Force of Gravity

Jan 20. 23 ②

$$\vec{F_g} = \frac{G m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

↑ constant. 39 → G on calc.

Frank 100kg → F_g ← F_g ← Jose 65kg

3m

$$F_g = \frac{G(100)(65)}{3^2} = 4.8 \times 10^{-8} N$$

100kg ↓

Sun $1.98 \times 10^{30} kg$

r = 700 000 000m

$$F_g = \frac{G(100)(1.98 \times 10^{30})}{(7.0 \times 10^8)^2} = 27000 N$$

$$a = \frac{G m_s}{r^2} = \frac{G(1.98 \times 10^{30})}{(7.0 \times 10^8)^2} = 270 \, m/s^2$$

$$\frac{270}{9.81} = 27 g's$$

↑ 27 times earth.

Earth

r = 6371000m

$m_e = 5.97 \times 10^{24} kg$

$$F_g = \frac{G(100)(5.97 \times 10^{24})}{(6.37 \times 10^5)^2} = 981.9 N$$

Jan. 20, 23

$$\vec{F_g} = \frac{G m_1 m_2}{\vec{r}^2} = m_1 a$$

$$\Downarrow$$

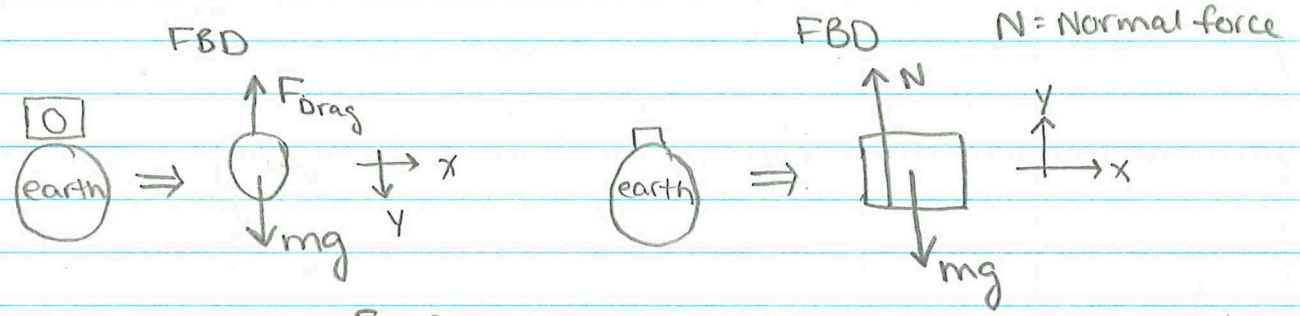
$$ma = \frac{Gm^2}{r^2} = \frac{G(5.97\times10^{24})}{(6.37\times10^{6}+1)^2} = 9.81\ m/s^2$$

$F_g = mg \qquad g = 9.81\ m/s^2\ (m)$

↖ only on the earth's surface

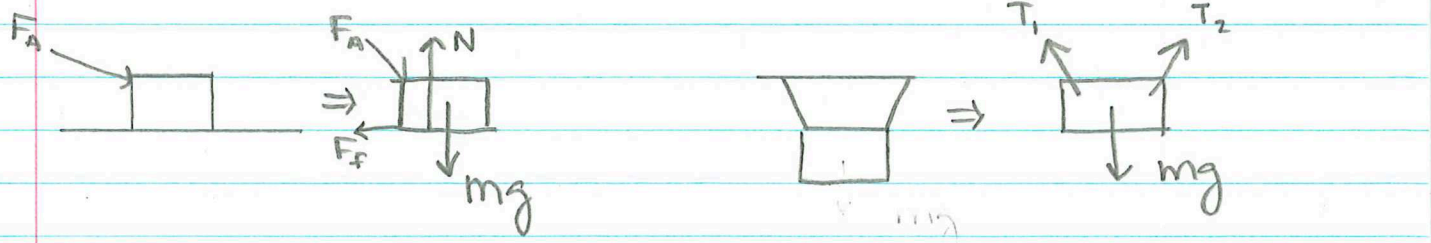
Free Body Diagrams (FBD)

