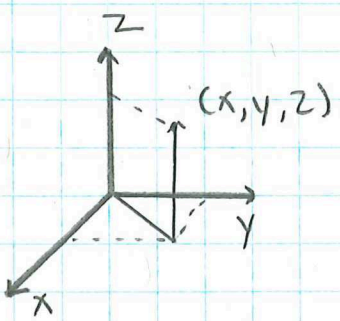
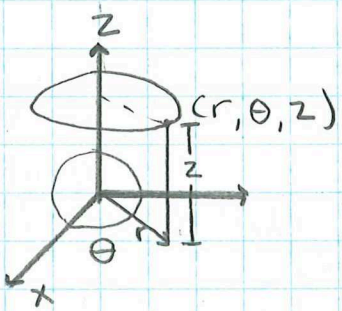
Circular Motion

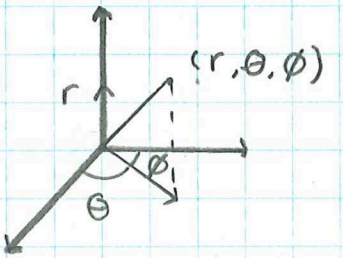
Feb. 17. 2023 ②



Cartesian

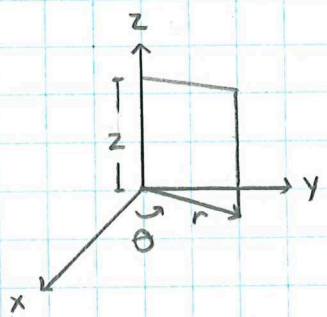


Cylindrical (polar)



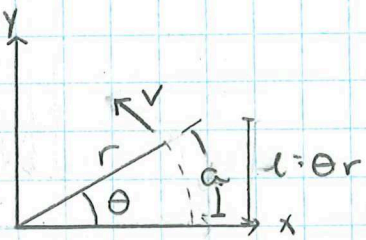
sphereical (space)

Cylindrical Coordinates



r and z are constant
only changing θ
it is a 1-d problem

Kinematics



(circular motion)

• = differential

•• = second derivative

x , y	r , θ (x)
$V_x = \frac{dx}{dt}$, $V_y = \frac{dy}{dt}$	$\dot{r} = \frac{dr}{dt}$ $\quad \omega = \frac{d\theta}{dt}$ (v)
$a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$	$\ddot{r} = \frac{d^2r}{dt^2}$
$\dot{x} = \frac{dx}{dt} = V_x$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (a)
$\ddot{x} = \frac{d^2x}{dt^2}$	

2π radians = 360°

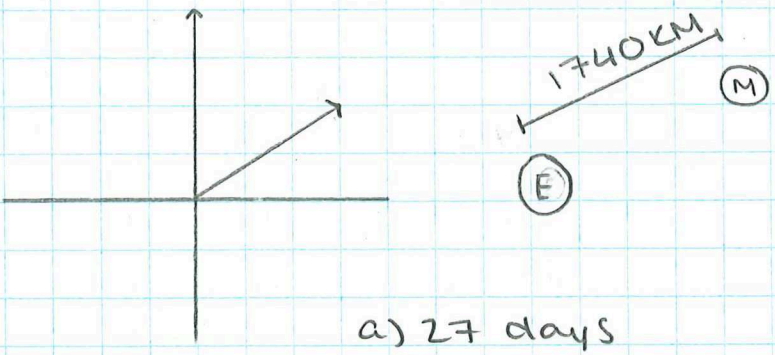
$$\theta = \frac{l}{r}$$

$$\frac{d}{dt}\theta = \frac{l}{r}\frac{d}{dt}$$

$$\frac{d\theta}{dt} = \frac{dl}{dt}\frac{1}{r}$$

$$\omega = \frac{v}{r}$$

2π radians = 360°



a) How long does it take the moon to rotate around the earth?

b) What is the moon's angular velocity? "w"

c) How fast is the moon going, relative to earth?

d) What is the moon's angular acceleration?

a) 27 days

b) $w = \frac{d\theta}{dt} = \frac{\theta_1 - \theta_0}{t_1 - t_0}$ ($\theta_1 = 2\pi$, $\theta_0 = 0$)

$$T = 27 \text{ days}\left(\frac{24 \text{ hrs}}{1 \text{ day}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)\left(\frac{60 \text{ sec}}{1 \text{ min}}\right) = 2332800 \text{s}$$

$$w = \frac{2\pi}{2332800} = 2.693 \times 10^{-6} \text{ rads/sec}$$

$$= 1.54 \times 10^{-4} \text{ °/sec}$$

c) $v = wr$

$= (2.693 \times 10^{-6})($ [illegible] $)$

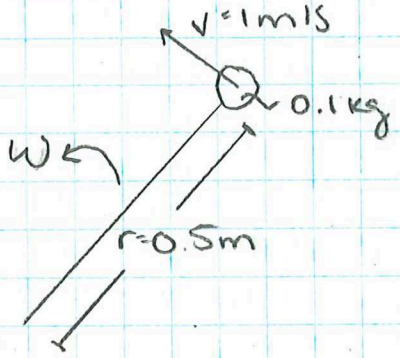
$$= 4.68 \text{m/s}\left(\frac{1 \text{ km}}{1000 \text{m}}\right)\left(\frac{60 \text{s}}{1 \text{min}}\right)\left(\frac{60 \text{ min}}{1 \text{ hr}}\right)$$

=

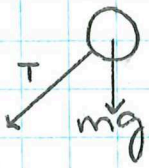
Distance	x	y	z	θ	r
Velocity	u	v	w	ω	$\dot{r}$
Acceleration	a_x	a_y	a_z	α	$\ddot{r}$

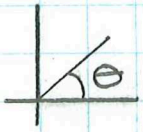
Circular Motion

Feb. 24, 2023

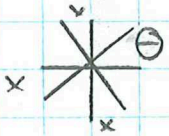


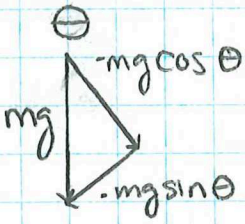
FBD





↖ coordinates in the center of rotation





$\Sigma F_r = ma_r = -T - mg\sin\theta$

~~$\Sigma F_\theta = ma_\theta = -mg\cos\theta$~~

$m\left(\frac{-v^2}{r}\right) = -T - mg\sin\theta$

$T = \frac{mv^2}{r} - mg\sin\theta$

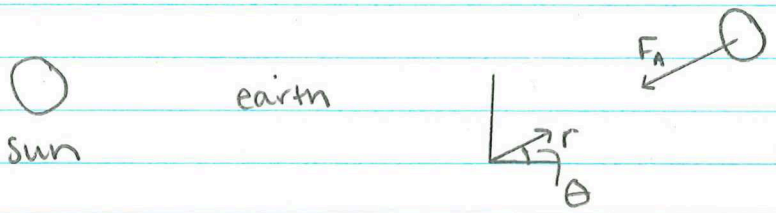
$a_r = \omega^2 r = \frac{-v^2}{r}$

$a_\theta \neq 0$

Orbital Problems

Feb. 24. 23 (3)

FBD



$$\Sigma F_r = ma_r = -\frac{Gmm}{r^2} \qquad a_r = \frac{-v^2}{r}$$

$$\Sigma F_\theta = m\vec{a}_\theta^{\,0} = 0$$

$$m_e\left(\frac{-v^2}{r}\right) = \frac{G m_s m_e}{r^2}$$

$$-v^2 = \frac{G m_s}{r}$$

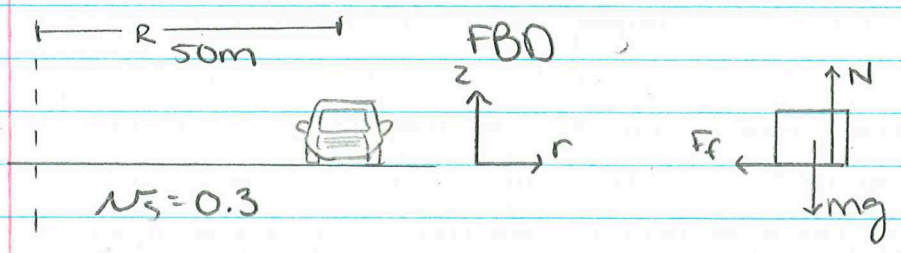
$$v = \sqrt{\frac{G m_s}{r}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11})(1.98\times10^{30})}{1.44\times10^{11}}}$$

$$v = 30\,000 \text{ m/s}$$

Banked Corners.

Feb. 24. 23 (4)



(1) $\Sigma F_r = ma_r = -F_f$

$a_r = \frac{-V^2}{r}$

(2) $\Sigma F_\theta = m\vec{a}_\theta^{\,0} = 0$

(3) $\Sigma F_z = m\vec{a}_z^{\,0} = N - mg$

Solve for N in (3)

$N = mg$ (3a)

Sub (3a) into 1, 4 and $F_f = \mu N$ (4)

$m\left(\frac{-V^2}{r}\right) = -\mu(mg)$

$V = \sqrt{r\mu g}$

$V = \sqrt{(50)(0.3)(9.81)} = 12 \text{ m/s}$

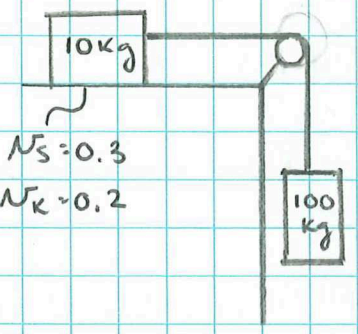
$12 \text{ m/s}\left(\frac{1 \text{ Km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right) = 44 \text{ Km/h}$

Try $\mu_s = 0.8$

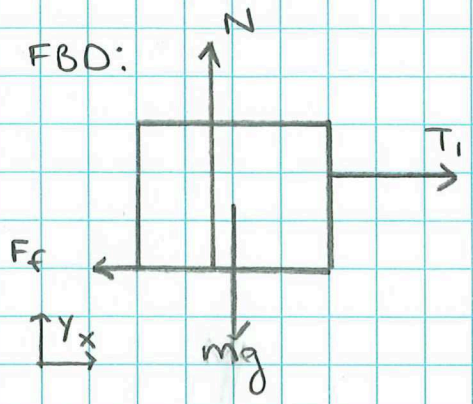
$N = 19.8 \text{ m/s}$

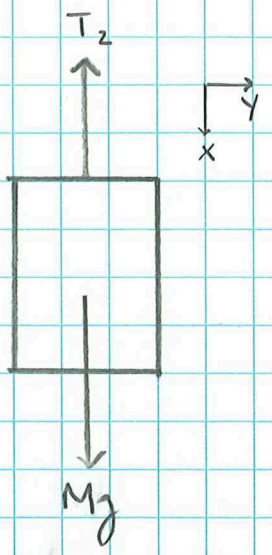
$V = 71 \text{ Km/h}$ ← This is why we bank corners

Determine tension in rope.



FBD:





(1) $\Sigma F_x = ma_{x_1} = T_1 - F_f$

(2) $\Sigma F_y = ma_{y_1}^{\,0} = N - mg$ $\qquad a_{x_1} = a_{x_2} = a_x$

(3) $\Sigma F_x = Ma_{x_2} = Mg - T_2$

(4) $\Sigma F_y = Ma_{y_2}^{\,0} = 0$

(2) Does it lift?

$ma_y = N - mg$

$0 = N - (10)(9.81)$

$N = 98.1$ ✓ Does not lift

Does it move? Assume $F_f = \mu_s N$

solve for T in (3)

$Ma_x = Mg - T$

$T = Mg - Ma_x$ (3a)

Sub (3a) into (1)

$ma_x = [Mg - Ma_x] - F_f$

$ma_x = [Mg - Ma_x] - 0.3(98.1)$

$10\,a_x = [100(9.81) - 100\,a_x] - 0.3(98.1)$

$10a_x + 100a_x = 981 - 29.43$

$[10+100]a_x = 981 - 29.43$

$a_x = \dfrac{981 - 29.43}{110}$

$a_x = 8.65\ m/s^2$ ✓ accelerates

$ma_x = [Mg - Ma_x] - \mu_s N$

$ma_x + Ma_x = Mg - \mu_s N$

$a_x = \dfrac{Mg - \mu_s N}{m+M}$

$a_x = \dfrac{100(9.81) - 0.3(98.1)}{10+100}$

$a_x = \dfrac{981 - 29.43}{110}$

$a_x = 8.65\ m/s^2$

Resolve using N_k for acceleration

$$ma_x = [Mg - Ma_x] + N_k N$$

$$[m+M]a_x = Mg - N_k N$$

$$a_x = \frac{Mg - N_k N}{m+M}$$

$$a_x = \frac{100(9.81) - 0.2(98.1)}{10+110}$$

$$a_x = \frac{981 - 19.62}{110}$$

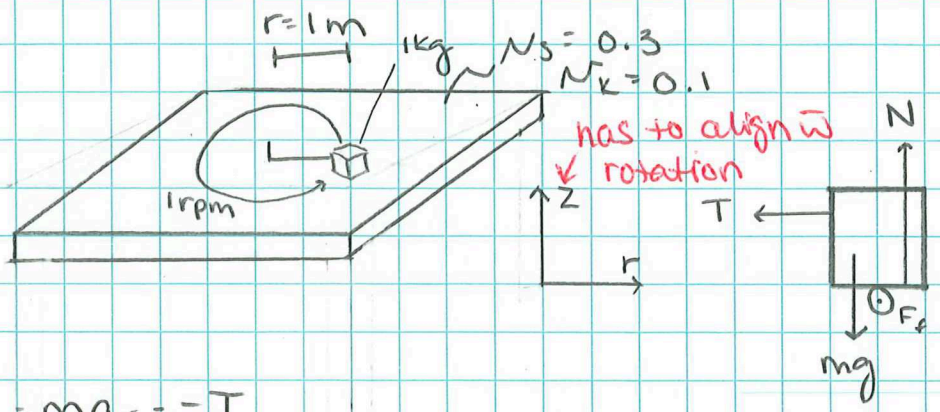
$$a_x = 8.74 \text{ m/s}^2$$

Find tension

$$T = Mg - Ma_x$$

$$T = 100(9.81) - (100)(8.74)$$

$$T = 981 - 874 = 107 \text{ N}$$



away from you ⊙

towards you ⊗

$\Sigma F_r = ma_r = -T$

$\Sigma F_\theta = ma_\theta = -F_f$

$\Sigma F_z = ma_z = N - mg$

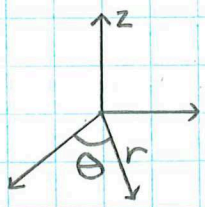
$a_r = -\omega^2 r$

$\omega = 1 \frac{\text{cycle}}{\text{min}}$ → convert to $\frac{\text{rads}}{\text{sec}}$

$$\omega = 1 \frac{\text{cycle}}{\text{min}} \left(\frac{2\pi \text{ rads}}{1 \text{ cycle}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$$

$$\omega = 0.105 \frac{\text{rads}}{\text{sec}}$$

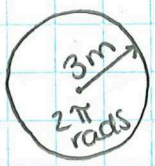
$$T = -ma_r$$
$$= -m[-\omega^2 r]$$
$$= m\omega^2 r = (1)(0.105)^2(1)$$
$$= 0.011 \text{ N}$$



