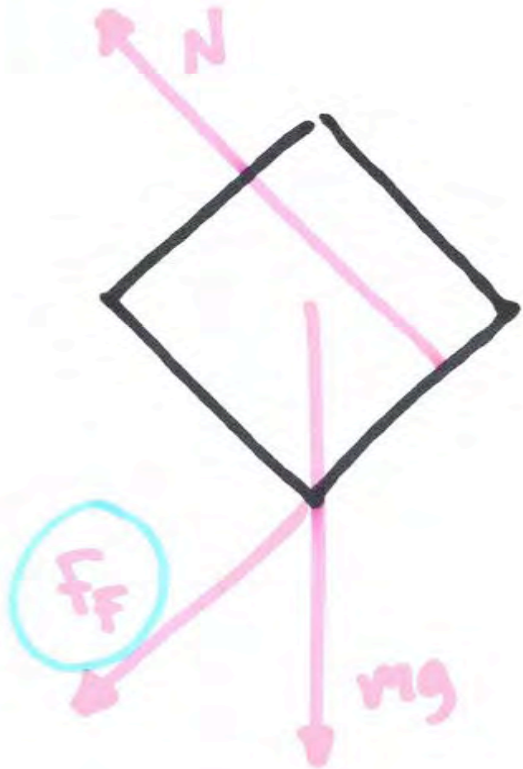
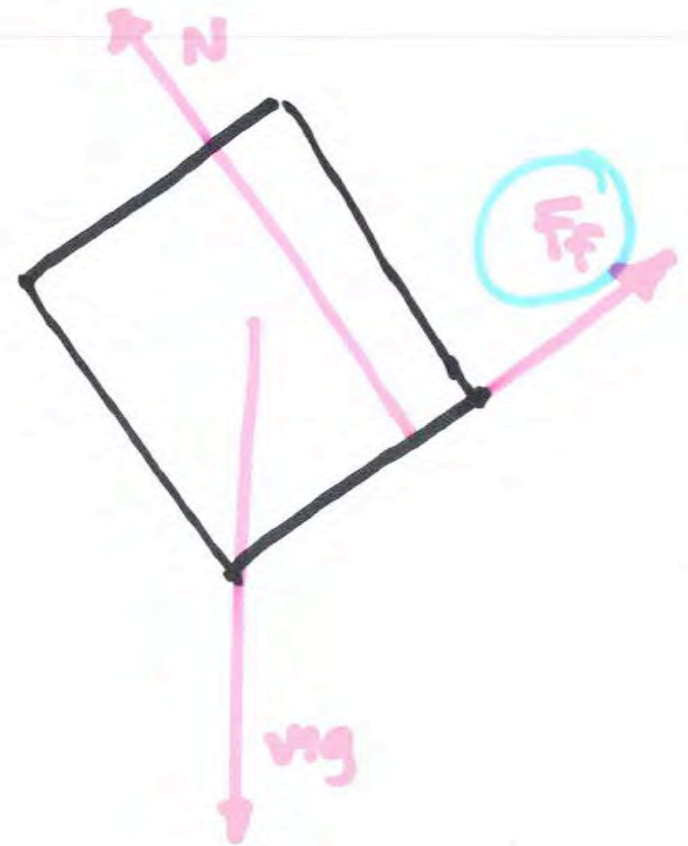


Banked turn.

max. velocity



min. velocity



Newton's 2nd Law:

max. v

$$\sum F_z = ma_z = N \cos \theta - mg - f_f \sin \theta$$

$$\sum F_r = ma_r = -N \sin \theta - f_f \cos \theta$$

min. v

$$\sum F_z = ma_z = N \cos \theta - mg + f_f \sin \theta$$

$$\sum F_r = ma_r = -N \sin \theta + f_f \cos \theta$$

if we are solving for max./min.

$$\boxed{f_f = \mu_s N}$$

← max friction possible.

$$\underline{\text{Max. } v}$$

$$N = \frac{mg}{\cos \theta - \mu_s \sin \theta}$$

sub N into (2)

$$m \left(-\frac{v^2}{r} \right) = - \left[\frac{mg}{\cos \theta - \mu_s \sin \theta} \right] \left[\sin \theta + \mu_s \cos \theta \right]$$

$$\boxed{v_{\text{max}}^2 = rg \frac{\sin \theta + \mu_s \cos \theta}{\cos \theta - \mu_s \sin \theta}}$$

$$\underline{\text{min. } v}$$

$$N = \frac{mg}{\cos \theta + \mu_s \sin \theta}$$

$$m \left(-\frac{v^2}{r} \right) = - \left[\frac{mg}{\cos \theta + \mu_s \sin \theta} \right] \left[\sin \theta - \mu_s \cos \theta \right]$$

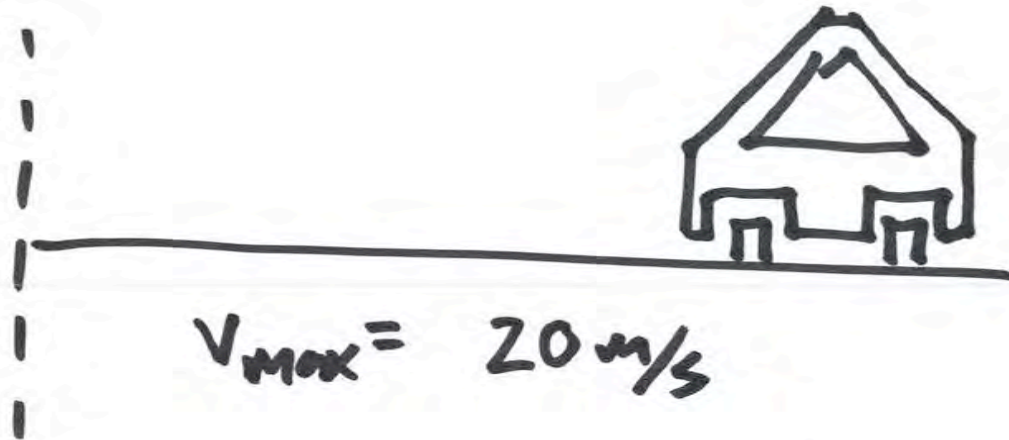
$$\boxed{v_{\text{min}}^2 = rg \frac{\sin \theta - \mu_s \cos \theta}{\cos \theta + \mu_s \sin \theta}}$$

examples:

$$r = 50 \text{ m}$$

$$\theta = 0^\circ$$

$$N_s = 0.8$$



$$V_{\max} = 20 \text{ m/s}$$

$$r = 50 \text{ m}$$

$$\theta = 30^\circ$$

$$N_s = 0.8$$

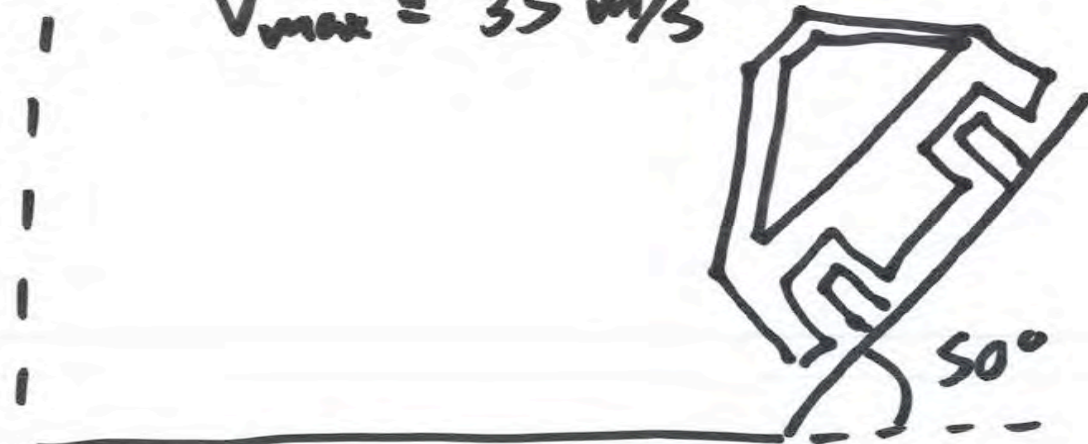


$$V_{\max} = 35 \text{ m/s}$$

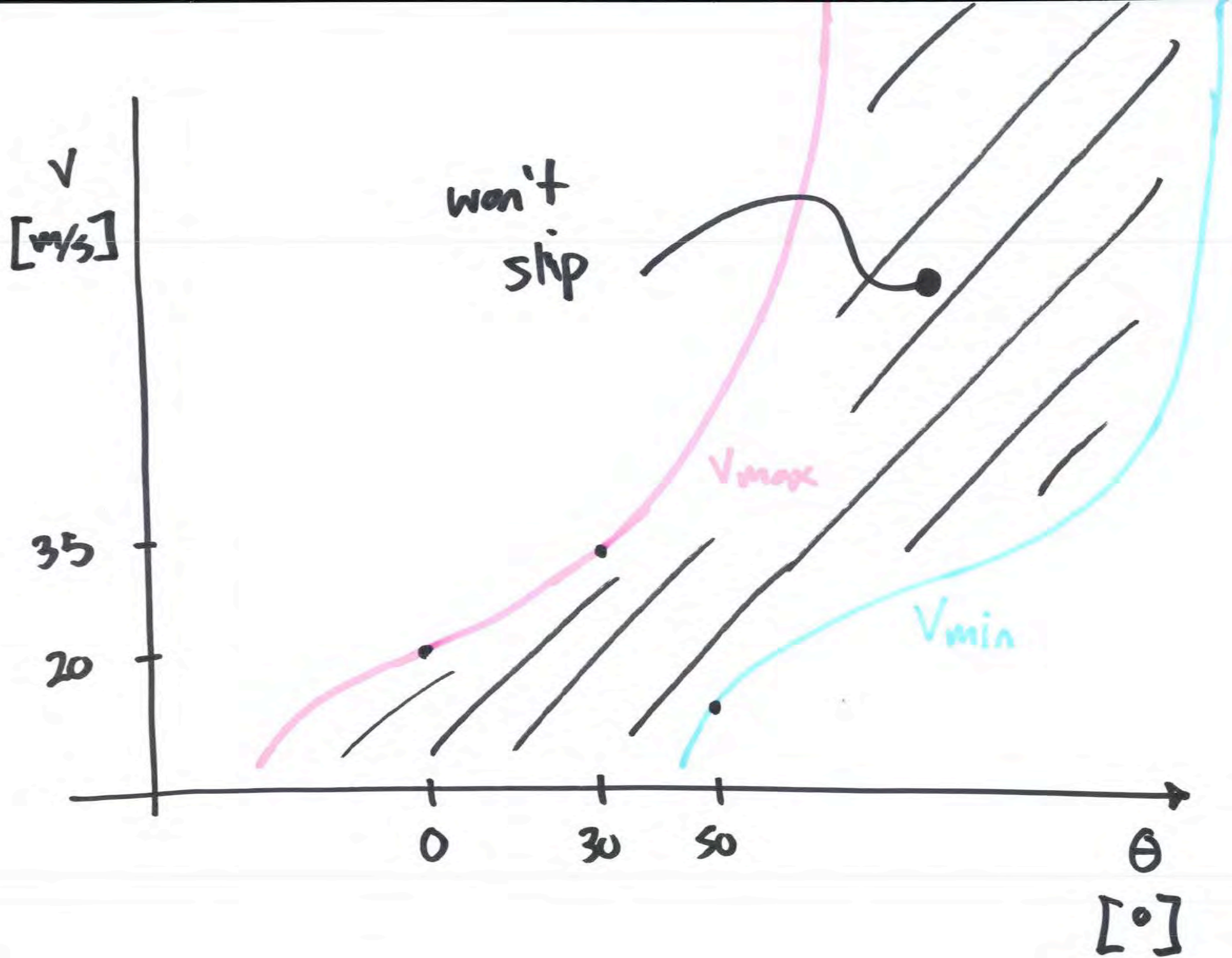
$$r = 50 \text{ m}$$

$$\theta = 50^\circ$$

$$N_s = 0.8$$



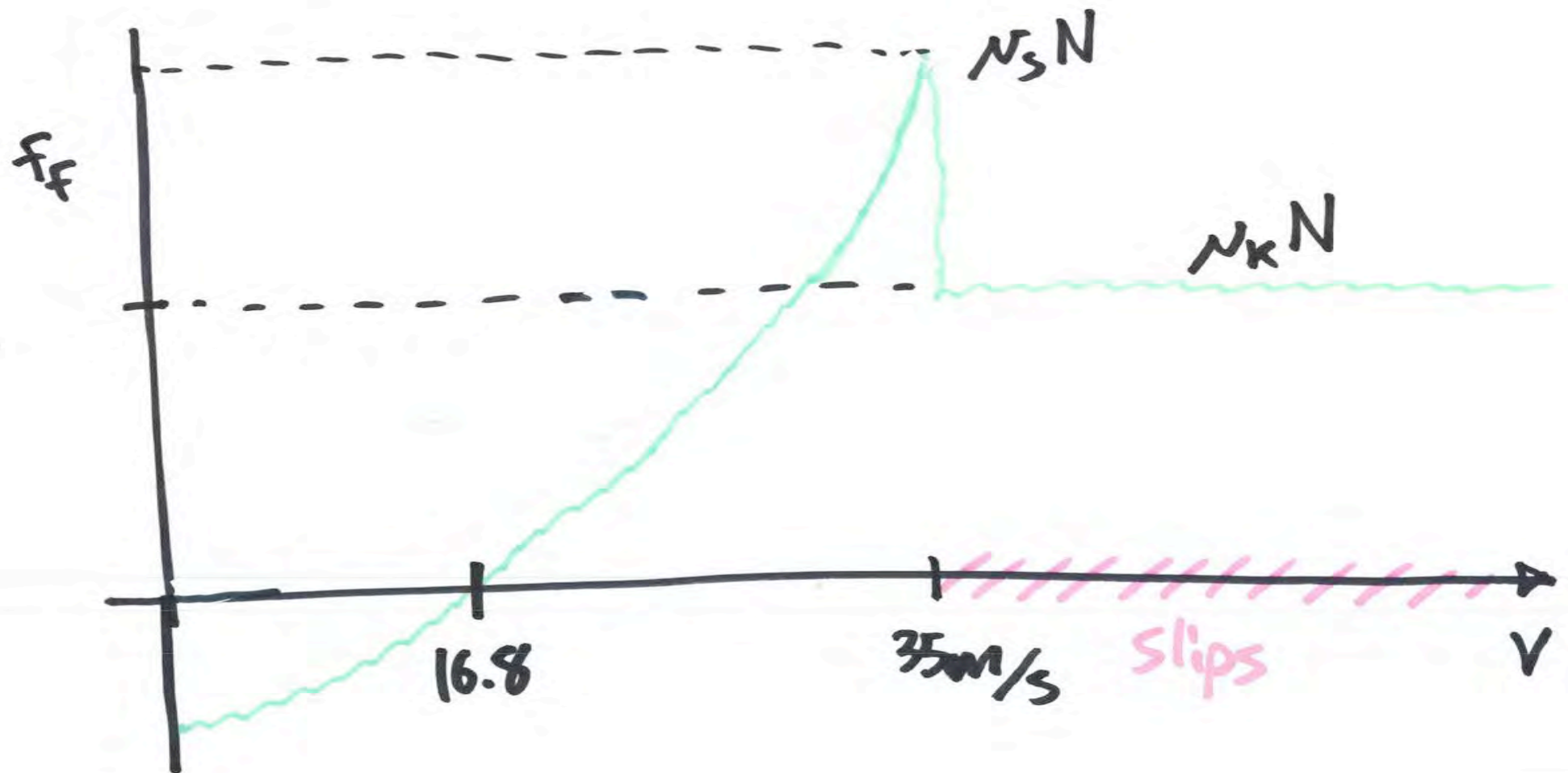
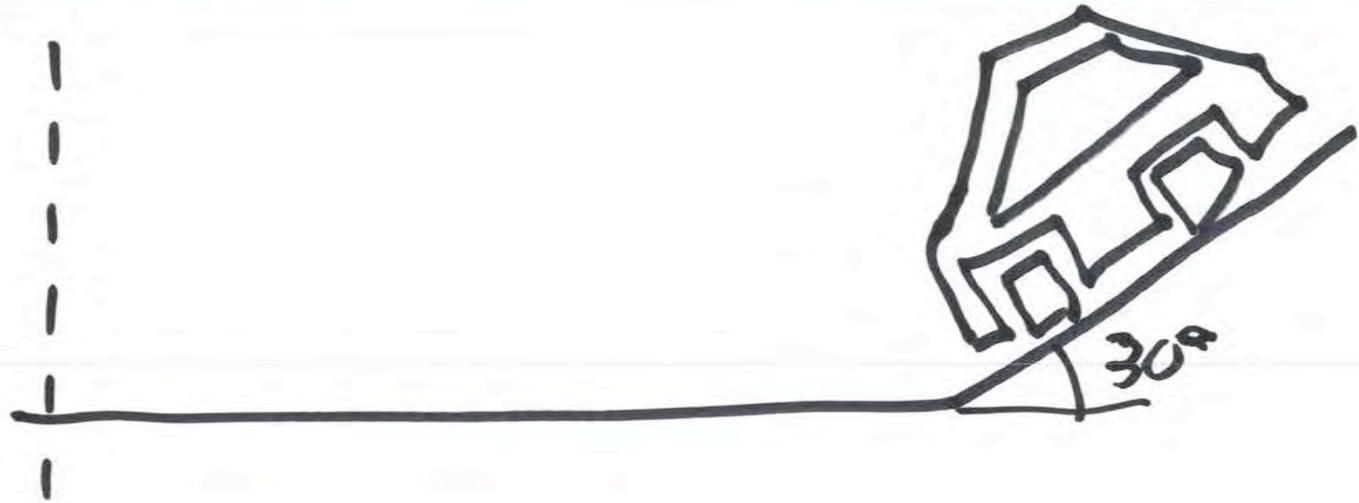
$$V_{\max} = 145 \text{ m/s}, V_{\min} = 10 \text{ m/s}$$



$$r = 50 \text{ m}$$

$$\theta = 30^\circ$$

$$\mu_s = 0.8$$



Extra:

how does F_f relate to v ?