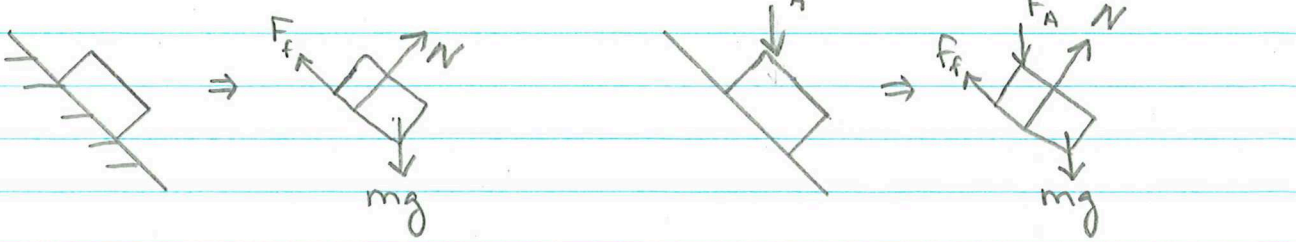
↳ Coordinate system

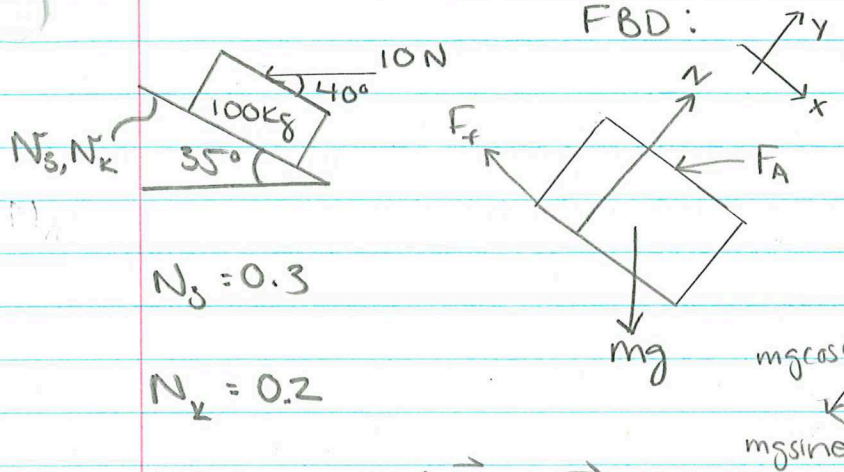


N = Normal force

F_f = friction $\qquad F_A$ = force applied $\qquad T$ = tension







$N_s = 0.3$

$N_k = 0.2$

$\sum \vec{F} = m\vec{a}$

$F_{fmax} = N\,N$

Newton:

(1) $\sum F_x = ma_x = mg\sin\theta - f_f - F_A\cos\phi$ (N / f_f)

(2) $\sum F_y = \cancel{ma_y}^{\,0} = N - mg\cos\theta - F_A\sin\phi$

Solve (2) for N:

$N = mg\cos\theta + F_a\sin\phi$ (2a)

Sub (2a) into (1)