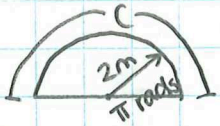
$\Sigma F_r = ma_r$
$\Sigma F_\theta = ma_\theta$
$\Sigma F_z = ma_z$

* for ω = constant

$a_r = -\omega^2 r = -\frac{v^2}{r}$



$C = 2\pi r$
$= 2\pi(3)$
$= 6\pi$

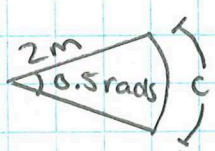


$C = \pi r$
$= \pi(2)$

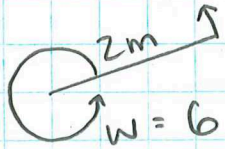
$C = \theta r$ distance m

$v = \omega r$ velocity m/s

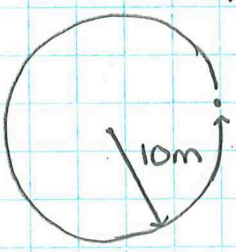
$a = \alpha r$ acceleration m/s^2



$C = \theta r$
$= (0.5)(2)$
$= 1$



$v = \omega r = 6(2)$
$= 12 m/s$



method 1:

$T = 40$ sec/cycle

determine w:

$C = (2\pi)(10)$

$= 20\pi$ [m]

$V = \frac{\Delta x}{\Delta t} = \frac{20\pi\ m}{40\ s} = 1.57$ [m/s]

$w = \frac{V}{r} = \frac{1.57}{10} = 0.157$ [rads/s]

$= 9°/s$

distance	x [m]	θ [rads]
velocity	v [m/s]	w [rads/s]

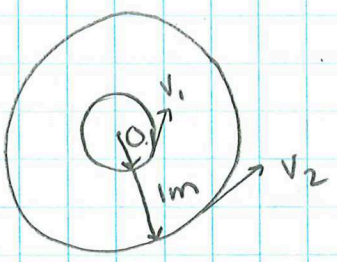
method 2:

$T = \left(\frac{40\ sec}{cycle}\right)\left(\frac{1\ cycle}{2\pi\ rads}\right)$

$T = \frac{40}{2\pi}\ \frac{sec}{rads}$

$\frac{1}{T} = \frac{2\pi}{40}\ \frac{rads}{sec}$

$= 0.157$ rads/sec

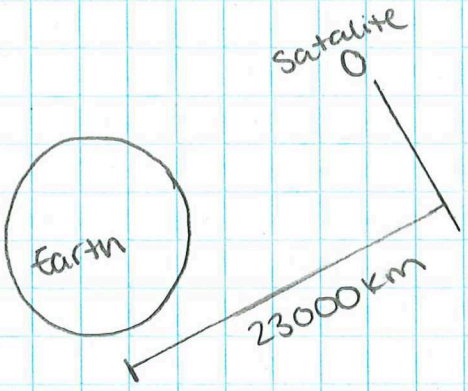


$$\omega = \frac{2\pi \text{ rads}}{1 \text{ sec}} = 2\pi \frac{\text{rads}}{\text{s}}$$

$$V_1 = \omega r_1 = 2\pi(0.1) = 0.63 \text{ m/s}$$

$$V_2 = \omega r_2 = (2\pi)(1) = 6.3 \text{ m/s}$$

Solve for the velocity of the satalite in km/h

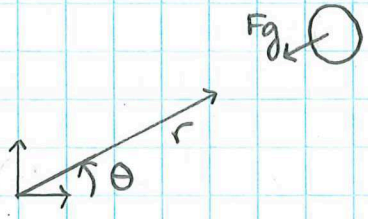


$m_e = 5.97 \times 10^{24}$ kg

$G = 6.67 \times 10^{-11}$ m^3/kgs^2

$V = [km/h]$

FBD:



$$\Sigma F_r = ma_r = -F_g$$
$$\Sigma F_\theta = ma_\theta = 0$$

$$m_s a_r = -\frac{G m_e m_s}{r^2}$$

$$\cancel{m_s}\left(-\frac{V^2}{r}\right) = \frac{G m_e \cancel{m_s}}{r^2}$$

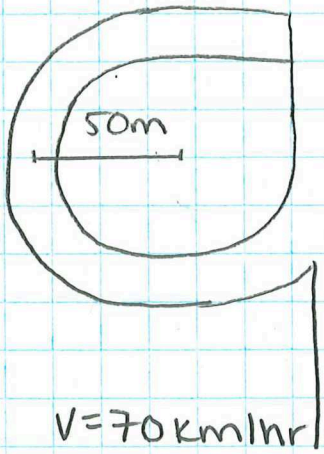
$$a_r = -\omega^2 r = -\frac{V^2}{r}$$

$$V^2 = \frac{G m_e}{r}$$

$$V = \sqrt{\frac{G m_e}{r}} = \sqrt{\frac{[6.67\times10^{-11}]\times[5.97\times10^{24}]}{23000}}$$

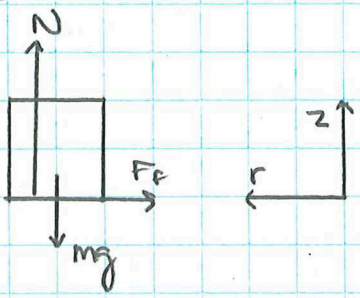
$$= 4160.88 \text{ m/s} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}}$$

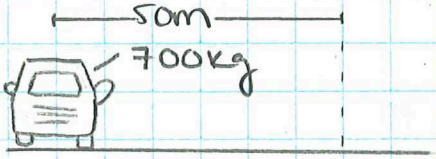
$$= 14979.2 \text{ km/h}$$



min μ_s to not slide

FBD:





Newton's 2nd Law

$\Sigma F_z = ma_z = N - mg$

$\Sigma F_r = ma_r = -F_f$

assume $a_z = 0$ (car not lifting off the ground)

$0 = N - mg$

$N = mg = (700)(9.81) = 6867\ N$

assume it is sliding $F_f = \mu N$

$ma_r = -\mu N$ $\qquad a_r = \frac{-v^2}{r}$

$m\left(\frac{-v^2}{r}\right) = -\mu_s N$

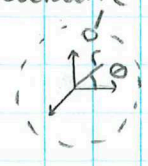
$\mu_s = \frac{mv^2}{Nr} = \frac{mv^2}{[mg]r} = \frac{v^2}{gr}$

$v = 70\,km/hr \times \frac{1000m}{1km} \times \frac{1hr}{3600s}$

$v = 19.4\,m/s$

$\mu_s = 0.77$

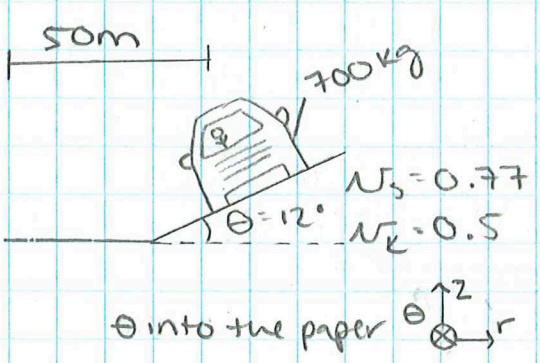
satalite (r, θ, z) $\quad \Sigma \vec{F} = m\vec{a}$ $\quad \Sigma F_r = ma_r$, $\Sigma F_\theta = ma_\theta$, $\Sigma F_z = ma_z$



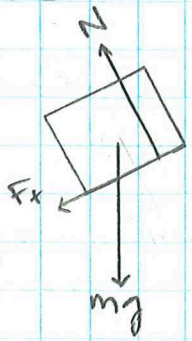
(top view)

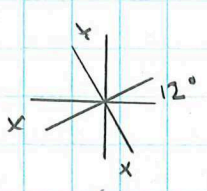
ω = constant

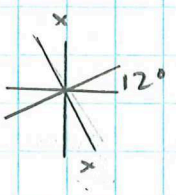
$a_r = -\omega^2 r = \frac{-v^2}{r}$

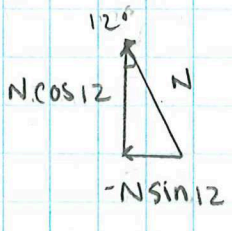


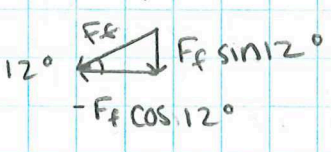
FBD











(1) $\Sigma F_r = ma_r = -N\sin 12 - F_f \cos 12$

$\Sigma F_\theta = ma_\theta = 0$

(2) $\Sigma F_z = ma_z = N\cos 12 - F_f \sin 12 - mg$

(3) Constant circular motion $\therefore a_r = \frac{-v^2}{r}$

(4) assume $a_z = 0$

(5) max. friction $f_f = \nu_s N$

Sub (3), (4) and (5) into (1)

$$m\left(\frac{-v^2}{r}\right) = -N\sin\theta - \nu_s N\cos\theta \quad (1a)$$

sub (3), (4), (5) into (2)

$$\theta = N\cos\theta - \nu_s N\sin\theta - mg$$

so $$N = \frac{mg}{\cos\theta - \nu_s \sin\theta} \quad (2a)$$

March 14.23 (2)

Sub (2a) into (1a)

$$\cancel{m}\left(-\frac{v^2}{r}\right) = \left[\frac{\cancel{m}g}{\cos\theta - \mu_s \sin\theta}\right]\left[-\sin\theta - \mu_s \cos\theta\right]$$

$$v = \sqrt{rg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

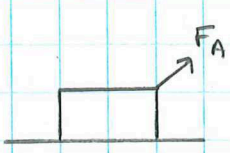
24 m/s

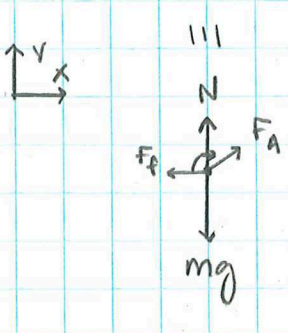
86 km/hr

Moments

Linear motion

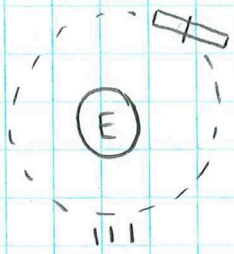
$\Sigma \vec{F} = m\vec{a}$



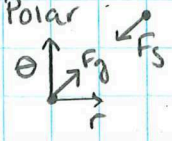


Circular motion

$\Sigma \vec{F} = m\vec{a}$



$a_r = \frac{-v^2}{r}$

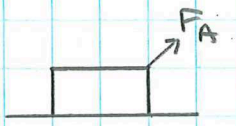


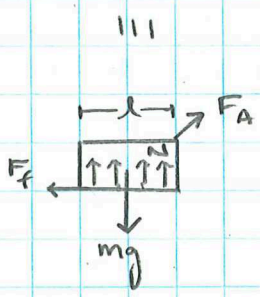
$wr = v$

$\theta r = c$

Object Rotation

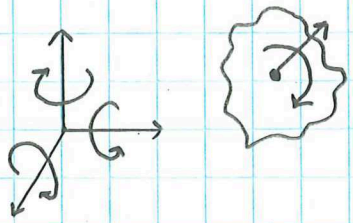
$\Sigma M_o = I_o \alpha_o$





How many equation to describe an object.

6



a_x, a_y, a_z

$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}$

$$\underbrace{\Sigma \vec{F}_{\perp} \cdot r}_{M_o} = \underbrace{\int r^2 \, dm}_{I_o} \cdot \alpha$$

ex. torque wrench

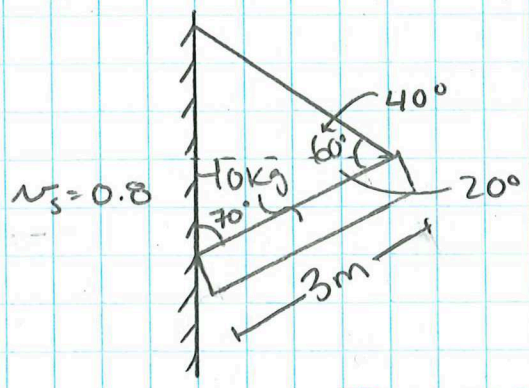
ex. figure skater

change distance

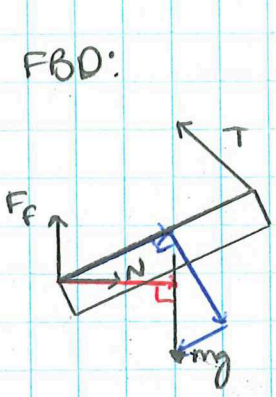
change distribution of mass

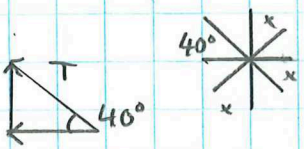
$\Sigma \vec{F} = m\vec{a}$

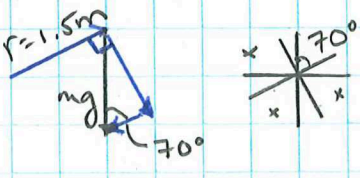
$\Sigma \vec{M}_o = I_o \vec{\alpha}_o$



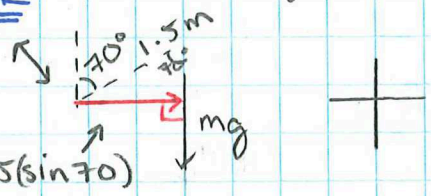
FBD:





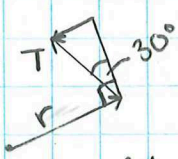


OR



$M_o = (1.5)(mg\sin 70)$

$M_o = [1.5(\sin 70)](mg)$



$M_o = (3)((T\cos(30))$

↑ b/c T is 3m from origin

assume not sliding, not rotating

(1) $\Sigma F_x = m\cancel{a_x}^{0} = N - T\cos(40)$

(2) $\Sigma F_y = m\cancel{a_y}^{0} = F_f + T\sin(40) - mg$

(3) $\Sigma M_o = \cancel{I_o \alpha_o}^{0} = +(1.5)(mg\sin 70) - 3(T\cos 30)$

Solve for T in (3)

$0 = 1.5(mg\sin 70) - 3(T\cos 30)$

$\frac{1.5\, mg \sin(70)}{3\cos(30)} = T$

$T = \frac{1.5(10)(9.81)\sin 70}{3\cos(30)}$

(3a) $T = 53\ N$

Sub 3a into (1)

$0 = N - T\cos 40$

$N = 53\cos 40$

$N = 40.7\ N$ ✓

Sub 3a into (2)

$0 = F_f + T\sin(40) - mg$

$F_f = -53\sin 40 + 10(9.81)$

$F_f = 63.9\ N$. ← force needed to sustain static

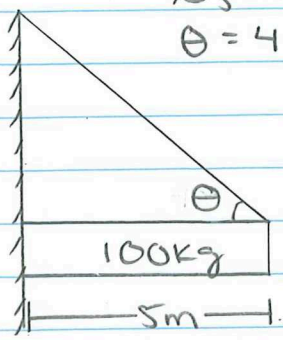
check

$F_{f\,max} = \mu_s N = 0.8(40.7)$

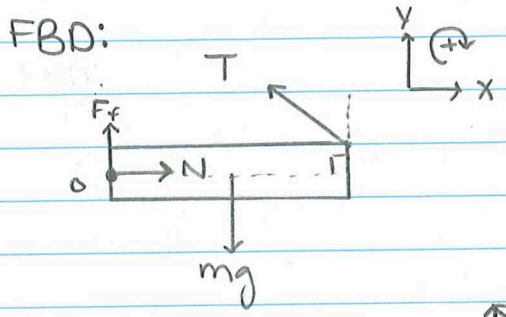
$= 32.6\ N$ ← smaller than F_f

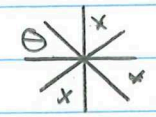
$\mu_s = 0.8$
$\theta = 40°$

Compare f_f with $f_{f\max}$.
Does it slide?



