into (2):

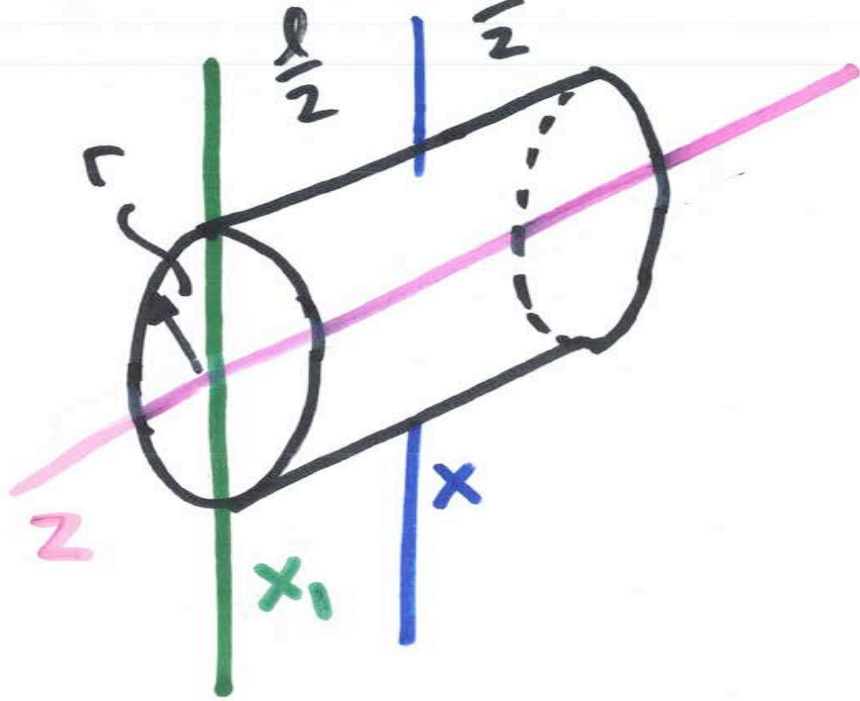
$$I_0\alpha_0 = [Mg - Mr\alpha_0]r$$

$$I_0\alpha_0 = Mgr - Mr^2\alpha_0$$

$$\alpha_0 = \frac{Mgr}{Mr^2 + I_0} = -\frac{a_y}{r}$$

What

I_0 ?



$$I_{xx} = \frac{1}{4} m r^2 + \frac{1}{12} m l^2$$

$$I_{x_1 x_1} = \frac{1}{4} m r^2 + \frac{1}{3} m l^2$$

$$I_{zz} = \frac{1}{2} m r^2$$

$$I_0 = I_{zz} = \frac{1}{2} m r^2$$

$$-\frac{a_y}{r} = \frac{Mg r}{Mr^2 + \frac{1}{2}mr^2}$$

$$a_y = - \frac{Mg r^2}{Mr^2 + \frac{1}{2}mr^2} = -g \frac{M}{M + \frac{1}{2}m}$$