$$-mv^2 = -r \tan \theta mg - r \tan \theta F_f \sin \theta - F_f r \cos \theta$$

$$F_f = \frac{r \tan \theta mg - mv^2}{-r \tan \theta \sin \theta - r \cos \theta}$$

$$F_f \sim v^2$$

Extra:

when is $F_f = 0$?

$$-mv^2 = -r \tan \theta mg - r \tan \theta \cancel{F_f \sin \theta} - \cancel{F_f r \cos \theta}$$

$$\cancel{mv^2} = r \tan \theta \cancel{mg}$$

$$V_{F_f=0} = \sqrt{rg \tan \theta}$$

$$= \sqrt{(50)(9.81) \tan(30^\circ)} = 16.8 \text{ m/s}$$

How does friction work?

$$F_f \propto N$$

$$F_f = \mu N$$

constant?



in real life:

$$\mu = f(\text{shape, temperature, h...})$$

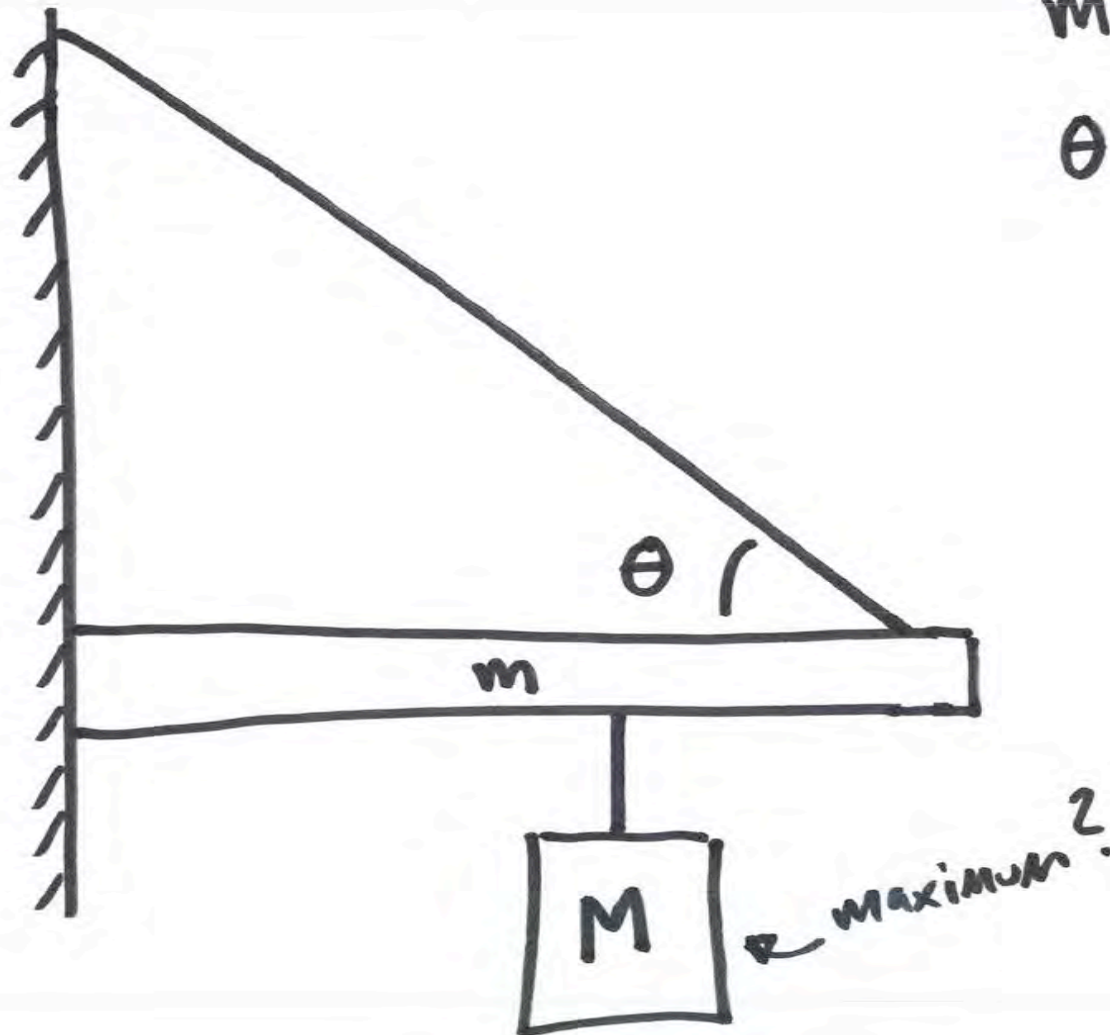
in physics book

$$\mu = f(\text{material})$$

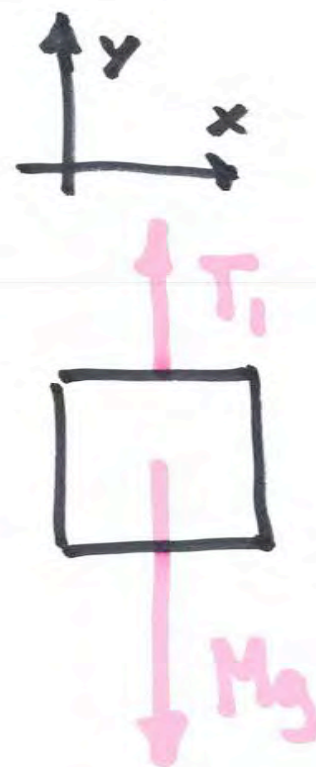
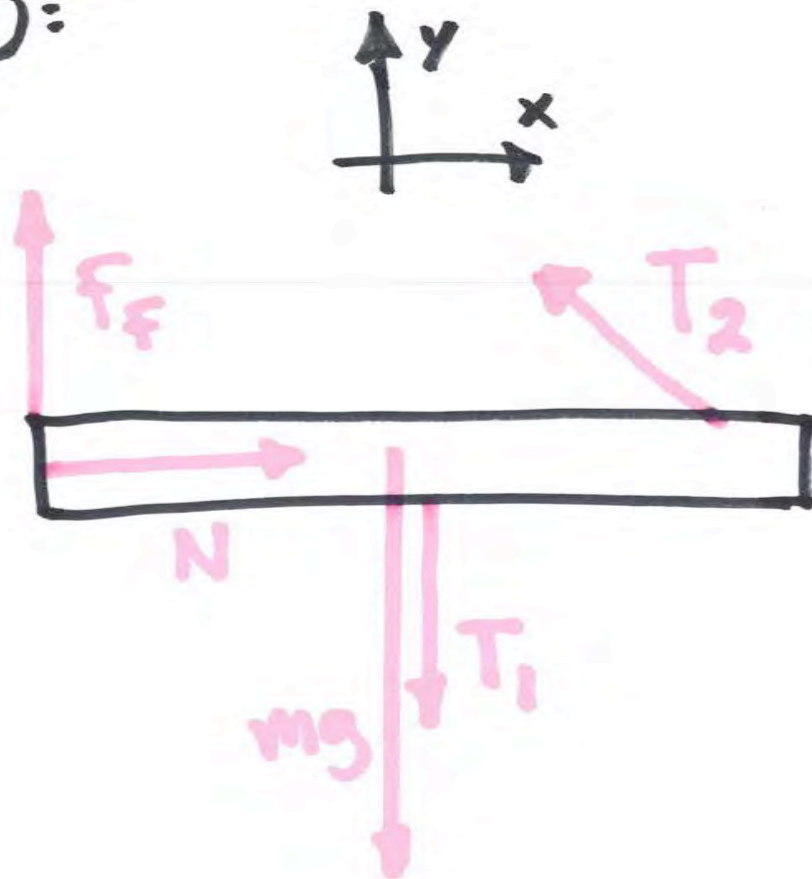
Whats the maximum weight before the rod slips

$$m = 10 \text{ kg}$$

$$\theta = 30^\circ$$



FBD:



Newton's 2nd Law:

$$\Sigma F_x = Ma_x = N - T_2 \cos \theta \quad (1)$$

$$\Sigma F_y = Ma_y = T_2 \sin \theta + F_f - mg - T_1 \quad (2)$$

$$\Sigma F_x = ma_x = 0$$

$$\Sigma F_y = ma_y = T_1 - Mg \quad (3)$$

looking for maximum weight, M

$$F_f = \mu_s N$$

(4)

solve (3):

$$T_1 = Mg$$

(3a)

solve (1):

$$N = T_2 \cos \theta$$

(1a)

sub (1a), (3a) & (4) into (2):

$$0 = T_2 \sin \theta + [\mu_s (T_2 \cos \theta)] - mg - Mg$$

$$0 = \underline{T_2} (\sin \theta + \mu_s \cos \theta) - g(m + \underline{M})$$

Need moments

FBD without moments look like this:



Moments

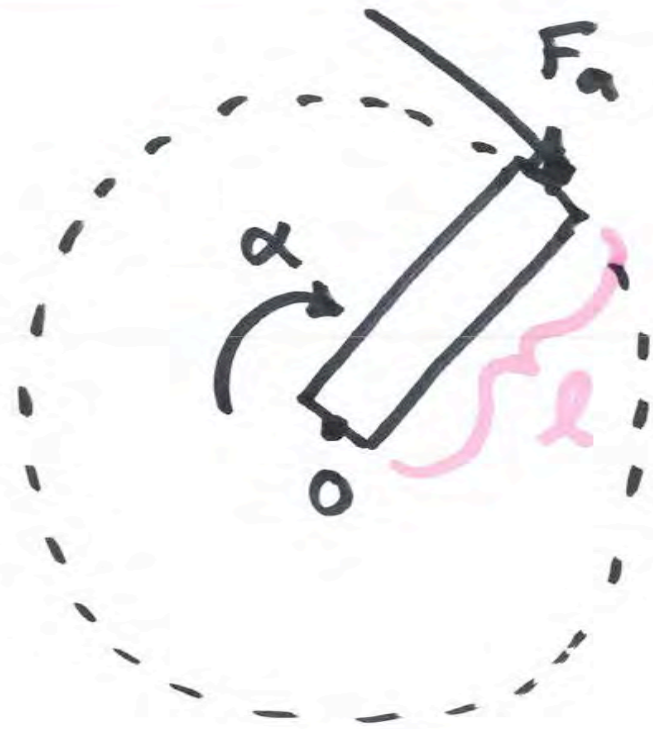
$$\Sigma \vec{F} = m \vec{a}$$

$$\int \Sigma \vec{F} \, d\tau = \int m \vec{a} \, d\tau \quad (\alpha r)$$

$\underbrace{\quad}_{M_0}$

$$\int \Sigma M_0 = \int (\alpha r) \alpha r \left(\frac{dm}{\ell} \right)$$

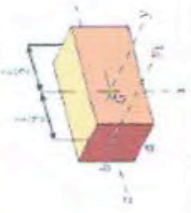
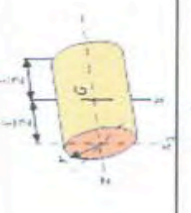
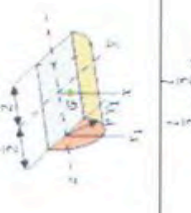
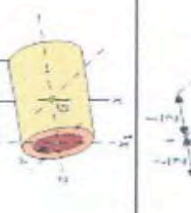
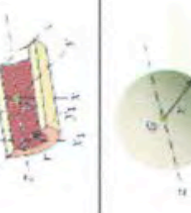
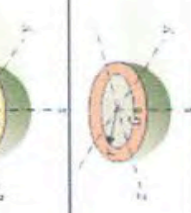
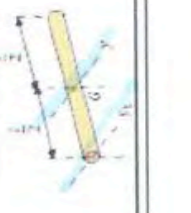
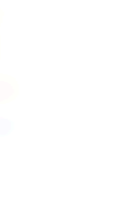
$$\Sigma M_0 = I_0 \alpha$$