FBD:

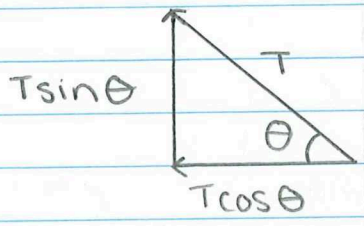




(1) $\sum F_x = m\cancel{a_x}^{0} = N - T\cos\theta$

(2) $\sum F_y = m\cancel{a_y}^{0} = F_f + T\sin\theta - mg$

(3) $\sum M_o = \cancel{I\alpha_o}^{0} = (2.5)(mg) - 5(T\sin\theta)$



@ $\theta = 40°$ solve (3) for T

$0 = 2.5(mg) - 5(T\sin 40)$

$T = \frac{2.5mg}{5\sin 40} = \frac{2.5(100)(9.81)}{5\sin 40}$

$T = 763.08$ (3a)

Sub (3a) into (1)

$0 = N - 763.08\cos 40$

$N = 584.55$ ✓ $a_x = 0$

Sub (3a) into (2)

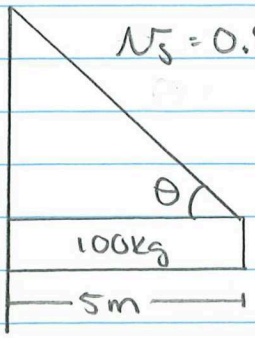
$0 = F_f + 763.08\sin 40 - (100)9.81$

$F_f = 490.5\,N$

Check $f_{f\max}$.

$F_{f\max} = \mu_s N = 0.8(584.55)$

$= 467.6\,N$ so the sign falls.



$\Sigma F_x = ma_x = N - T\cos\theta$
$\Sigma F_y = ma_y = F_f + T\sin\theta - mg$
$\Sigma M_o = I_o\alpha_o = 2.5mg - 5T\sin\theta$

$F_f max = N_S N = 0.8(584.55) = 467.6N$
Set $f_f = N_S N$ sub (4) into (2)
$0 = [N_S N] + T\sin\theta - mg$

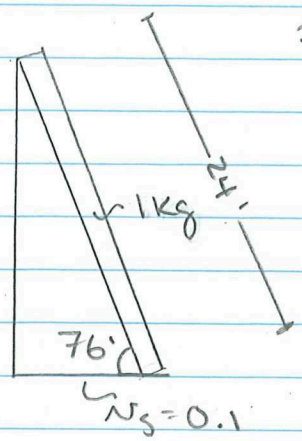
sub (1) into (2a) $\quad 0 = T(N_S\cos\theta + \sin\theta) - mg$
$0 = N_S[T\cos\theta] + T\sin\theta - mg$

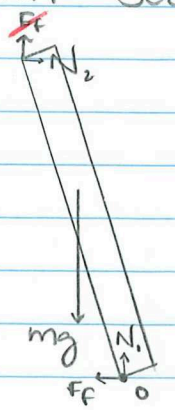
sub (3) into (2b)
$0 = \left[\frac{2.5mg}{5\sin\theta}\right](N_S\cos\theta + \sin\theta) - mg$
$0 = \frac{2.5mg}{5}\left(\frac{N_S\cos\theta}{\sin\theta} + 1\right) - mg \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$
$0 = \frac{2.5mg}{5}\left(\frac{N_S}{\tan\theta} + 1\right) - mg$
$\frac{N_S}{\tan\theta} + 1 = \frac{5mg}{2.5mg}$
$\frac{N_S}{\tan\theta} = 1$
$\tan\theta = N_S$

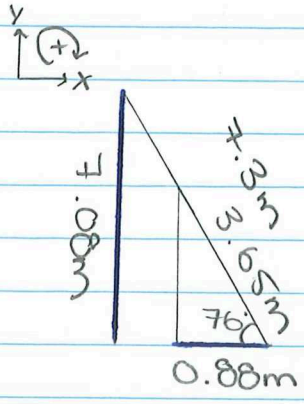
$\theta = \tan^{-1}(0.8) = 38.65°$

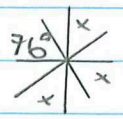
$24' \div 3.3 = 7.3\text{m}$

$3.3' = 1\text{m}$ Does it slide?





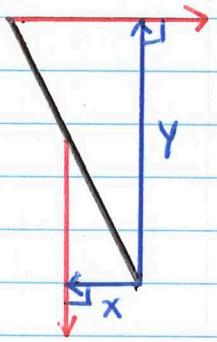




$\sum F_x = m\cancel{a_x}^{0} = N_2 - F_f$

$\sum F_y = m\cancel{a_y}^{0} = N_1 - mg$

$\sum M_o = \cancel{I_o \alpha_o}^{0} = -0.88\,mg + 7.1\,N_2$



Solve (3) for N_2

$0 = -0.88\,mg + 7.1\,N_2$

$N_2 = \dfrac{0.88(1)(9.81)}{7.1} = 1.22\,N$

Sub (3a) into (1)

$F_f = N_2 = 1.22\,N$

Solve (2) for N_1

$N_1 = mg = (1)(9.81) = 9.81\,N$

Check $f_{p\,max}$

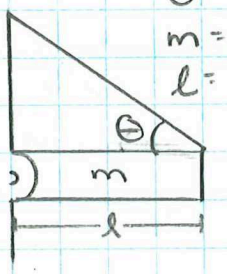
$F_{f\,max} = \mu_s N_1 = 0.1(9.81) = 0.981\,N \rightarrow$ slides down.

What does μ_s have to be to not move?

$F_f = 1.22 = \mu_s N_1$

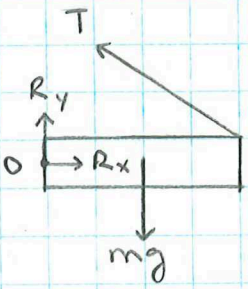
$\mu_s = \dfrac{1.22}{9.81} = 0.12$ Needs to be 0.12 to not move.

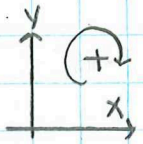
March 21.23

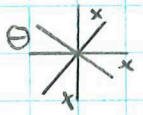


$\theta = 40°$
$m = 100kg$
$l = 5m$

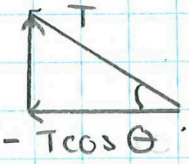
FBD







$+ T\sin\theta$

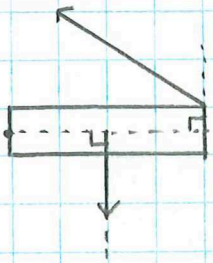


$-T\cos\theta$

(1) $\sum F_x = \cancel{ma_x}^{\,0} = R_x - T\cos\theta$

(2) $\sum F_y = \cancel{ma_y}^{\,0} = T\sin\theta + R_y - mg$

(3) $\sum M_o = \cancel{I_o\alpha}^{\,0}_{\,o} = mg\left(\frac{l}{2}\right) - T\sin\theta\,(l)$



solve (3) for T

$$0 = mg\left(\frac{l}{2}\right) - T\sin\theta\,(l)$$

$$0 = \frac{mg}{2} - T\sin\theta$$

$$T = \frac{mg}{2\sin\theta} = 763\,N \quad \checkmark \text{ confirm acceleration is zero.}$$

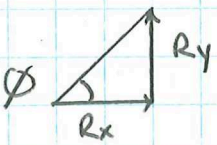
Sub (3a) into (1)

$$R_x = \frac{mg\cos\theta}{2\sin\theta} = 585\,N$$

Sub (3a) into (2)

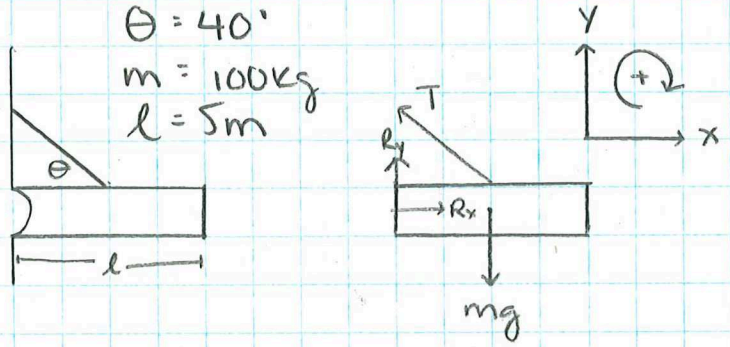
$$R_y = mg - \frac{mg\sin\theta}{2\sin\theta} = \frac{mg}{2} = 490.5\,N$$

Reaction force (NOt on test)



$$\emptyset = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 39.97°$$

March 21 23 (2)



$$\Sigma F_x = ma_x = R_x - T\cos\theta$$
$$\Sigma F_y = ma_y = R_y - T\sin\theta - mg$$
$$\Sigma M_0 = I_0\alpha_0 = mg\left(\frac{l}{2}\right) - T\sin\theta\left(\frac{l}{2}\right)$$

Solve (3) for T:

$$T = \frac{mg}{\sin\theta} = 1526\,N$$

Sub (3a) into (1)

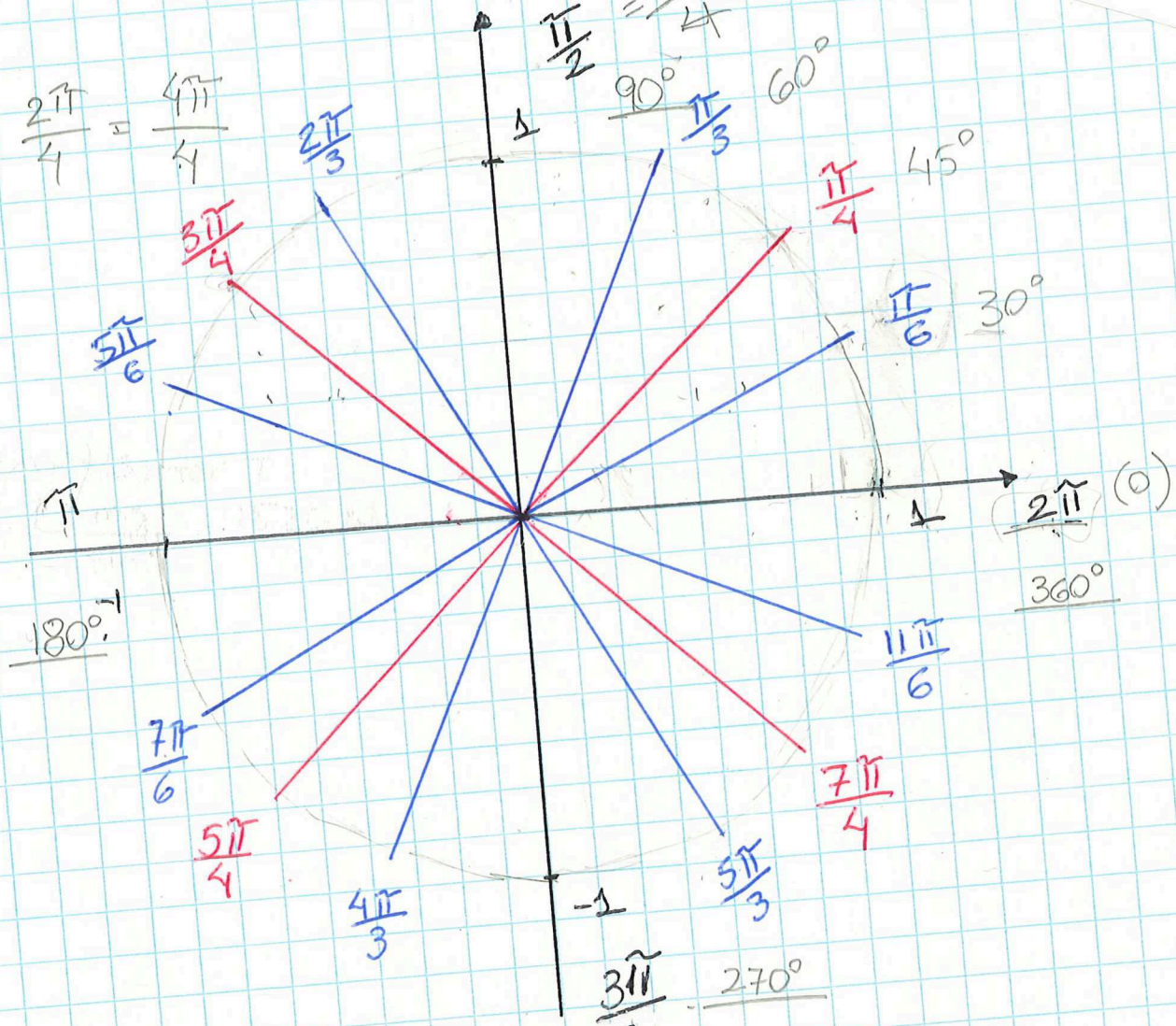
$$R_x = \frac{mg\cos\theta}{\sin\theta} = 1170\,N$$

Sub (3a) into (2)