$$a_y = -6.54 \text{ m/s}^2$$

$$M = 10 \text{ kg} ; m = 10 \text{ kg}$$

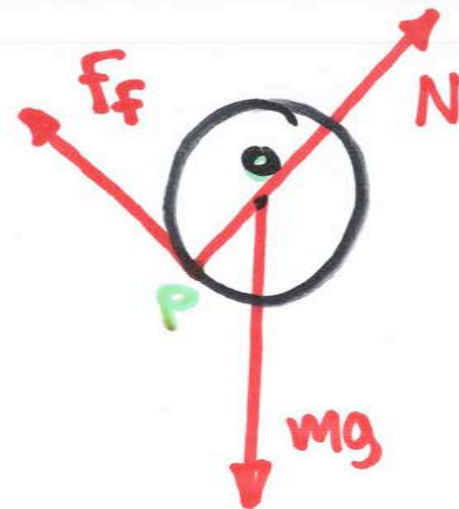
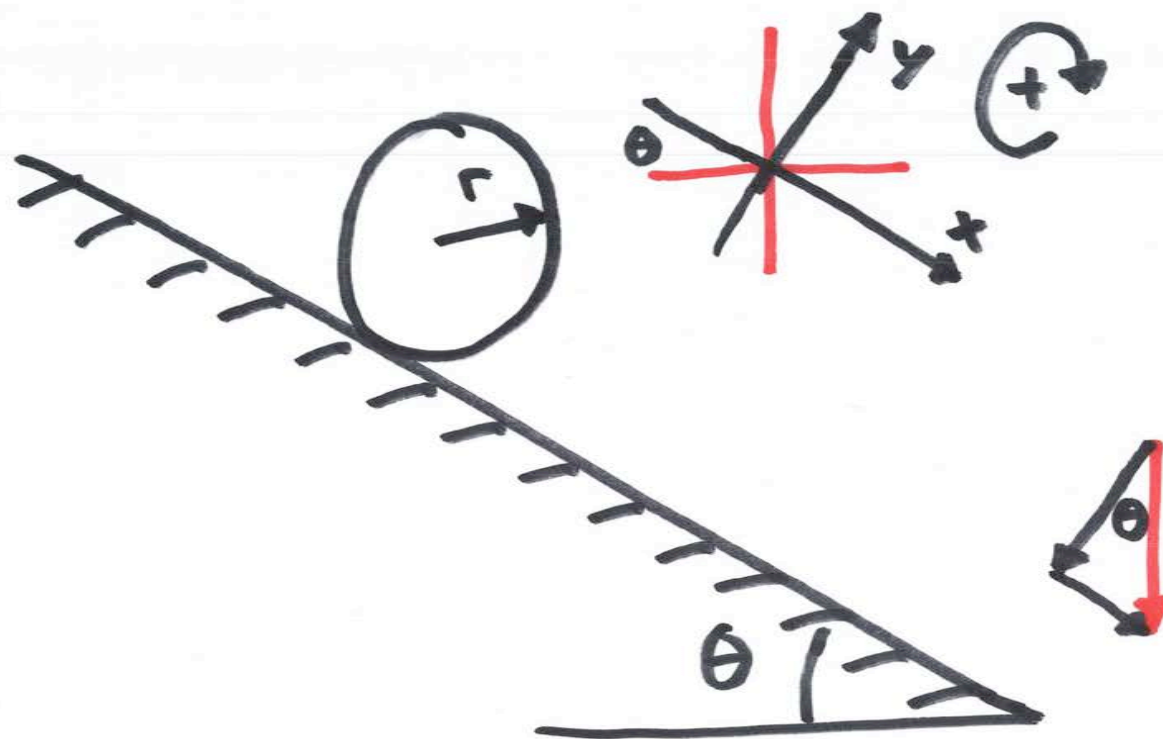
$$r = 1 \text{ m}$$

$$a_y = -9.81 \text{ m/s}^2$$

$$M = 10 \text{ kg} , m = 0 \text{ kg}$$

$$a_y = -1.6 \text{ m/s}^2$$

$$M = 10 \text{ kg} , m = 100 \text{ kg}$$



$$\Sigma F_x = ma_x = mg \sin \theta - f_f \quad (1)$$

$$\Sigma F_y = \cancel{ma_y^0} = N - mg \cos \theta \quad (2)$$

$$\Sigma M_o = I_o \alpha_o = f_f r \quad (3)$$

$$\Sigma M_p = I_p \alpha_p = mg \sin \theta \cdot r \quad (3)$$

Solve (2)

$$N = mg \cos \theta.$$

sub (1) into (3):

$$I_0 \alpha_0 = [mg \sin \theta - \mu_{\max}] r \quad (3a)$$

$$I_0 = \frac{1}{2} m r^2$$

$$\boxed{\alpha_0 = \frac{a_x}{r}}$$

sub into (3a)

$$\left[\frac{1}{2} m r^2 \right] \left[\frac{a_x}{r} \right] = [mg \sin \theta - \mu_{\max}] r$$

$$\frac{1}{2} m a_x = m g \sin \theta - m a_x$$

$$a_x = \frac{2}{3} g \sin \theta$$

sub into (1):

$$m \left[\frac{2}{3} g \sin \theta \right] = m g \sin \theta - F_f$$

$$F_f = \frac{1}{3} m g \sin \theta$$

max friction

$$F_{f_{\max}} = \mu_s N$$

$$= \mu_s m g \cos \theta$$

$$m = 100 \text{ kg}$$

$$\mu_s = 0.3$$

$$\theta = 30^\circ$$

$$r = 0.2 \text{ m}$$

$$F_f = \frac{1}{3} mg \sin \theta = 163.5 \text{ N}$$

$$F_{f_{\max}} = \mu_s mg \cos \theta = 254.9 \text{ N}$$

rolls without slipping.

$$a_x = \frac{2}{3} g \sin \theta = 3.27 \text{ m/s}^2$$

$$I_p = \frac{1}{2} m r^2 + m r^2$$

(parallel
axis
theorem)

sub into (3)

$$\left[\frac{1}{2} m r^2 + m r^2 \right] \alpha_p = m g \sin \theta r$$

$$\frac{3}{2} m r^2 \alpha_p = m g \sin \theta r$$

$$\boxed{r \alpha_p = \frac{2}{3} g \sin \theta}$$

Momentum:

$$\sum m \cdot v$$

$$\frac{dm}{dt} \cdot v + m \cdot \frac{dv}{dt}$$

$$\frac{dm}{dt} \cdot v + m a$$

$$\sum F = ma$$