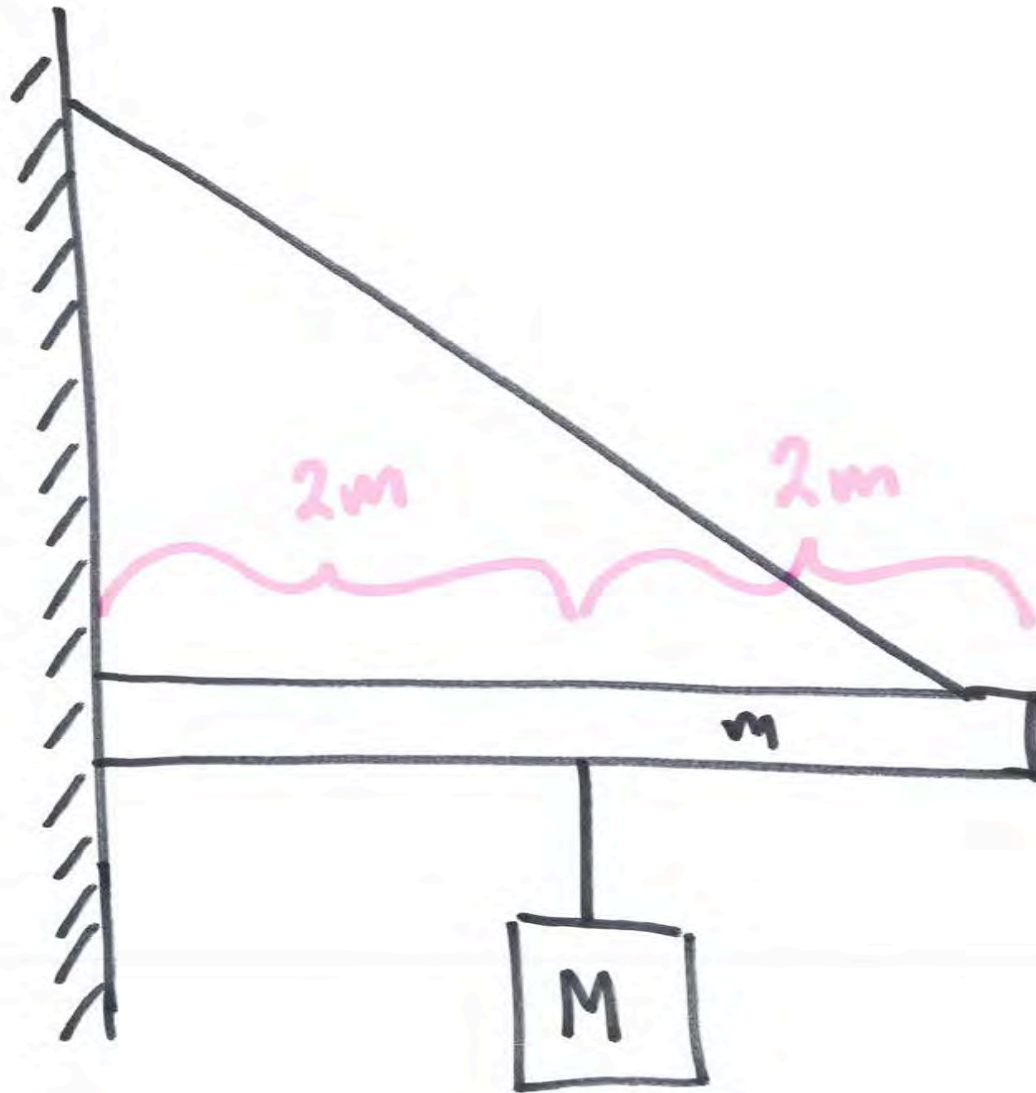
$$\int r^2 dm = I_0$$

$$M_0 = F_a r$$

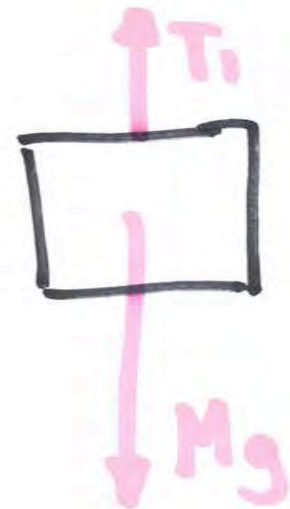
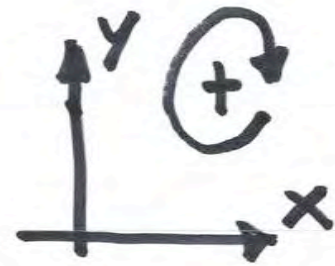
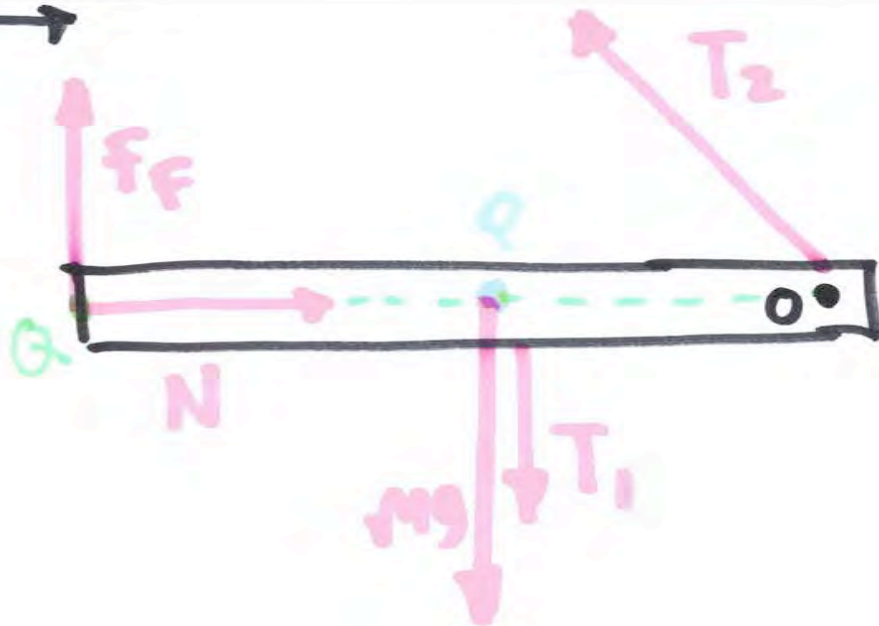
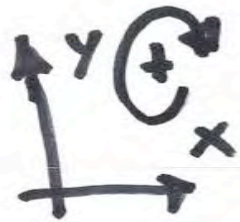
Moment of Inertia Table

Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + b^2)$ $I_{yy} = \frac{1}{12} m(b^2 + c^2)$ $I_{zz} = \frac{1}{12} m(a^2 + c^2)$ $I_{xy} = I_{yz} = I_{zx} = \frac{1}{12} mabc$
Circular Cylinder		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} mh^2$ $I_{yy} = I_{zz} = \frac{1}{4} mr^2 + \frac{1}{12} mh^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} mh^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} mh^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} mh^2$ $I_{zz} = mr^2$
Sphere		—	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{83}{320} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{83}{320} mr^2$
Uniform Slender Rod		—	$I_{xx} = \frac{1}{12} mL^2$ $I_{yy} = I_{zz} = \frac{1}{12} mL^2$

What's the maximum weight before the rod slips?



FBD:



Newton's 2nd Law:

$$\sum F_x = M \cancel{\alpha_x} = N - T_2 \cos \theta \quad (1)$$

$$\sum F_y = M \cancel{\alpha_y} = T_2 \sin \theta + F_f - mg - T_1 \quad (2)$$

$$\sum M_o = I_o \cancel{\alpha_o} = (F_f)(4) - (mg)(2) - (T_1)(2) \quad (3)$$

"
($F_f \cdot r$)

$$\sum M_p = I_p \cancel{\alpha_p} = (F_f)(2) - (T_2 \sin \theta)(2)$$

$$\sum M_Q = I_Q \cancel{\alpha_Q} = (mg)(2) + (T_1)(2) - (T_2 \sin \theta)(4)$$

$$\sum F_x = \cancel{Ma_x} = 0$$

$$\sum F_y = \cancel{Ma_y} = T_1 - Mg \quad (4)$$

looking for maximum, $F_f = \mu_s N$ (5)

solve (4) & (1):

$$N = T_2 \cos \theta, \quad T_1 = Mg$$

sub (1a), (4a), (5) into (2):

$$0 = T_2 \sin \theta + \mu_s [T_2 \cos \theta] - mg - Mg$$

$$0 = T_2 (\sin \theta + \mu_s \cos \theta) - mg - Mg$$

$$T_2 = \frac{mg + Mg}{\sin \theta + \mu_s \cos \theta} \quad (2a)$$

sub (2a) into (3):

$$0 = 4 \overset{f_f}{\mu_s} [T_2 \cos \theta] - 2mg - 2[Mg]$$

$$0 = 4 \left[\frac{mg + Mg}{\sin \theta + \mu_s \cos \theta} \right] [\mu_s \cos \theta] - 2mg - 2Mg$$

$$0 = 4(mg + Mg) \mu_s \cos \theta - 2g(m + M)(\sin \theta + \mu_s \cos \theta)$$

$$0 = 4g \mu_s \cos \theta (m + M) - 2g(\sin \theta + \mu_s \cos \theta)(m + M)$$

indeterminat.

Try without $F_f = \mu_s N$:

sub (1a) & (4a) into (2):

$$0 = T_2 \sin \theta + F_f - mg - Mg$$

$$F_f = Mg + mg - T_2 \sin \theta \quad (2a)$$

sub (2a) into (3):

$$0 = [Mg + mg - T_2 \sin \theta](4) - 2mg - 2Mg$$

$$0 = 4Mg + 4mg - 4T_2 \sin \theta - 2mg - 2Mg$$

$$0 = 2Mg + 2mg - 4T_2 \sin \theta$$

$$T_2 = g \frac{m+M}{2 \sin \theta}$$

$$F_f = Mg + mg - \left[g \frac{m+M}{2 \sin \theta} \right] \sin \theta$$

$$= Mg + mg - \frac{1}{2}mg - \frac{1}{2}Mg$$

$$F_f = \frac{g}{2} (m+M)$$

$$\begin{aligned}
 F_F &= N_3 N \\
 &= N_3 [T_2 \cos \theta] \\
 &= N_3 \cos \theta \left[g \frac{m+M}{2 \sin \theta} \right]
 \end{aligned}$$

$$F_F = \frac{g}{2} (m+M)$$

$$F_{F_{\max.}} = N_3 \frac{\cos \theta}{\sin \theta} \frac{g}{2} (m+M)$$

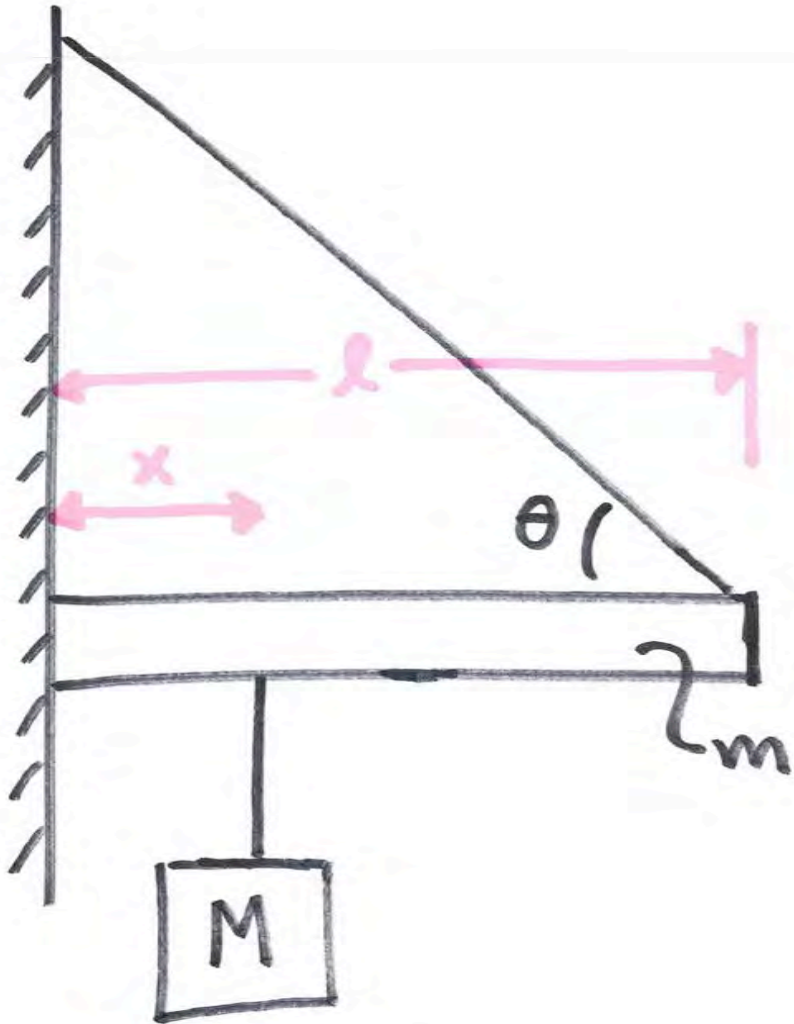
$$N = \frac{\cos \theta}{\sin \theta} \frac{g}{2} (m+M)$$