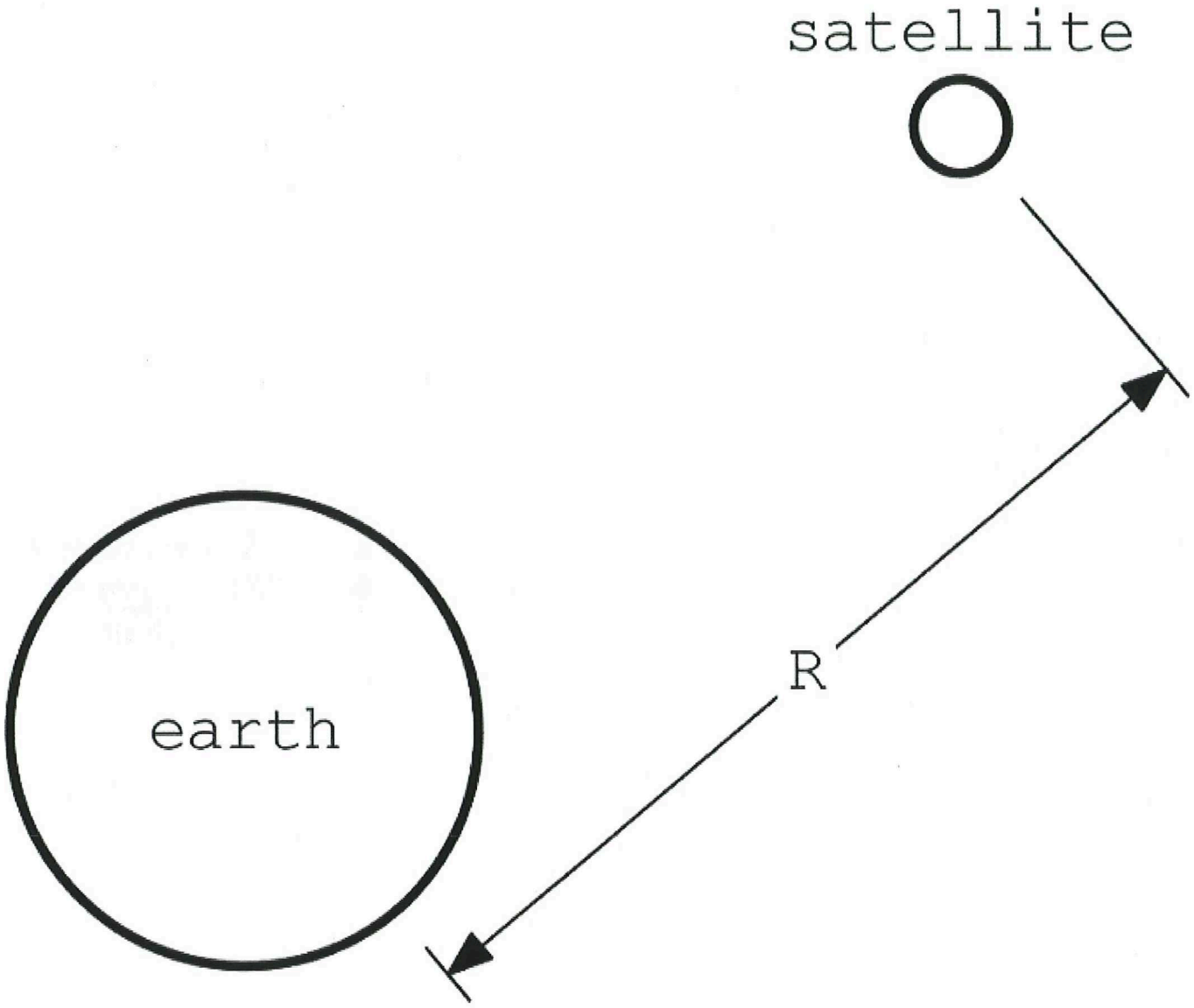
$$R_y = mg - T\sin\theta$$
$$= mg - \left(\frac{mg}{\sin\theta}\right)\sin\theta$$
$$= 0$$

Ex 1

$\frac{2\pi}{4} + \frac{2\pi}{4} = \frac{4\pi}{4}$



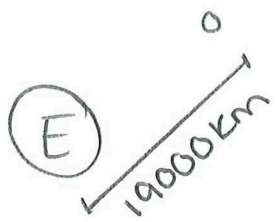
$\frac{2\pi}{4} + \frac{2\pi}{4} + \frac{2\pi}{4} = \frac{6\pi}{4}$



Solve for the velocity of the satellite in km/h to 0 decimal places, given:

mass of earth=5.970e+24kg, R=19000km, and G=6.670e-11m^3/kgs^2

Solve for the velocity of the satalite



mass earth = 5.970×10^{24} kg

$G = 6.670 \times 10^{-11}\ m^3/kgs^2$

FBD:



$\Sigma F_r = ma_r = -FG$

$\Sigma F_\theta = ma_\theta = 0$

$a_r = \frac{-v^2}{r}$

$m_s a_r = -FG$

$m_s\left(\frac{-v^2}{r}\right) = -\frac{Gme\,m_s}{r^2}$

$-FG = -\frac{Gmm}{r^2}$

Change R to m

$19000\,km \times \frac{1000m}{1\,km} = 19,000,000m$

$v^2 = \frac{Gm_e}{r}$

$v = \sqrt{\frac{Gm_e}{r}}$

$v = \sqrt{\frac{6.670 \times 10^{-11} \times 5.970 \times 10^{24}}{19,000,000}}$

$v = 4577.97$ m/s

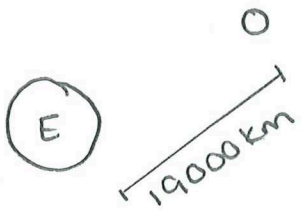
Convert to km/hr

$4577.97\frac{m}{s} \times \frac{1km}{1000m} \times \frac{60s}{1min} \times \frac{60min}{1hr} = 16480.7$ ✓

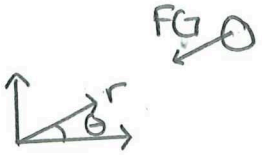
Solve for the velocity of the satalite.

mass Earth = 5.970×10^{24} kg

$G = 6.670 \times 10^{-11}\ m^3/kgs^2$



FBD:



$\Sigma F_r = ma_r = -FG$

$\Sigma F_\theta = ma_\theta = 0$

$a_r = \frac{-v^2}{r}$

$-FG = -\frac{Gmm}{r^2}$

$m_s a_r = -FG$

$m_s\left(\frac{-v^2}{r}\right) = -Gm_e m_s$

$\frac{-v^2}{r} = \frac{-Gm_e}{r^2}$

$v^2 = \frac{Gm_e}{r}$

$v = \sqrt{\frac{Gm_e}{r}}$

$v = \sqrt{\frac{6.670 \times 10^{-11} \cdot 5.970 \times 10^{24}}{19000kg}}$

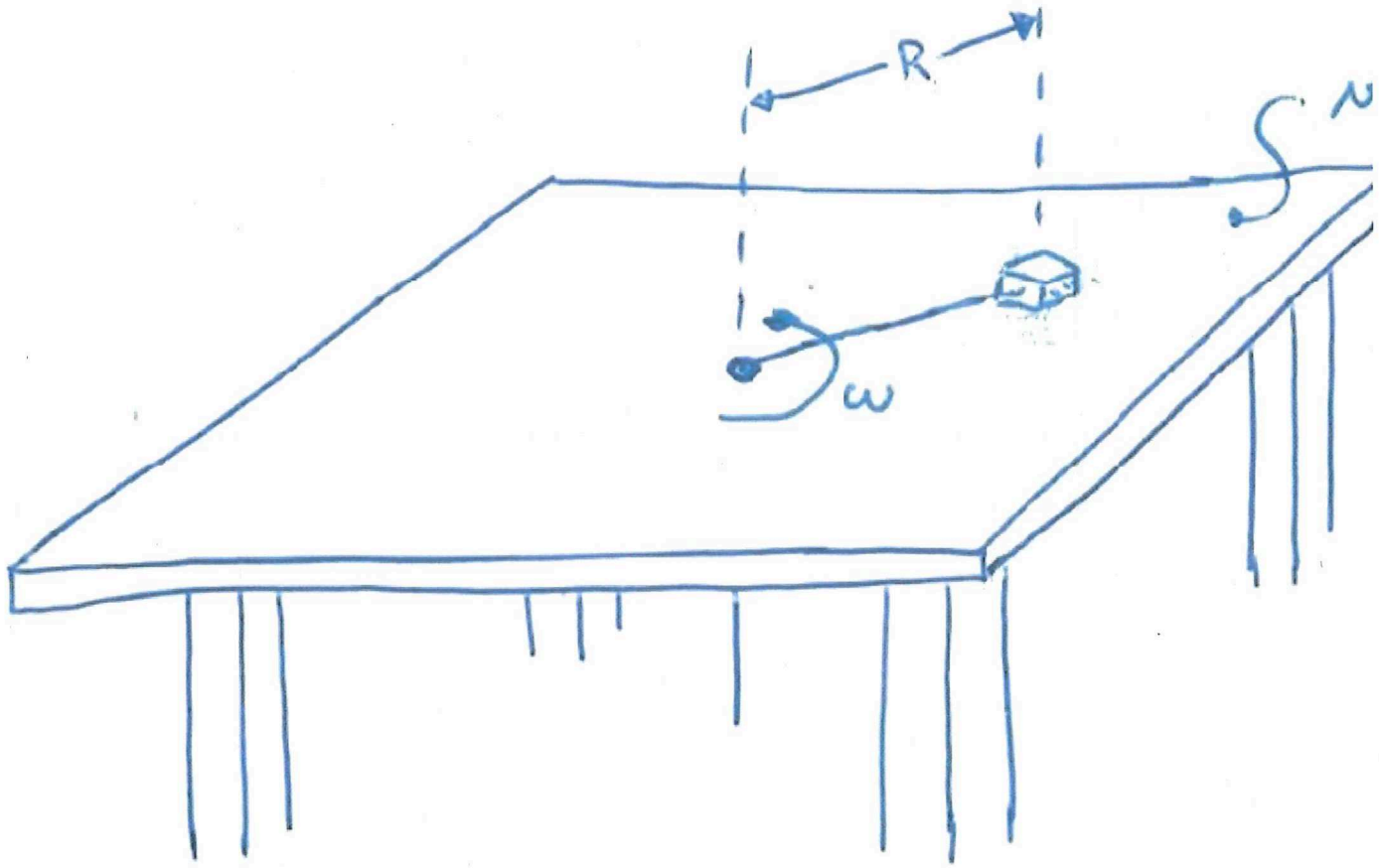
$v = \sqrt{2.0957 \times 10^{10}}$

$v = \sqrt{20957842110}$

144768 X

A = 16481

Question 2



Solve for the tension in the attached rope : [Newtons] to 1 decimal place, given:

$m=4\,kg$, $R=2.0\,m$, $\omega=40\,rpm$,

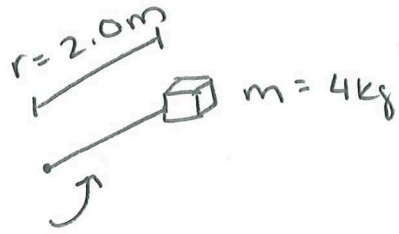
$mu_s=0.3$ and $mu_k=0.2$

Selected Answer: 14.2

Correct Answer: 140.4

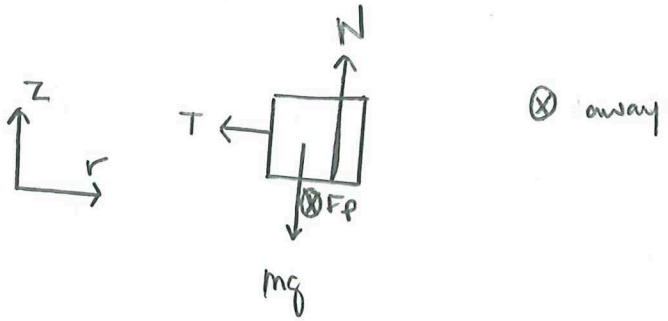
Answer range +/- 0.2 (140.2 - 140.6)

Solve for Tension in the attached rope.



$\mu_s = 0.3$

$\mu_k = 0.2$



$\Sigma F_r = ma_r = -T$

$\Sigma F_\theta = ma_\theta = +F_f$

$\Sigma F_z = ma_z = N - mg$

$\omega = 1 \frac{\text{cycle}}{\text{min}}$ → Convert to $\frac{\text{rads}}{\text{sec}}$

$$\omega = 40\frac{\text{cycle}}{\text{min}}\left(\frac{2\pi\text{rads}}{1\text{cycle}}\right)\left(\frac{1\text{min}}{60\text{s}}\right)$$

$\omega = 1.3\overline{3} \frac{\text{rads}}{\text{sec.}}$

$$a_r = -\omega^2 r = \frac{-v^2}{r}$$

$ma_r = -T$

$T = -ma_r$

$= -m[-\omega^2 r]$

$= m\omega^2 r$

$= 4(1.33)^2 2$

$= 14.22\,N$

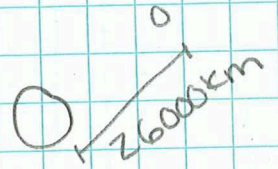
A = 140.4

Quiz

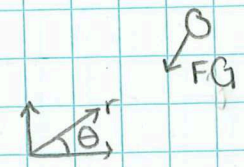
Solve for the velocity of the satalite

$m_e = 5.970 \times 10^{24}\ kg$

$G = 6.670 \times 10^{-11}\ m^3/kg\,s^2$



FBD:



$\Sigma F_r = ma_r = -FG$

$\Sigma F_\theta = ma_\theta = 0$

$m_s a_r = -FG$

$-FG = -\frac{Gmm}{r^2}$

$m_s\left(\frac{-v^2}{r}\right) = -\frac{Gm_e m_s}{r^2}$

$a_r = \frac{-v^2}{r}$

$v^2 = \frac{Gm_e}{r}$

$26000\,km \times \frac{1000\,m}{1\,km} = 26{,}000{,}000\,m$

$v = \sqrt{\frac{6.670\times10^{-11} \cdot 5.970\times10^{24}}{26{,}000{,}000}}$

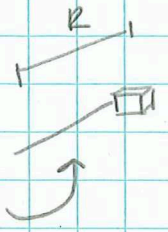
$v = 3913.482612\ m/s$

Convert to km/hr

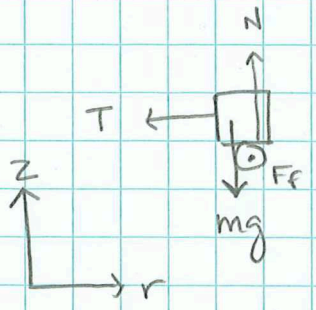
$3913.48\ \frac{m}{s}\left(\frac{1}{1000m}\right)\left(\frac{60s}{1min}\right)\left(\frac{60m}{1hr}\right) = 14089\ km/h$ ✓

Solve for the tension

Quiz



$m = 3\,kg$
$R = 2.0\,m$
$\omega = 35\,rpm$



$\mu_s = 0.3$
$\mu_k = 0.2$

$\Sigma F_r = ma_r = -T$
$\Sigma F_\theta = ma_\theta = -F_f$
$\Sigma F_z = ma_z = N - mg$

$\omega = \frac{1\ cycle}{min} \rightarrow \frac{rads}{sec.}$

$\omega = 35\ \frac{cycle}{min}\left(\frac{2\pi\ rads}{1\ cycle}\right)\left(\frac{1\ min}{60\ s}\right)$

$= 3.665\ \frac{rads}{s}$

$a_r = -\omega^2 r = -\frac{v^2}{r}$

$ma_r = -T$

$m(-\omega^2 r) = -T$

$T = -m(-\omega^2 r)$

$= m\omega^2 r$

$= 3(3.665)^2(2)$

$T = 80.6\ N$ ✓

$\Sigma F_x = ma_x$ $\begin{cases} a_x = 0 \\ a_y \neq 0 \end{cases}$

$\Sigma F_y = ma_y$

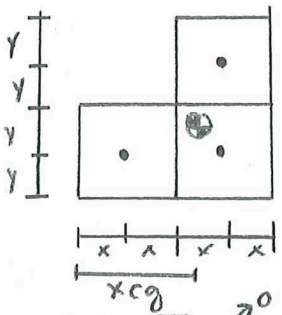
$\Sigma F_r = ma_r$

$\Sigma F_\theta = ma_\theta \qquad a_r = \omega^2 r$

$\Sigma M_0 = I_0 \alpha_0$

$\alpha = 0 \quad \alpha \neq 0$

Centre of Mass



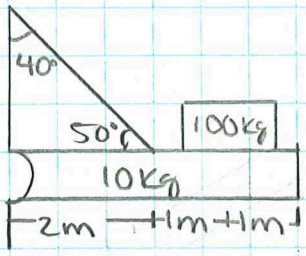
x_{cg} = center of geometry.

$\Sigma M_0 = I_0 \cancel{\alpha_0}^{0} = mg(x) + mg(3x) + mg(3x)$

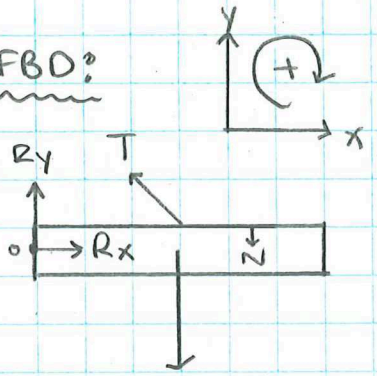
$= mg(x_i)$

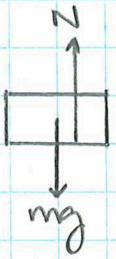
$= Mx_{cg}$

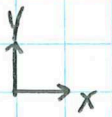
$$\vec{x}_{cg} = \frac{\Sigma m_i \vec{x}_i}{\Sigma m_i}$$



FBD:







(1) $\sum F_x = m\cancel{a_x^{\rightarrow 0}} = 0$

(2) $\sum F_y = m\cancel{a_y^{\rightarrow 0}} = N - mg$

(3) $\sum F_x = m\cancel{a_x^{\rightarrow 0}} = Rx - T\cos 50$

(4) $\sum F_y = m\cancel{a_y^{\rightarrow 0}} = R_y + T\sin 50 - Mg - N$

(5) $\sum M_o = \cancel{I\alpha^{\rightarrow 0}} = Mg(2) + N(3) - T\sin 50(2)$

(2) $0 = N - mg$

$N = mg$

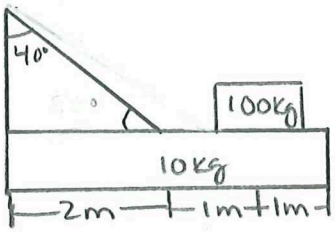
(5) $0 = Mg(2) + N(3) - T\sin 50(2)$

$T\sin 50(2) = Mg(2) + N(3)$

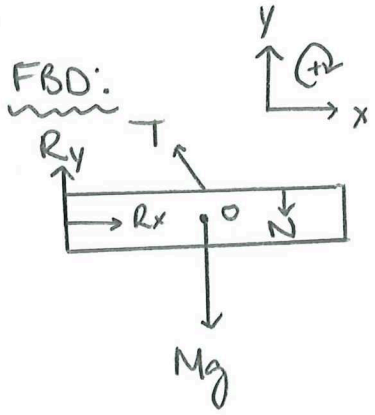
$$T = \frac{Mg(2) + N(3)}{\sin 50(2)}$$

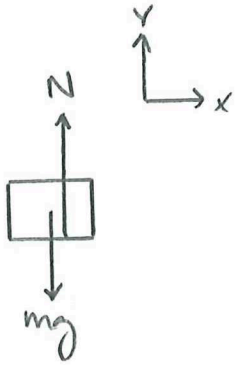
(4) $0 = R_y + T\sin 50 - Mg - N$

$R_y = Mg + N - T\sin 50$



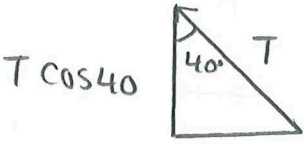
FBD:





(1) $\Sigma F_x = m\vec{a}_x^{\,0} = 0$

(2) $\Sigma F_y = m\vec{a}_y^{\,0} = N - mg$



(3) $\Sigma F_x = m\vec{a}_x^{\,0} = R_x - T\sin 40$

(4) $\Sigma F_y = m\vec{a}_y^{\,0} = R_y + T\cos 40 - N - Mg$

(5) $\Sigma M_O = I_O\vec{\alpha}^{\,0} = R_y(2) + (N)(1)$

solve for N in (2)

(2) $N = mg = 981\ N$

solve for Ry in (4)

(5) $0 = R_y(2) + N(1)$

$R_y = \frac{-N}{2} = -490.5\ N$

solve for T in (5)

(4) $0 = R_y + T\cos 40 - N - Mg$

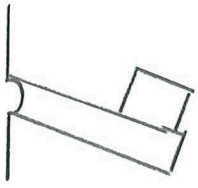
$T = \frac{N + Mg - R_y}{\cos 40} = 2048.97\ N$

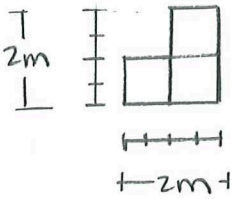
Solve for R_x in (3)

(3) $0 = R_x - T\sin 40$

$R_x = T\sin 40 = 1317\ N$

$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = 1405\ N$



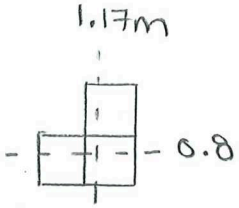


$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{(m)(0.5) + (m)(1.5)}{3m}$$

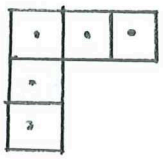
$$= 1.17$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i}$$



$$= \frac{(m)(0.5)(2) + (m)(1.5)}{m+m+m}$$

$$= 0.83$$



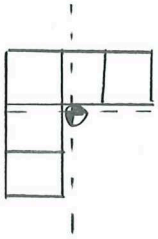
$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{(m)(0.5)(3) + (m)(1.5) + (m)(2.5)}{5m}$$

$$= \frac{1.5 + 1.5 + 2.5}{5}$$

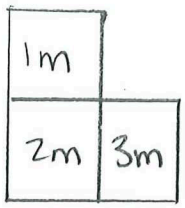
$$= 1.1$$

$$y_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$



$$= \frac{m(0.5) + m(1.5) + m(2.5)(3)}{5m}$$

$$= 1.9$$



$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

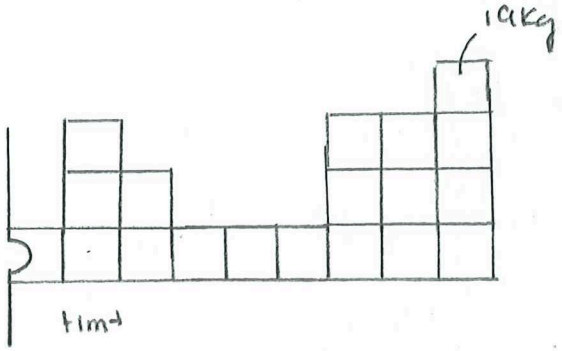
$$= \frac{(1m+2m)(0.5) + (3m)(1.5)}{6m}$$

$$= 1$$

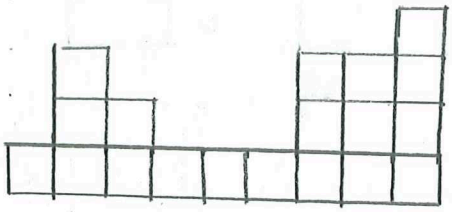
$$y_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

$$= \frac{(2m+3m)(0.5) + (m)(1.5)}{6m}$$

$$= 0.66$$

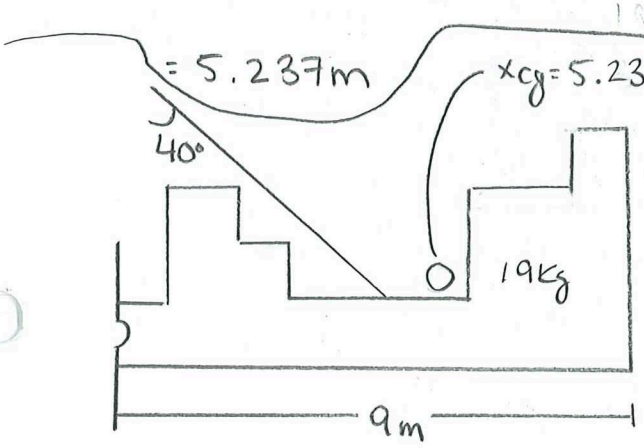


$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma x_i}$$

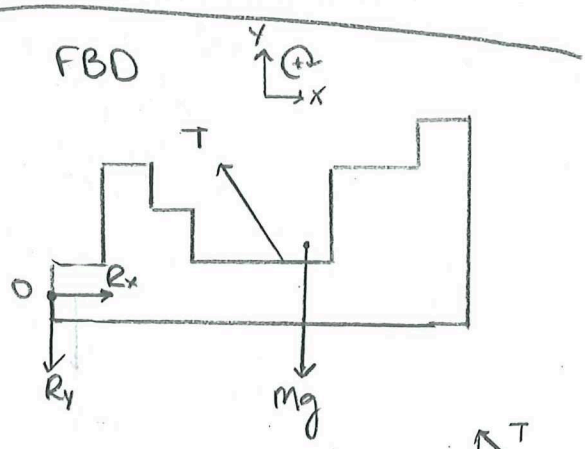


$$= \frac{(0.5)+(3)(1.5)+(2)(2.5)+(3.5)+(4.5)+(5.5)+(3)(6.5+7.5)+(4)(8.5)}{19}$$

$= 5.237\,m$



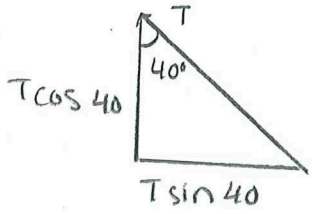
FBD



$\Sigma F_x = ma_x^{\to 0} = R_x - T\sin 40$

$\Sigma F_y = ma_y^{\to 0} = T\cos 40 - R_y - Mg$

$\Sigma M_o = I_o\alpha_o^{\to 0} = Mg(5.237) - T\cos 40(4.5)$



$0 = Mg(5.237) - T\cos 40(4.5)$

$T = \frac{Mg(5.237)}{\cos 40(4.5)} = \frac{19(9.81)(5.237)}{\cos(40)(4.5)} = 283.16\,N$

$0 = R_x - T\sin 40$

$R_x = T\sin 40 = 182\,N$

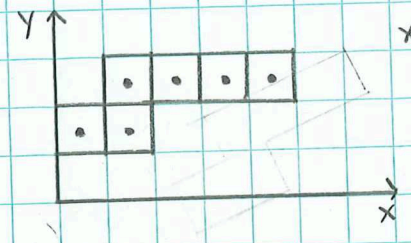
$0 = T\cos 40 - R_y - Mg$

$R_y = T\cos 40 - Mg = 30.5\,N$

$|\vec{R}| = \sqrt{(30.5)^2 + (182)^2} = 184.5\,N$

Mar. 24. 23 ①

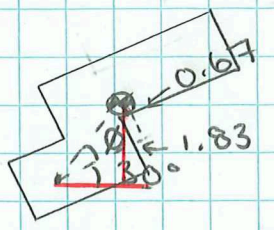
$$\vec{x}_{cg} = \sum m_i \vec{x}_i$$



$$x_{cg} = \frac{m(0.5) + 2m(1.5) + m(2.5) + m(3.5) + m(4.5)}{6m}$$

$$= \frac{14}{6} = 2.33$$

Redo. Mar. 24. 23 (2)

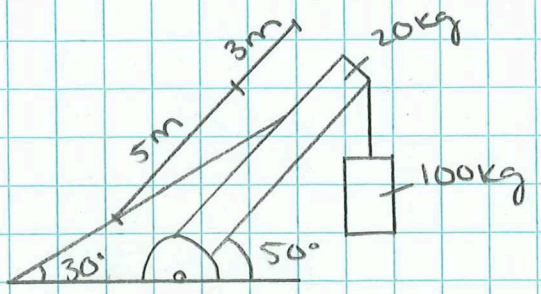


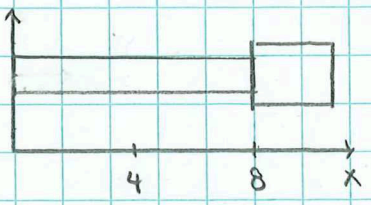
$$\sqrt{(0.67)^2 + (1.83)^2} = 1.949$$

$$\emptyset = \tan^{-1}\left(\frac{0.67}{1.83}\right) = 20.1°$$

$$\emptyset_2 = 30° + 20.1° = 50.1°$$

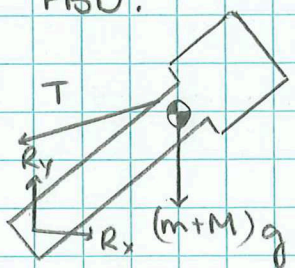
Determine $|\vec{R}|$

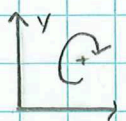




$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{(20)(4) + (100)(8)}{120} = 7.\overline{3}$$

FBD:





(1) $\Sigma F_x = m\vec{a}_x^{\,0} = R_x - T\cos 30$

(2) $\Sigma F_y = m\vec{a}_y^{\,0} = R_y - T\sin 30 - (m+M)g$

(3) $\Sigma M_0 = I_0\vec{\alpha}^{\,0} = (m+M)g\sin 40(7.\overline{3}) - T\sin(20)(5)$

Assume not moving: solve (3) for T

$$0 = (m+M)g\sin 40(7.3) - T\sin(20)(5)$$

$$T = \frac{(m+M)g\sin 40(7.3)}{\sin(20)(5)}$$

(3a) $$T = \frac{(120)(9.81)\sin(40)(7.3)}{\sin(20)(5)} = 3230.12\ N$$

Sub (3a) into (1)

$0 = R_x - T\cos 30$

$R_x = 3230.12(\cos 30)$

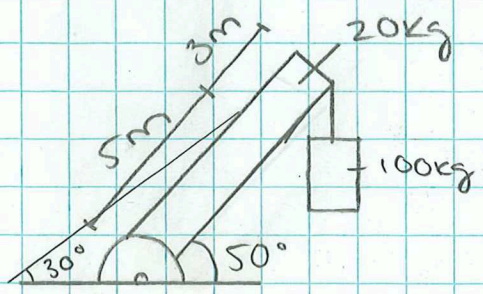
$R_x = 2797.37\ N$

Sub (3a) into (2)

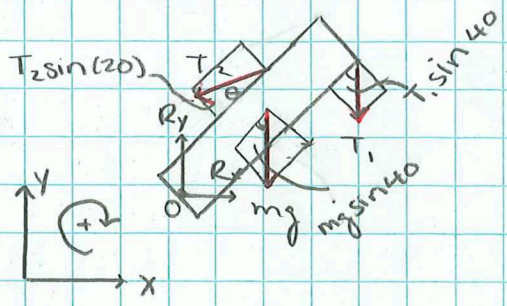
$0 = R_y - T\sin(30) - (m+M)g$

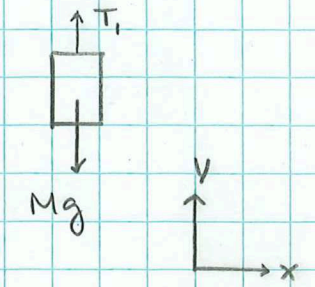
$R_y = 3230.12\sin(30) + (120)(9.81)$

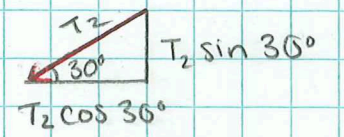
$R_y = 2792.26\ N$



FBD:





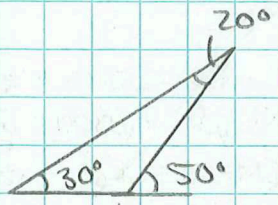


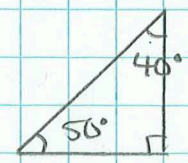
(4) $\sum F_y = M\cancel{a_y}^{0} = T_1 = Mg$

(1) $\sum F_x = ma_x = R_x - T_2 \cos 30$

(2) $\sum F_y = ma_y = R_y = -T_2 \sin 30 - T_1 - mg$

(3) $\sum M_o = I_o \alpha_o = mg \sin(40)(4) + T_1 \sin(40)(8) - T_2 \sin(20)(5)$





Solve (4) for T_1

$T_1 = Mg = 981\ N$

Sub (4a) into (3)

$$T_2 = \frac{mg \sin(40)(4) + (981)(\sin(40))(8)}{5 \sin(20)} = 3245\ N$$

Sub (3a) into (1)

$R_x = T_2 \cos(30) = 2810\ N$

Sub (3a) into (2)

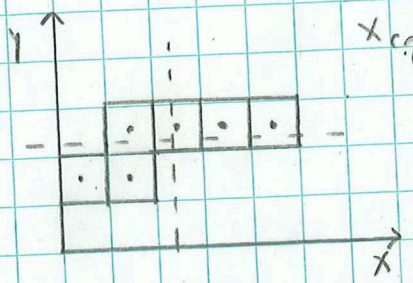
$R_y = T_1 + T_2 \sin 30 + mg = 2800\ N$

$|\vec{R}| = 3970\ N$

Redo

Mar. 24. 23 ①

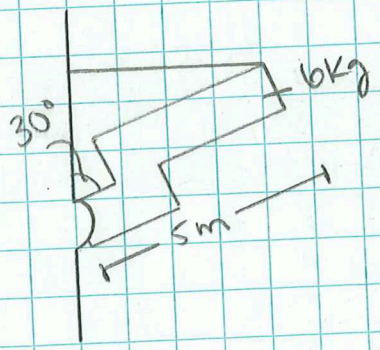
$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

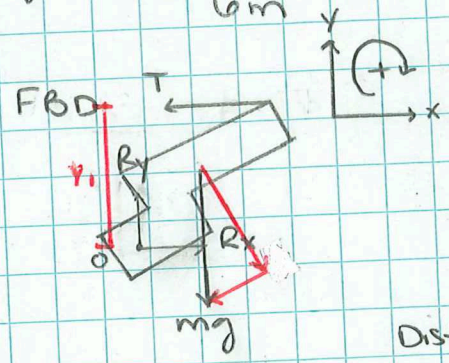


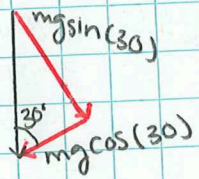
$$x_{cg} = \frac{m(0.5) + 2m(1.5) + m(2.5) + m(3.5) + m(4.5)}{6m}$$

$$= \frac{14}{6} = 2.33$$

$$y_{cg} = \frac{2m(1.5) + 4m(2.5)}{6m} = \frac{13}{6} = 2.17$$







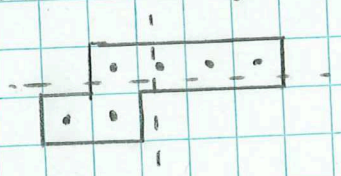
(1) $\sum F_x = ma_x = R_x - T$

(2) $\sum F_y = ma_y = R_y - mg$

(3) $\sum M_o = I_o \alpha_o = mg\sin(30)(1.83) - mg\cos(30)(0.67) - T(3.55)$

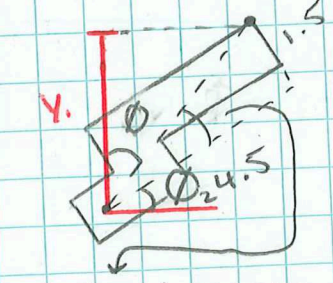
(3) Solve

Distance from origin



$2.33 - 0.5 = 1.83$

$2.17 - 1.5 = 0.67$



$$\tan^{-1}\left(\frac{1.5}{4.5}\right) = 18.4°$$

$\phi = 30° + 18.4° = 48.4°$

$\phi_2 = 90° - \phi = 41.56°$

$y_1 = 4.74\cos(41.56) = 3.5$

$\Sigma M_0 = I_0 \alpha_0 =$

$\downarrow$

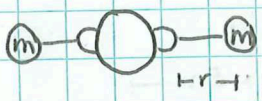
$I = \int r^2 dm$

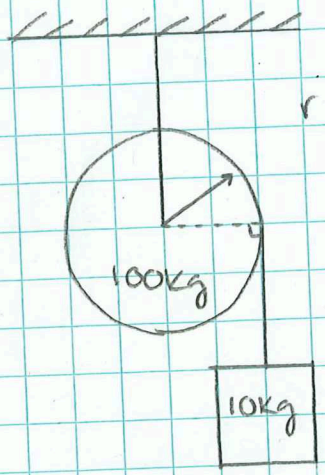
$= \Sigma m_i r_i^2$

$\downarrow$

$F_{\perp} \cdot l_0$



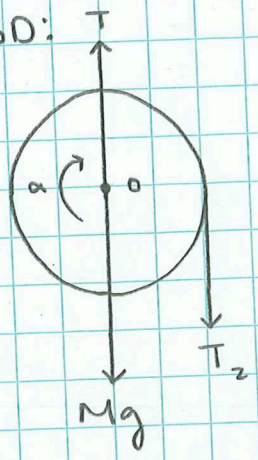


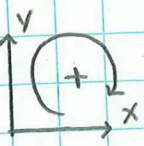


what is the block's acceleration? Mar. 29. 23 ②

r = 0.5

FBD:







Pulley

(1) $\Sigma F_y = \cancel{Ma_{y/M}}^{0} \quad T - Mg - T_2$

(2) $\Sigma M_0 = I_0 \alpha_0 = T_2 (r)$

Mass

(3) $\Sigma F_y = ma_{ym} = T_2 - mg$

Kinematic

(4) $-a_y = \alpha r$

$\alpha = -\frac{a_y}{r}$

solve for T_2 in (3)

$ma_y = T_2 - mg$

$T_2 = ma_y + mg$

sub (3a) into (2)

$I_0 \alpha_0 = T_2 (r)$

$I_0 \alpha_0 = [ma_y + mg](r)$

$\left(\frac{1}{2} Mr^2\right) \alpha_0 = [ma_y + mg] r$

solve for α_0

$I_0 = I_{zz} = \frac{1}{2} Mr^2$

$= \frac{1}{2} (100)(0.5)^2$

$= 12.5$

sub (4) into (2a)

$\left(\frac{1}{2} M \cancel{r^2}\right)\left[\frac{-a_y}{\cancel{r}}\right] = ma_y \cancel{r} + mg \cancel{r}$

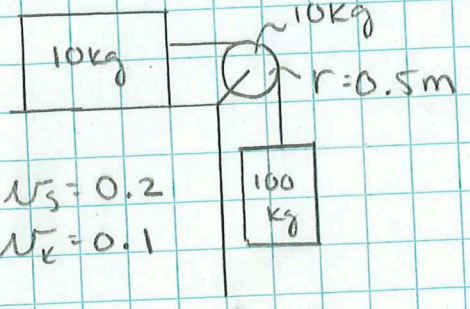
$-\frac{Ma_y}{2} = ma_y + mg$

$-\frac{Ma_y}{2} - ma_y = mg$

$-a_y \left[\frac{M}{2} + m\right] = mg$

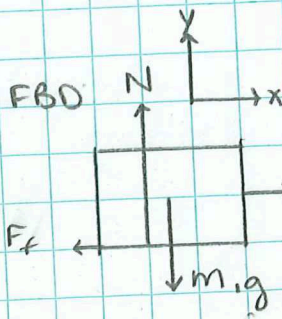
$a_y = \frac{-mg}{\frac{M}{2} + m} = \frac{(-10)(9.81)}{\frac{100}{2} + 10} = -1.6 \text{ m/s}^2$

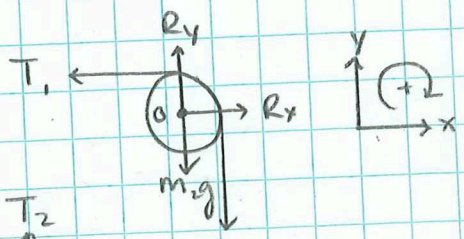
neg b/c coordinate system

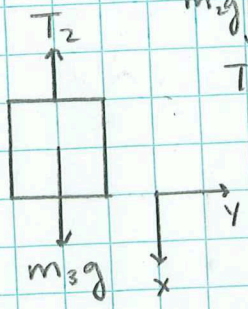


$\mu_s = 0.2$

$\mu_k = 0.1$







(1) $\sum F_x = m_1 a_{x_1} = T_1 - F_f$

(2) $\sum F_y = m_1 a_{y_1}^{\,0} = N - m_1 g$

(3) $\sum F_x = m_2 a_{x_2}^{\,0} = R_x - T_1$

(4) $\sum F_y = m_2 a_{y_2}^{\,0} = R_y - m_2 g - T_2$

(5) $\sum M_o = I_o \alpha_o = T_2(r) - T_1(r)$

(6) $\sum F_x = m_3 a_{x_3} = m_3 g - T_2$

(7) moving $F_f = \mu N$

kinematics

(8) $a_{x_1} = a_{x_3} = a_x$

(9) $a_x = \alpha_o r$

(10) $I_o = I_{zz} = \frac{1}{2} m r^2$

solve (2) for $N \rightarrow N = m_1 g = (10)(9.81) = 98.1\,N$ ✓ $(a_{y_1} = 0)$

solve (1) for $T_1 \rightarrow T_1 = m_1 a_x + F_f \rightarrow T_1 = m_1 a_x + \mu N$

solve (6) for $T_2 \rightarrow T_2 = m_3 g - m_3 a_x$

sub (1a), (6a) & (9) into (5) $\rightarrow I_o\left[\frac{a_x}{r}\right] = [m_3 g - m_3 a_x] r - [m_1 a_x + \mu N] r$

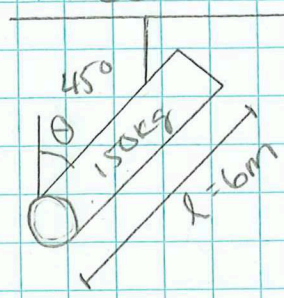
$$I_o[a_x] = m_3 g r^2 - m_3 a_x r^2 - m_1 a_x r^2 - \mu N r^2$$

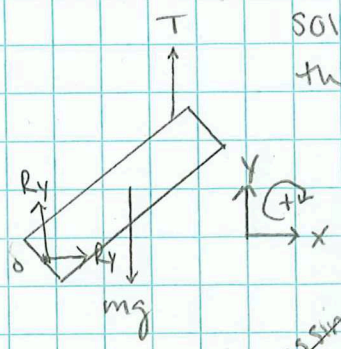
sub (10) into above eqn $\left[\frac{1}{2} m_2 r^2\right] a_x = m_3 g r^2 - m_3 a_x r^2 - m_1 a_x r^2 - \mu N r^2$

$$\left[\frac{1}{2} m_2\right] a_x + m_3 a_x + m_1 a_x = m_3 g - \mu N \rightarrow a_x \left[\frac{1}{2} m_2 + m_3 + m_1\right] = m_3 g - \mu N$$

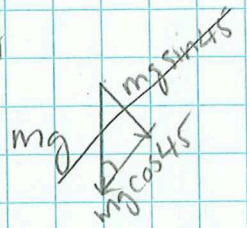
$$a_x = \frac{m_3 g - \mu N}{m_1 + \frac{m_2}{2} + m_3} \rightarrow a_x = \frac{m_3 g - \mu m_1 g}{m_1 + \frac{m_2}{2} + m_3} \rightarrow a_x = \left(\frac{m_3 - \mu m_1}{m_1 + \frac{m_2}{2} + m_3}\right) g$$

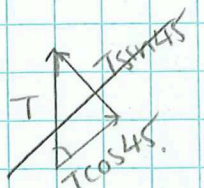
Quick Moments





Solve for the magnitude of the reaction force





$\Sigma F_x = ma_x^{\,0} = R_x$

$\Sigma F_y = ma_y^{\,0} = R_y + T - mg$

$\Sigma M_O = I_O\alpha_O^{\,0} = T(6) + mg(3)$

$0 = T(6) - mg(3)$

$T = \frac{mg(3)}{6}$

$T = \frac{150(9.81)(3)}{6} = 735.75\,N$

$0 = R_x = 0$

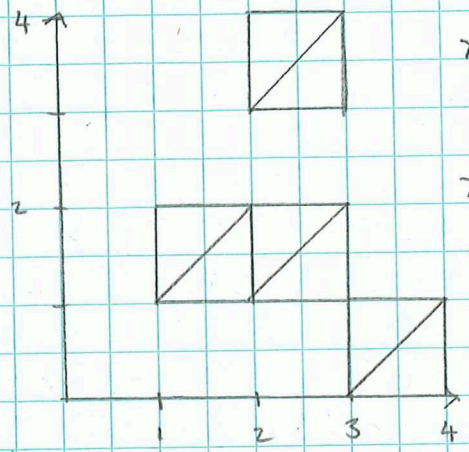
$0 = R_y + 735.75 - mg$

$R_y = (150)(9.81) - 735.75$

$R_y = 735.75\,N$ ✓

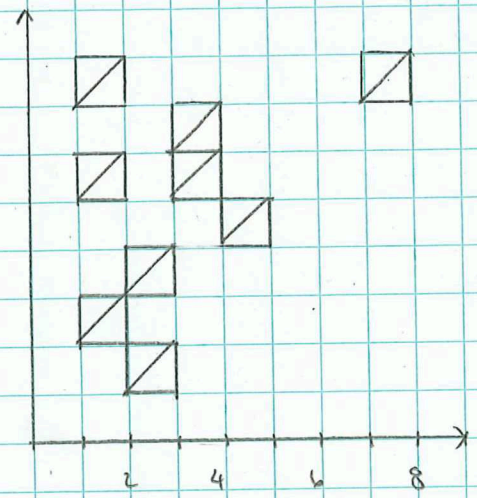
Quiz center of gravity

calculate x_{cg}

① 

$$x_{cg} = \frac{1m(1.5) + 2m(2.5) + 1m(3.5)}{4m}$$

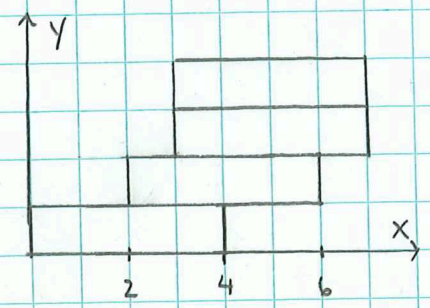
$x_{cg} = 2.5$ ✓



$$x_{cg} = \frac{3m(1.5) + 2m(2.5) + 2m(3.5) + m(4.5) + m(7.5}{9m}$$

$x_{cg} = 3.17$ ✓

March 31.23 ①



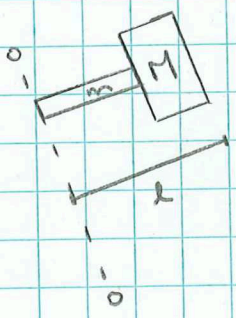
$$x_{cg} = \frac{(4m)(2) + 4m(4) + 4m(5)(2)}{16m}$$