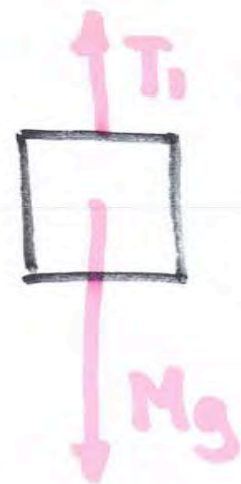
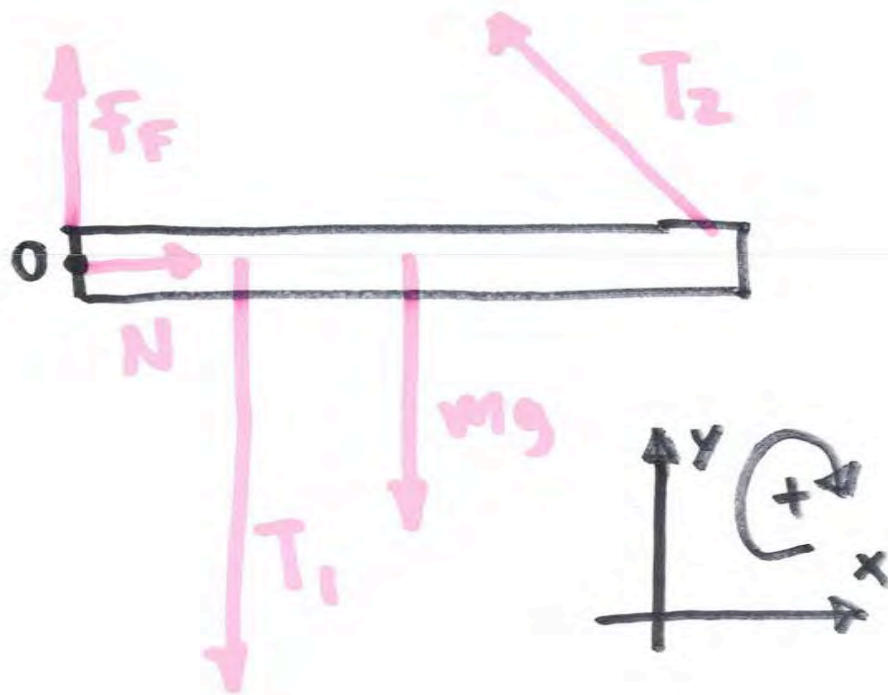
if we set $F_F = F_{F_{\max.}}$:

$$\frac{g}{2} (m+M) = N_3 \frac{\cos \theta}{\sin \theta} \frac{g}{2} (m+M)$$

$$\sin \theta = N_3 \cos \theta$$

More general:





$$\sum F_x = ma_x = N - T_2 \cos \theta$$

$$\Rightarrow N = T_2 \cos \theta$$

$$\sum F_y = ma_y = F_f + T_2 \sin \theta - T_1 - mg \quad (1)$$

$$\sum M_o = I_o \alpha_o = T_1 x + mg \frac{l}{2} - T_2 \sin \theta l \quad (2)$$

$$\sum F_x = Ma_x = 0$$

$$\sum F_y = Ma_y = T_1 - Mg$$

$$\Rightarrow T_1 = Mg$$

$$F_f + T_2 \sin \theta - Mg - mg = 0 \quad (1a)$$

$$Mgx + mg \frac{l}{2} - T_2 \sin \theta l = 0 \quad (2a)$$

sub (1a) into (2a)

$$Mgx + mg \frac{l}{2} - [Mg + mg - F_f] l = 0$$

$$Mgx - Mgl + \frac{l}{2} mg - mgl + F_f l = 0$$

$$Mg(x-l) - \frac{l}{2} mg = -F_f l$$

$$F_f = \frac{mg}{2} + Mg \frac{l-x}{l}$$

$$F_f + T_2 \sin \theta - Mg - mg = 0 \quad (1a)$$

$$T_2 = \frac{1}{\sin \theta} [Mg + mg - F_f]$$

$$= \frac{1}{\sin \theta} \left[Mg + mg - \frac{mg}{2} - Mg \frac{l-x}{l} \right]$$

$$\boxed{T_2 = \frac{1}{\sin \theta} \left[\frac{mg}{2} + Mg \frac{x}{l} \right]}$$

$$1 - \frac{l-x}{l}$$

$$1 - \frac{l}{l} + \frac{x}{l}$$

$$F_{F_{\max}} = N_S N$$

$$= N_S [T_2 \cos \theta]$$

$$= N_S \cos \theta \left[\frac{1}{\sin \theta} \left(\frac{mg}{2} + Mg \frac{x}{L} \right) \right]$$

$$F_{F_{\max}} = N_S \frac{\cos \theta}{\sin \theta} \left(\frac{mg}{2} + Mg \frac{x}{L} \right)$$

when is $f_f = f_{fmax}$.

$$\frac{mg}{2} + Mg \frac{l-x}{l} = N_s \frac{\cos \theta}{\sin \theta} \left(\frac{mg}{2} + Mg \frac{x}{l} \right)$$

$$Mg \frac{l-x}{l} - N_s \frac{\cos \theta}{\sin \theta} Mg \frac{x}{l} = N_s \frac{\cos \theta}{\sin \theta} \frac{mg}{2} - \frac{mg}{2}$$

$$Mg - Mg \frac{x}{l} - N_s \frac{\cos \theta}{\sin \theta} Mg \frac{x}{l} = \dots$$

$$Mg \left(1 - \frac{x}{l} - N_s \frac{\cos \theta}{\sin \theta} \frac{x}{l} \right) = \dots$$

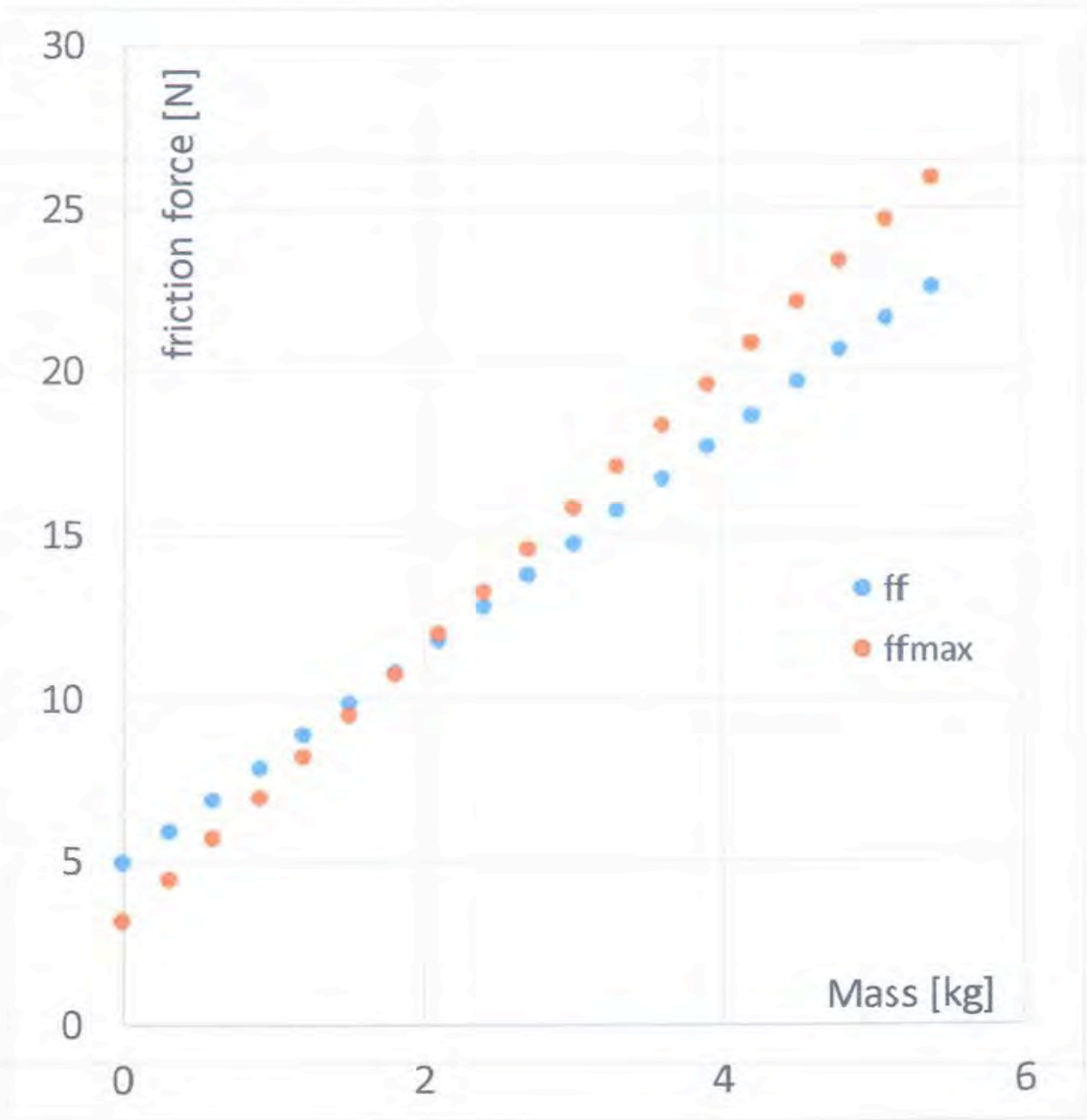
$$M = \frac{\frac{m_0}{2} \left(N_s \frac{\cos \theta}{\sin \theta} - 1 \right)}{g \left(1 - \frac{x}{l} - N_s \frac{\cos \theta}{\sin \theta} \frac{x}{l} \right)}$$