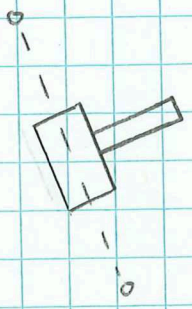
$$= \frac{8m + 16m + 40m}{16m} = 4$$

Moment of Inertia

$$I_o = \int r^2 dm$$

$$= \Sigma r_i^2 m_i$$





$$I = \left(\frac{l}{2}\right)^2 m + l^2 M$$

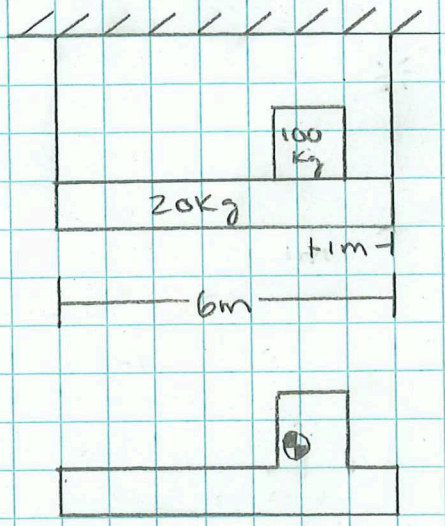
Big moment of Inertia

$$I = \left(\frac{l}{2}\right)^2 m$$

Small moment of Inertia

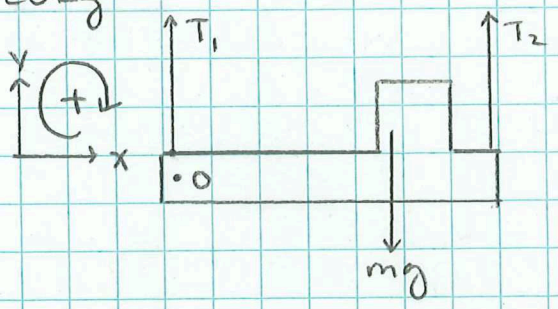
March 31, 23 (2)

Determine tension in both ropes.



Center of mass

$$x_{cg} = \frac{20kg(3) + 100kg(5)}{120kg} = 4.\overline{6}$$



$\Sigma F_x = m\cancel{a_x}^{\,0} = 0$

$\Sigma F_y = m\cancel{a_y}^{\,0} = T_1 + T_2 - mg$

$\Sigma M_o = \cancel{I_o\alpha}^{\,0}_{\,o} = mg(4.6) - T_2(6)$

$0 = mg(4.6) - T_2(6)$

$T_2(6) = mg(4.\overline{6})$

$$T_2 = \frac{120(9.81)(4.\overline{6})}{6}$$

$T_2 = 915.6\ N$

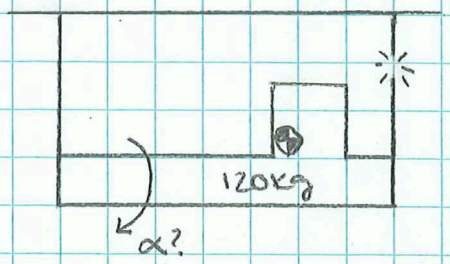
$0 = T_1 + T_2 - mg$

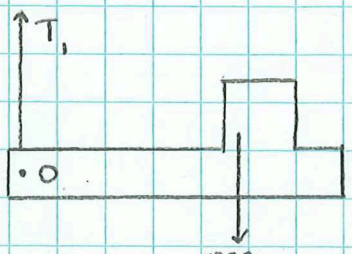
$T_1 = mg + T_2$

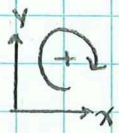
$T_1 = (120)(9.81) - 915.6$

$T_1 = 261.6\ N$

March 31, 23 (3)







$\Sigma F_x = m\vec{a}_x^{\,0} = 0$

$\Sigma F_y = ma_y = T_1 - mg$

$\Sigma M_0 = I_0\alpha_0 = mg(4.\overline{6})$

Moment of inertia from O.

$I_0 = \sum r_i^2 m_i$

$= \underset{\text{(rod)}}{\left(\frac{1}{3}\right)(20)(6^2)} + \underset{m}{(100)}\underset{r^2}{(5^2)}$

$= 2740\ kgm^2$

$I_0\alpha_0 = mg(4.\overline{6})$

$\alpha_0 = \frac{mg(4.\overline{6})}{I_0}$

$\alpha_0 = \frac{120(9.81)(4.\overline{6})}{2740}$

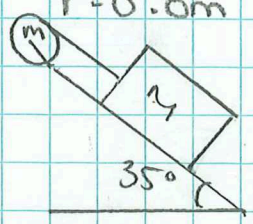
$\alpha_0 = 2$

$x = \theta r \quad 2\ 1/s^2$

$v = \omega r$

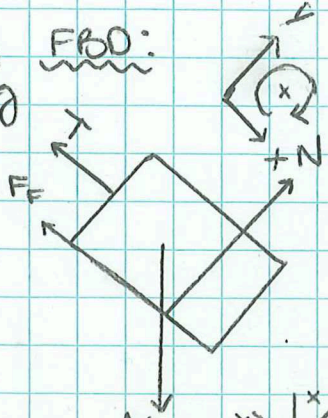
$a = \alpha r$

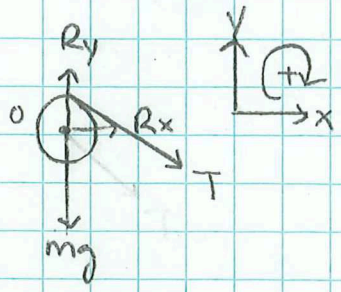
Mar. 31. 23 ④



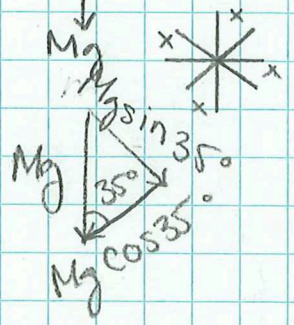
$m = 20\,kg$
$M = 20\,kg$

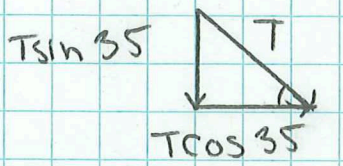
FBD:





$\nu_s = 0.2$
$\nu_k = 0.1$





4 $\Sigma F_x = Ma_x = Mg\sin 35 - T - F_f$

5 $\Sigma F_y = \cancel{Ma_y^{\,0}} = N - Mg\cos 35$

1 $\Sigma F_x = \cancel{Ma_x^{\,0}} = R_x + T\cos 35$

2 $\Sigma F_y = \cancel{Ma_y^{\,0}} = R_y - T\sin 35 - Mg$

3 $M_o = I_o\alpha_o = Tr$

$a_x = \alpha r$ assume moving $F_f = \nu N$

sub (5) into (4)

$Ma_x = Mg\sin(35) - T - \nu[Mg\cos 35]$

$T = Mg(\sin 35 - \nu\cos 35) - Ma_x$

sub (4a) into (3):

$I_o\alpha_o = [Mg(\sin 35 - \nu\cos 35) - Ma_x]r$

sub $(a_x = \alpha r)$ & $I_o = \frac{1}{2}mr^2$

$$\left[\frac{1}{2}mr^2\right]\left[\frac{a_x}{r}\right] = Mgr(\sin 35 - \nu\cos 35) - Ma_x r$$

$$a_x\left(\frac{m}{2} + M\right) = Mg(\sin 35 - \nu\cos 35)$$

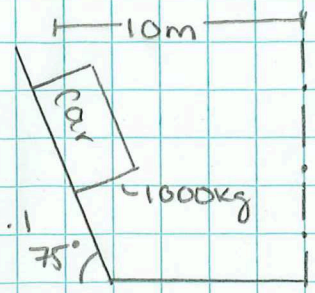
$$a_x = \frac{Mg\sin 35 - \nu\cos 35}{M + \frac{m}{2}}$$

solve $\Rightarrow \nu_s = 2.6$

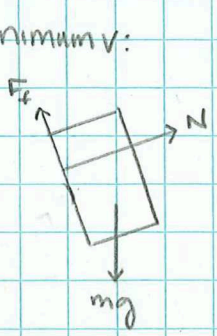
solve $\Rightarrow \nu_k = 3.2\ m/s^2$

Apr. 4, 23 ①

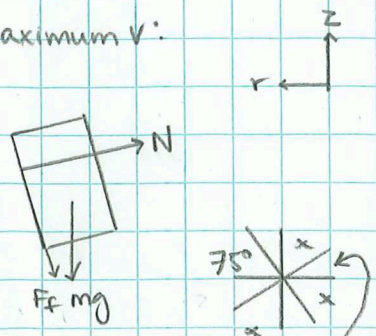
Determine velocity range where the car does not slide



minimum v:



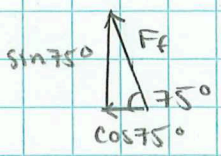
maximum v:

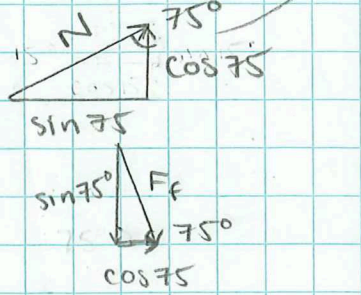


(3) $a_r = -\omega^2 r = \frac{-v^2}{r}$ (4) $f_f = \mu N$ static friction

$\Sigma F_z = ma_r^{\,0} = F_f \sin 75 + N\cos 75 - mg$

$\Sigma F_r = ma_r = F_f \cos 75 - N\sin 75$





sub (4) into (1)

$0 = F_f \sin 75 + N\cos 75 - mg$

$0 = \mu N \sin 75 + N\cos 75 - mg$

$0 = N(\mu \sin 75 + \cos 75) - mg$

$N = \frac{mg}{\mu \sin 75 + \cos 75}$

max v:

$\Sigma F_z = ma_z = N\cos 75 - F_f \sin 75 - mg$

$\Sigma F_r = ma_r = -N\sin 75 - F_f \cos 75$

sub (4) and (3) into (2)

$m\left(\frac{-v^2}{r}\right) = (\mu N)\cos 75 - N\sin 75$

$-\frac{mv^2}{r} = N(\mu\cos 75 - \sin 75)$

sub (1a) into (2)

$-\frac{mv^2}{r} = \left[\frac{mg}{\mu \sin 75 + \cos 75}\right](\mu\cos 75 - \sin 75)$

$-\frac{mv^2}{r} = mg\left[\frac{\mu\cos 75 - \sin 75}{\mu \sin 75 + \cos 75}\right]$

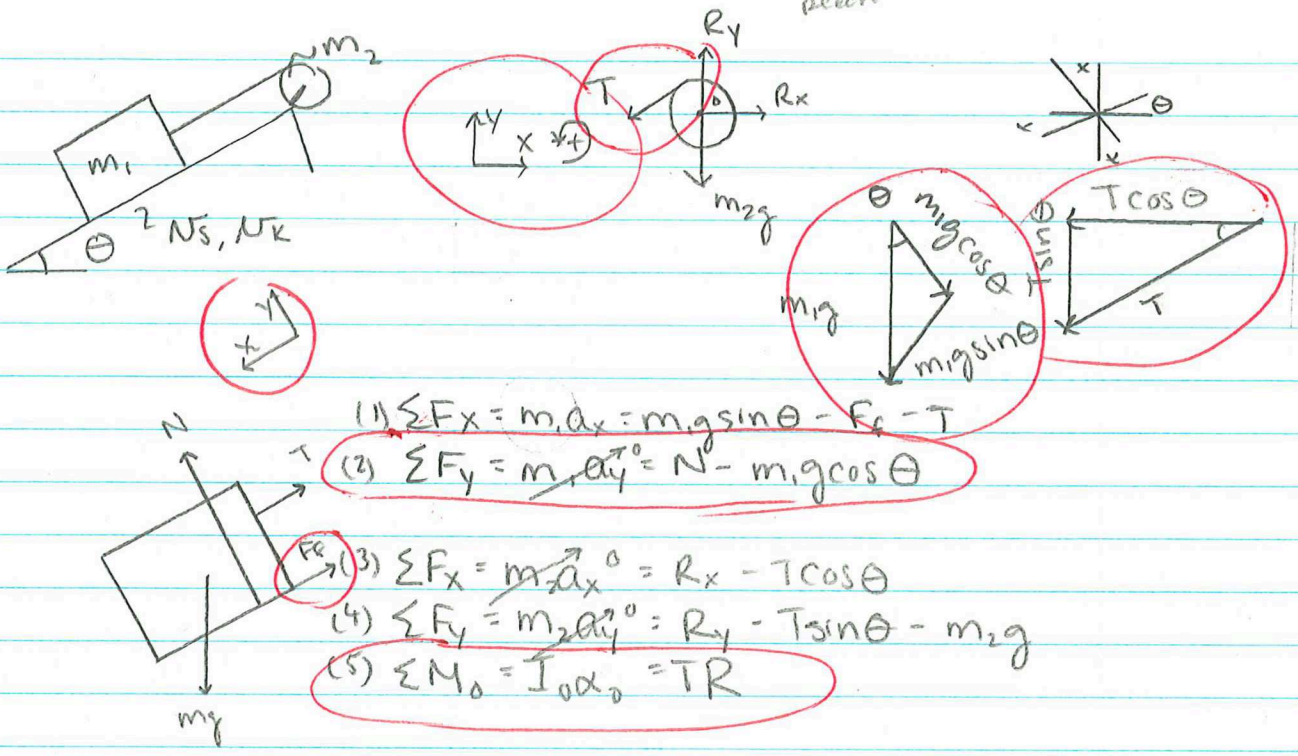
$v = \sqrt{-rg\frac{\mu\cos 75 - \sin 75}{\mu \sin 75 + \cos 75}}$

$v = 16\,m/s$ ✓

$v = 24\,m/s$

201.5

2 each 1st
3 each
each 5 each!



(1) $\Sigma F_x = m_1 a_x = m_1 g \sin\theta - F_f - T$

(2) $\Sigma F_y = m_1 a_y^{\,0} = N - m_1 g \cos\theta$

(3) $\Sigma F_x = m_2 a_x^{\,0} = R_x - T\cos\theta$

(4) $\Sigma F_y = m_2 a_y^{\,0} = R_y - T\sin\theta - m_2 g$

(5) $\Sigma M_0 = I_0 \alpha_0 = TR$

Circular motion: $\alpha R = a = a_x$ (7)

moving (6) $f_f = \mu N$

moment of inertia: $I_0 = \frac{1}{2} m_2 R^2$ (8)

eqn (3) and (4) are shit.

sub (2) into (6) and then into (1)

$ma_x = m_1 g \sin\theta - [\mu [m_1 g \cos\theta]] - T$

$T = m_1 g(\sin\theta - \mu\cos\theta) - ma_x$ (1a)

sub (1a), (7) & (8) into (5)

$\left[\frac{1}{2} m_2 R^2\right]\left[\frac{a_x}{R}\right] = [m_1 g(\sin\theta - \mu\cos\theta) - m_1 a_x] R$

$\frac{1}{2} m_2 a_x + m_1 a_x = m_1 g(\sin\theta - \mu\cos\theta)$

$a_x = m_1 g \dfrac{\sin\theta - \mu\cos\theta}{\frac{1}{2} m_2 + m_1}$

$a_x = 2.15\,\mu_s$

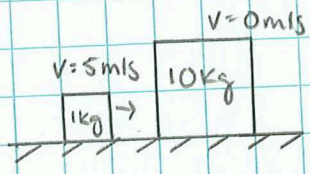
$a_x = 3.0\,\mu_k$

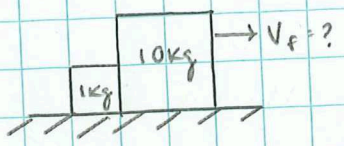
Momentum

$\vec{F} = m\vec{a}$

- $M_0 = I_0\alpha_0$ (Moments)
- $m_1\vec{v}_1 - m_0\vec{v}_0 = \int \vec{F}\,dt$ (momentum)
- $\vec{F} = \vec{v}\frac{dm}{dt} + m\vec{a}$

Example:





$m_0v_0 + M_0v_0$

$(m+M)v_f$

$(m_0v_0 + M_0v_0) - ((m+M)v_f) = 0$

elastic collision = E is conserved

inelastic collision = E is non-conserved

$$v_f = \frac{m_0v_0 + M_0v_0}{m_0 + M_0} = \frac{(1)(5) + (10)(0)}{1+10}$$

$$v_f = \frac{5}{11} = 0.4\overline{5}\ m/s$$

Kinematics: $\vec{v} = \frac{d\vec{x}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$ = Constant → 3 equations

Newton's 2nd Law: $\vec{F} = \frac{d}{dt}(m\vec{v})$ — moments / momentum \ impulse
(dynamics)

$= m\vec{a}$

Thermodynamics:

Zeroth Law: | T_1 | T_2 | T_3 | if $T_1 = T_2$ & $T_2 = T_3$ ∴ $T_1 = T_3$

First Law: $\Delta E = Q - W$, $W = \vec{F} \cdot \Delta\vec{x}$

Second Law: $\Delta S > \emptyset$, entropy $> \emptyset$

Third Law: $T \not\to -273.15°C$
$\not\to \emptyset K$

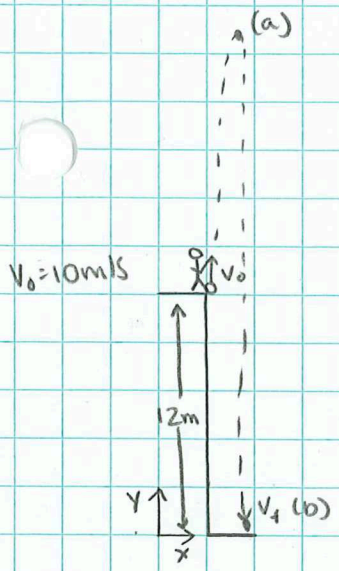
$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}k\Delta x^2 + IV\Delta t + \text{chemistry} + mc^2$

(current, voltage)

(kinetic) (gravitational) (spring) (electrical)

$+ mc\Delta T$
(internal energy)

April 12.23 ③



Kinematics

$y_0 = 12$

$y_a =$

$y_b = 0$

$V_{oy} = 10$

$V_{ay} = 0$

$V_{by} =$

$a_y = -9.81$

$$\vec{V} = \vec{V}_0 + at$$

$$\vec{x} = \vec{x}_0 + \vec{V}_0 t + \frac{1}{2} a t^2$$

$$\vec{V} = \vec{V}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$V_{ay}^2 = V_{oy}^2 + 2a_y(y_a - y_0)$ (V_{ay}, y_0 → 0)

$-V_{oy}^2 = 2a_y y_a - 2a_y y_0$

$2a_y y_0 - V_{oy}^2 = 2a_y y_a$

$$y_a = \frac{2a_y y_0 - V_{oy}^2}{2a_y}$$

$$y_a = \frac{2(-9.81)(12) - (10^2)}{2(-9.81)}$$

$y_a = 17.1 m$

$V_{by}^2 = V_{oy}^2 + 2a_y(y_0 - y_0)$ (y_0 → 0)

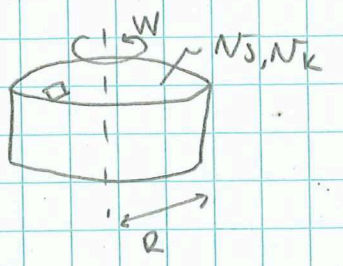
$V_{by} = \sqrt{V_{oy}^2 - 2a_y y_0}$

$V_{by} = \sqrt{(10)^2 - 2(-9.81)(12)}$

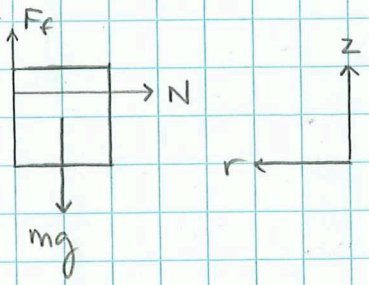
$V_{by} = \pm 18.3$

$V_{by} = -18.3 \ m/s$

Test 2 Review Question 2



FBD



(1) $\Sigma F_r = ma_r = -N$

(2) $\Sigma F_z = m\cancel{a_z}^{0} = f_f - mg$

(3) $a_r = -\omega^2 r$

(4) $f_f = \mu N$

Sub (4) into (2)

$0 = \mu N - mg$

(2a) $N = \frac{mg}{\mu}$

Sub (2a) and (3) into (1)

$\cancel{m}(-\omega^2 r) = \frac{\cancel{m}g}{\mu}$

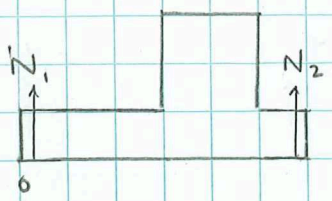
$\omega = \sqrt{\frac{g}{r\mu}} = \sqrt{\frac{9.81}{(2.5)(0.2)}} = 4.43$

$\omega = 4.43 \frac{rads}{s}\left(\frac{60s}{1min}\right)\left(\frac{1\,cycle}{2\pi\,rads}\right) = 42.3\ cycle/min$

$= 42.3\ rpm$

Test 2 Review Question 3

April 14, 23 (2)



$$x_{cg} = \frac{(6)(3) + (4)(4)}{10} = 3.4\,m$$

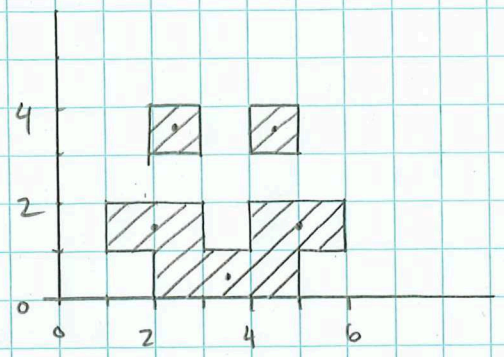
(1) $\Sigma F_x = (m+M)a_x = 0$

(2) $\Sigma F_y = (m+M)a_y = N_1 + N_2 - (m+M)g$

(3) $\Sigma M_0 = I_0 \alpha_0 = (m+M)g(3.4) - N_2(6)$

$$N_2 = \frac{(m+M)g(3.4)}{6}$$

sub N_2 into (2) to find N_1.



$$x_{cg} = \frac{2kg(2) + 3kg(3.5) + 2kg(5) + 1kg(2.5) + 1kg(4.5)}{9kg}$$

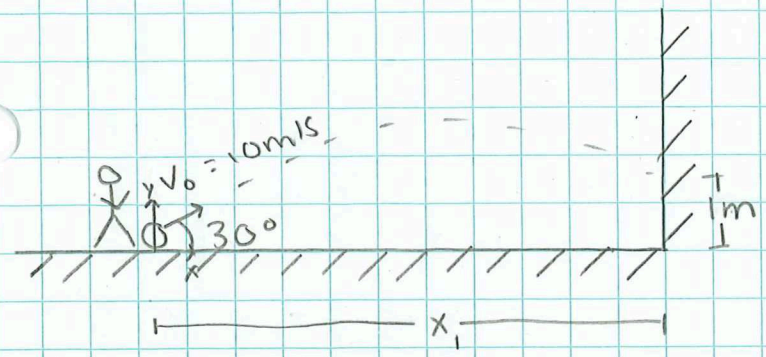
$$x_{cg} = \frac{4 + 10.5 + 10 + 2.5 + 4.5}{9} = 3.5$$

$$y_{cg} = \frac{3kg(0.5) + 4kg(1.5) + 2kg(3.5)}{9}$$

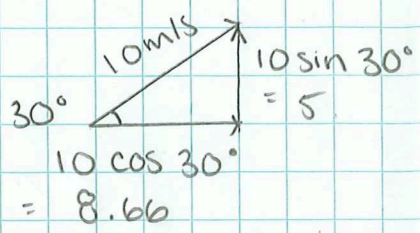
$$y_{cg} = \frac{1.5 + 6 + 7}{9} = 1.6$$

Final test
cg
kinematics
easy forces
hard forces

I_o, N, θ, R



x		y		t
$x_0 = 0$		$y_0 = 0$		$t =$
$x_1 =$		$y_1 = 1$		
$V_{0x} = 8.66$		$V_{0y} = 5$		
$V_{1x} = 8.66$		$V_{1y} =$		
$a_x = 0$		$a_y = -9.81$		



② solve for time.

$V_{1y} = V_{0y} + at$

$t = \dfrac{V_{1y} - V_{0y}}{a_y}$

$t = \dfrac{(-2.32) - (5)}{-9.81} = 0.7465$

③ solve for V_{1x}

$x_1 = x_0 + V_{0x}t + \cancel{\frac{1}{2}at^2}^{\,0}$

$x_1 = 0 + 8.66(0.746)$

$x_1 = 6.5 m$

V_{1x}

① $V_{1y}^2 = V_{0y}^2 + 2a(y_1 - y_0)$

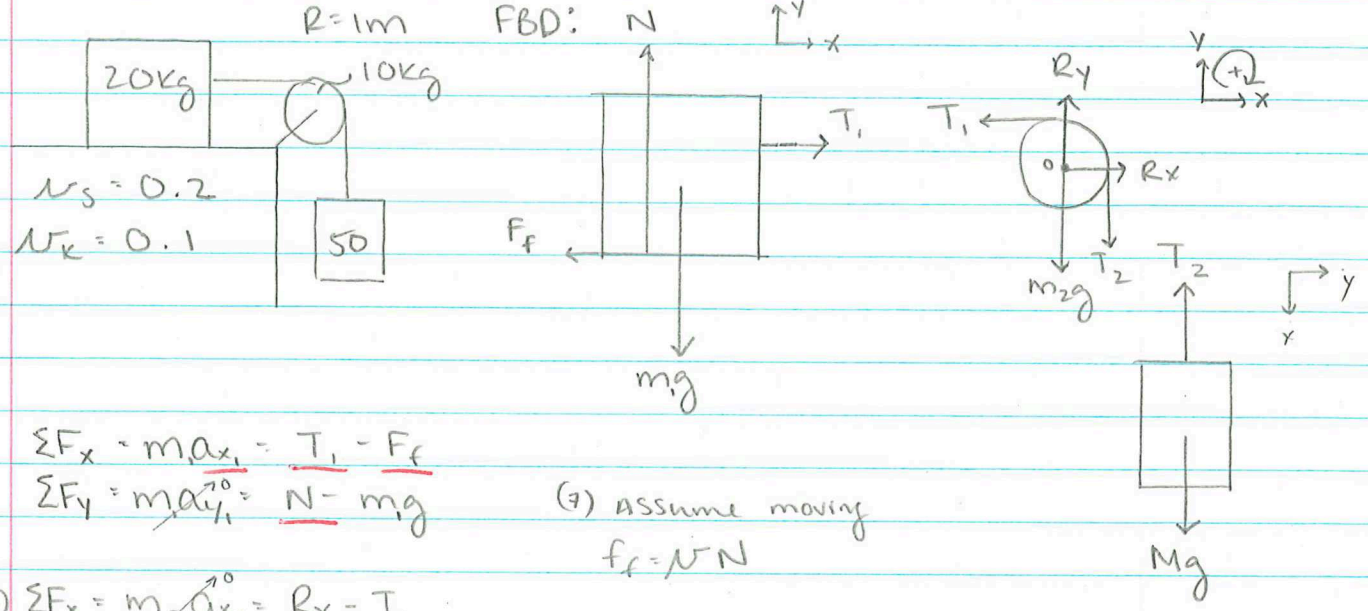
$V_{1y}^2 = (5)^2 + (2)(-9.81)(1 - 0)$

$V_{1y}^2 = 25 - 19.62$

$V_{1y} = \pm 2.32$

$V_{1y} = -2.32$

Solve for the acceleration. April 17, 23 ③



(1) $\Sigma F_x = m_1 a_{x_1} = T_1 - F_f$

(2) $\Sigma F_y = m_1 a_{y_1}^{\nearrow 0} = N - m_1 g$

(7) assume moving

$f_f = \mu N$

(3) $\Sigma F_x = m_2 a_{x_2}^{\nearrow 0} = R_x - T_1$

(4) $\Sigma F_y = m_2 a_{y_2}^{\nearrow 0} = R_y - m_2 g - T_2$

(8) circular motion

(5) $\Sigma M = I_o \alpha_o = T_2(1) - T_1(1)$ $\quad \alpha_o r = a_x$

(6) $\Sigma F_x = M a_{x_3} = Mg - T_2$

(9) unstretched rope

$a_{x_1} = a_{x_3} = a_x$

① solve (2) for N

$0 = N - m_1 g$

$N = 20(9.81) = 196.2$

② Sub N into (1)

$ma_x = T_1 - \mu N$

$20(a_x) = T_1 - 0.2(196.2)$

$T_1 = 20(a_x) + 0.2(196.2)$

③ Sub T_1 into (6)

$Ma_x = Mg - T_2$

$50(a_x) = 50(9.81) - T_2$

$T_2 = 50(9.81) - 50(a_x)$

④ sub T_1 and T_2 into (5)

$I_o \alpha_o = T_2(1) - T_1(1)$

$I_o a_o = [Mg - Ma_x]R - [m_1 a_x + \mu m_1 g]R$

$I_o = \frac{1}{2} m_2 R^2$ $\quad \alpha_o R = a_x$

$\frac{1}{2} m_2 R a_x = MgR - Ma_x R - m_1 a_x R - \mu m_1 g R$

$\frac{1}{2} m_2 a_x + Ma_x + m_1 a_x = Mg - \mu m_1 g$

$a_x\left(\frac{1}{2} m_2 + M + m_1\right) = Mg - \mu m_1 g$

$$a_x = \frac{Mg - \mu m_1 g}{\frac{1}{2} m_2 + M + m_1}$$