$$M = \frac{m}{2} \frac{N_s \frac{\cos \theta}{\sin \theta} - 1}{1 - \frac{x}{l} \left(1 + N_s \frac{\cos \theta}{\sin \theta} \right)}$$

Hanging Mass In Middle Of Bar

Parameters	
g	9.81 m/s2
m	1 kg (mass of bar)
x	2 m
L	3 m
theta	25 degree
mu_s	0.3
M = 1.866 kg	

Solution with respect to M		
M [kg]	ff [N]	ffmax [N]
0	5	3
0.3	5.9	4.4
0.6	6.9	5.7
0.9	7.8	6.9
1.2	8.8	8.2
1.5	9.8	9.5
1.8	10.8	10.7
2.1	11.8	12.0
2.4	12.8	13.3
2.7	13.7	14.5
3.0	14.7	15.8
3.3	15.7	17.0
3.6	16.7	18.3
3.9	17.7	19.6
4.2	18.6	20.8
4.5	19.6	22.1
4.8	20.6	23.4
5.1	21.6	24.6
5.4	22.6	25.9



Case $\frac{x}{l} = \frac{1}{2} \quad (x = \frac{l}{2})$

$$M = \frac{m}{2} \frac{\sqrt{3} \frac{\cos \theta}{\sin \theta} - 1}{1 - \frac{1}{2} - \frac{1}{2} \sqrt{3} \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{m}{2} \frac{\sqrt{3} \frac{\cos \theta}{\sin \theta} - 1}{\frac{1}{2} - \frac{1}{2} \sqrt{3} \frac{\cos \theta}{\sin \theta}}$$

$$= \frac{m}{2} \frac{\cancel{\sqrt{3} \frac{\cos \theta}{\sin \theta}} - 1}{-\frac{1}{2} \left(\cancel{-1 + \sqrt{3} \frac{\cos \theta}{\sin \theta}} \right)}$$

$$= \frac{m}{2} \left(-\frac{2}{1} \right) = -m.$$

$M = -m$

Back to original problem:

$$0 = 4g\mu_3 \cos \theta (m+M) - 2g(\sin \theta + \mu_3 \cos \theta)(m+M)$$

indeterminant...

$$0 = 2g(m+M) [2\mu_3 \cos \theta - (\sin \theta + \mu_3 \cos \theta)]$$

there are 2 ways to make this zero:

$$m+M = 0$$

$\Rightarrow$

$$\boxed{M = -m}$$

$$2\mu_3 \cos \theta - (\sin \theta + \mu_3 \cos \theta) = 0$$

$\Rightarrow$

$$\boxed{\sin \theta = \mu_3 \cos \theta}$$

Both solⁿ are found here within.

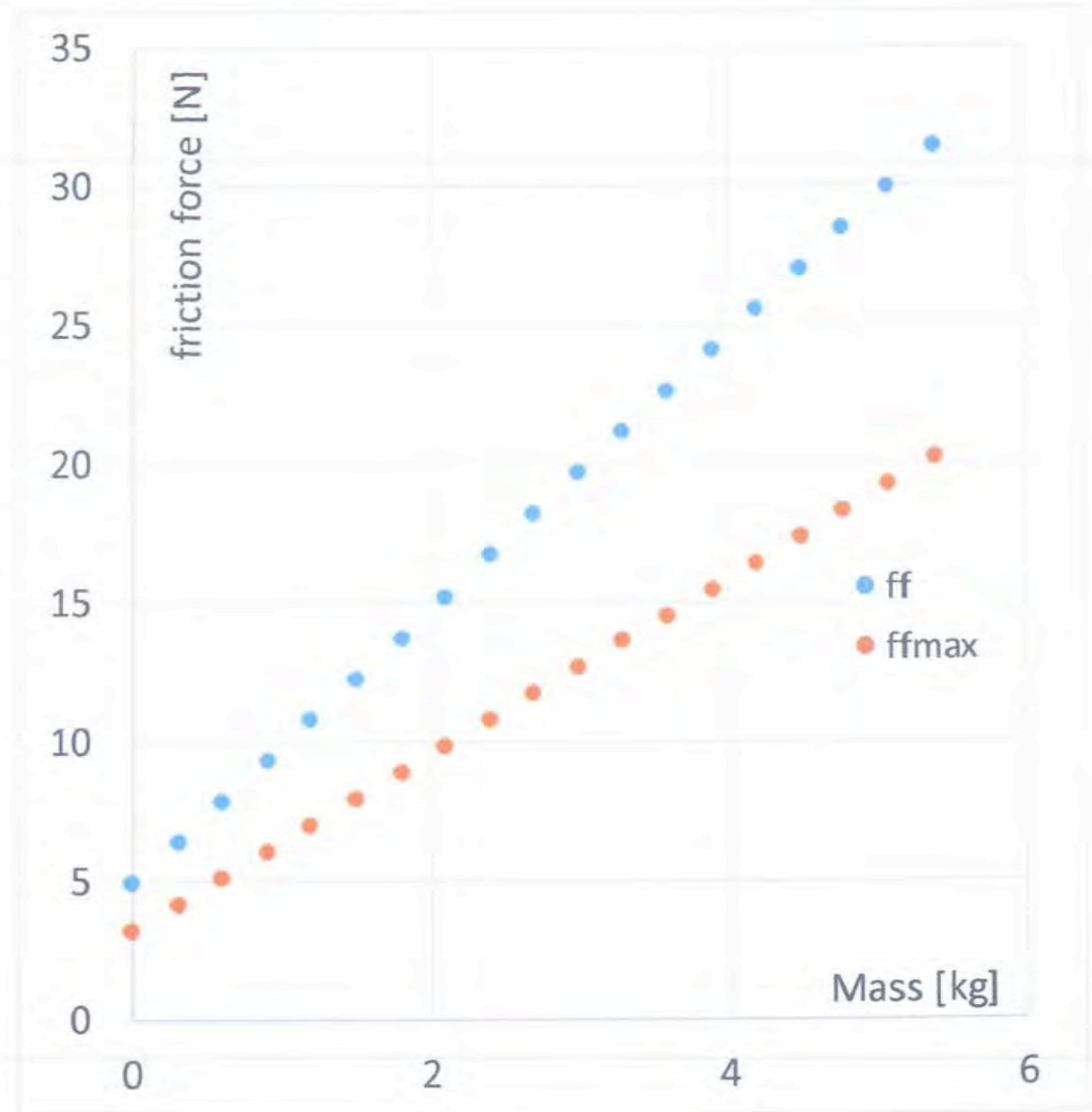
Hanging Mass In Middle Of Bar

Parameters

g	9.81 m/s ²
m	1 kg (mass of bar)
x	2 m
L	4 m
theta	25 degree
mu_s	0.3
M =	-1.000 kg

Solution with respect to M

M [kg]	ff [N]	ffmax [N]
0	5	3
0.3	6.4	4.1
0.6	7.8	5.0
0.9	9.3	6.0
1.2	10.8	6.9
1.5	12.3	7.9
1.8	13.7	8.8
2.1	15.2	9.8
2.4	16.7	10.7
2.7	18.1	11.7
3.0	19.6	12.6
3.3	21.1	13.6
3.6	22.6	14.5
3.9	24.0	15.5
4.2	25.5	16.4
4.5	27.0	17.4
4.8	28.4	18.3
5.1	29.9	19.2
5.4	31.4	20.2



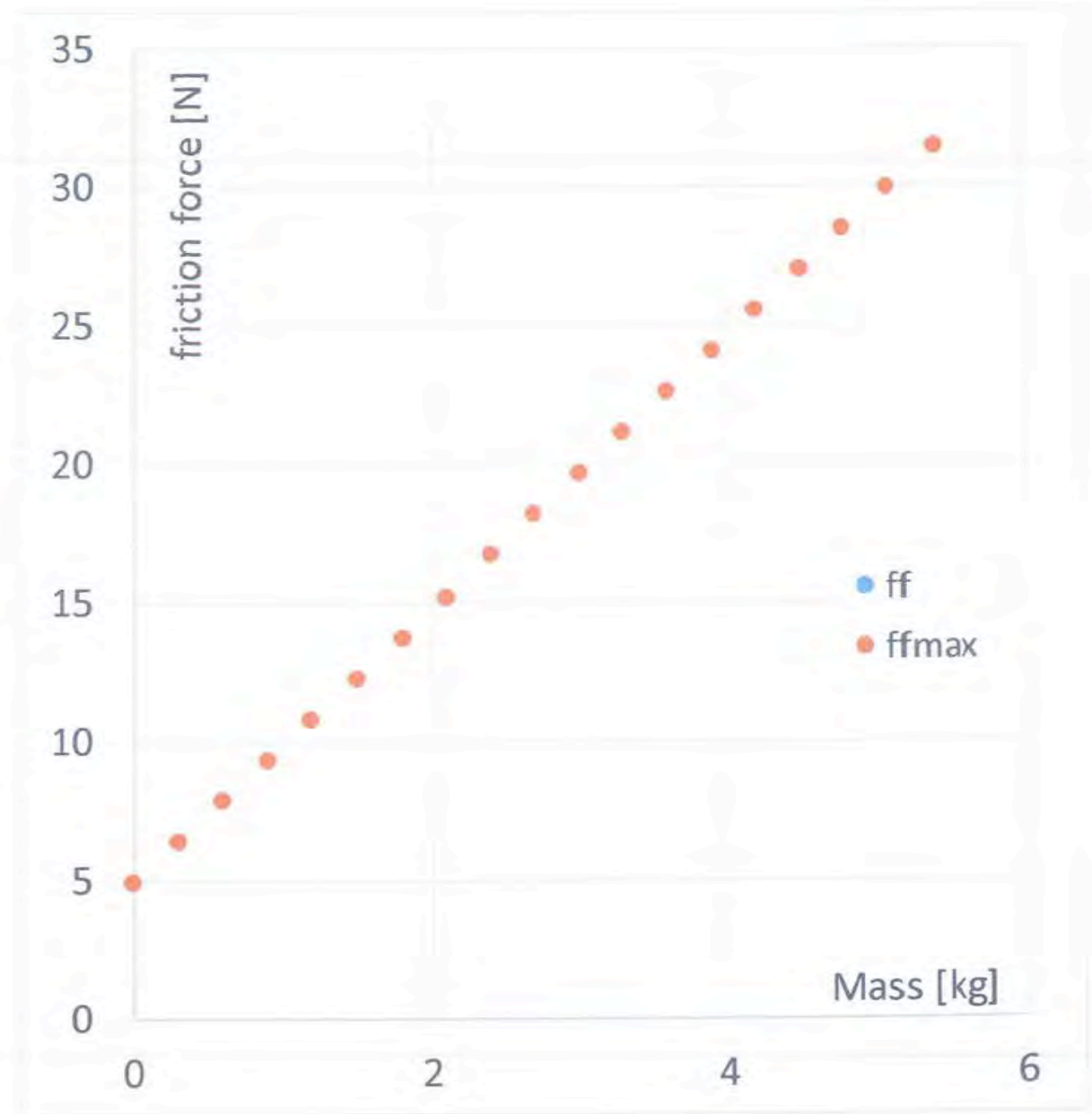
Hanging Mass In Middle Of Bar

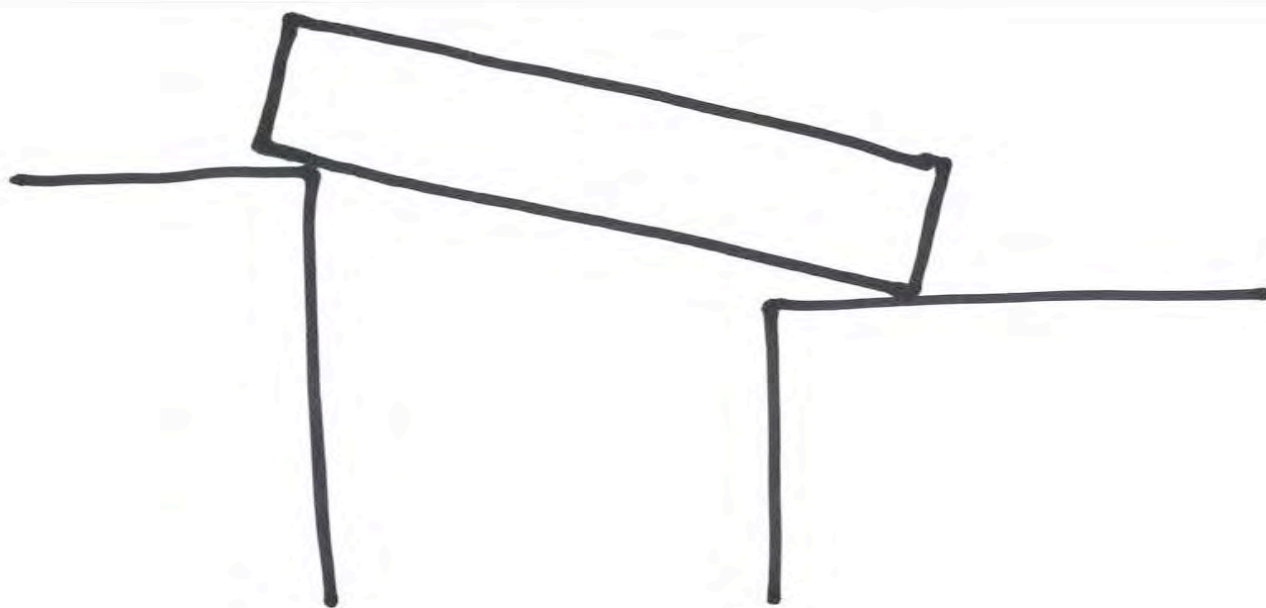
Parameters

g	9.81 m/s ²
m	1 kg (mass of bar)
x	2 m
L	4 m
θ	16.7 degree
μ_s	0.3
$M =$	-1.000 kg
$\theta =$	16.6992442 degrees

Solution with respect to M

M [kg]	ff [N]	ff_{max} [N]
0	5	5
0.3	6.4	6.4
0.6	7.8	7.8
0.9	9.3	9.3
1.2	10.8	10.8
1.5	12.3	12.3
1.8	13.7	13.7
2.1	15.2	15.2
2.4	16.7	16.7
2.7	18.1	18.1
3.0	19.6	19.6
3.3	21.1	21.1
3.6	22.6	22.6
3.9	24.0	24.0
4.2	25.5	25.5
4.5	27.0	27.0
4.8	28.4	28.4
5.1	29.9	29.9
5.4	31.4	31.4





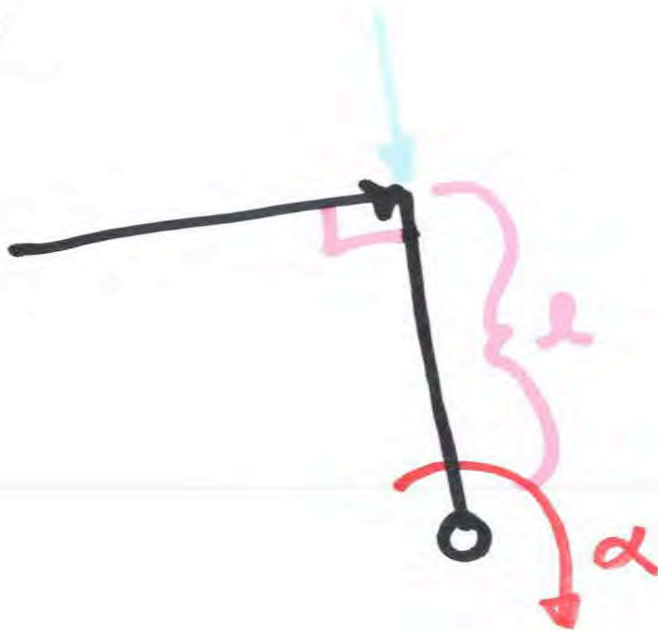
indeterminate ...

Force:



linear

Torque:
(moment)



rotational