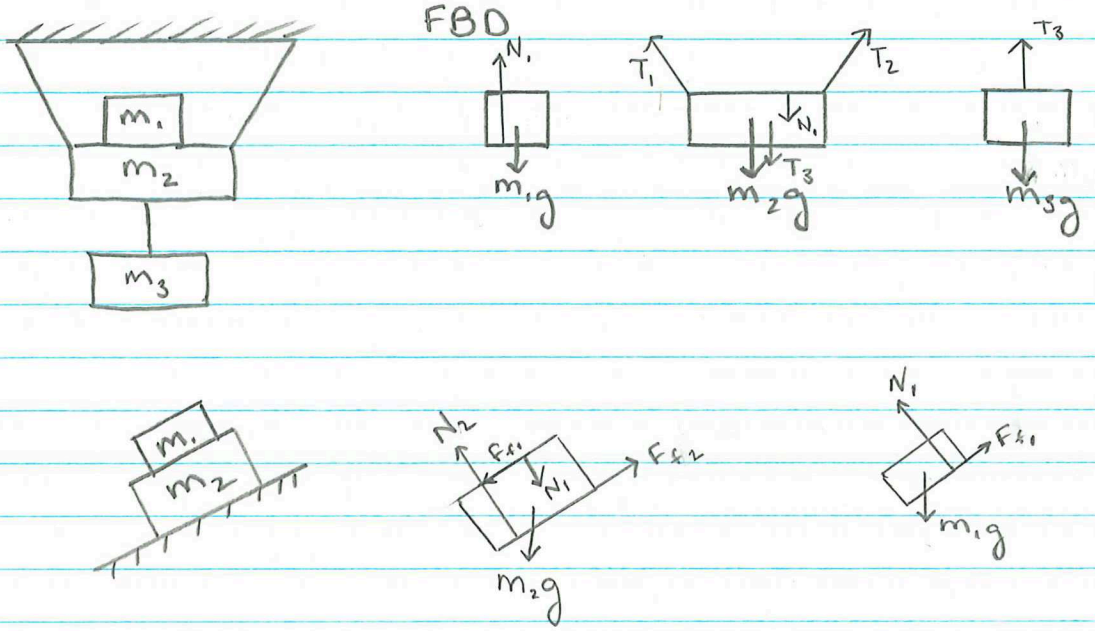
$ma_x = mg\sin\theta - N[mg\cos\theta + F_a\sin\phi] - F_a\cos\phi$

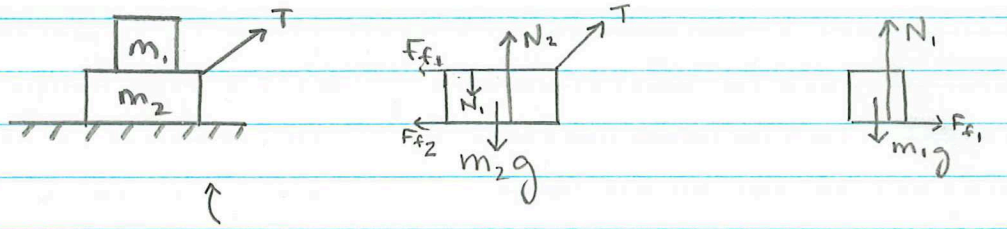
$$a_x = \frac{mg\sin\theta - N\,mg\cos\theta - N\,F_a\sin\phi - F_a\cos\phi}{m}$$

$N = N_s$ (does it move?)

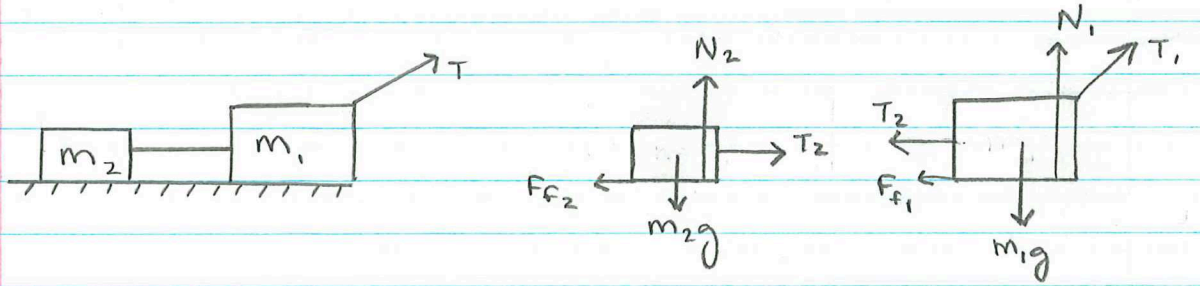
$N = N_k$ (the acceleration)

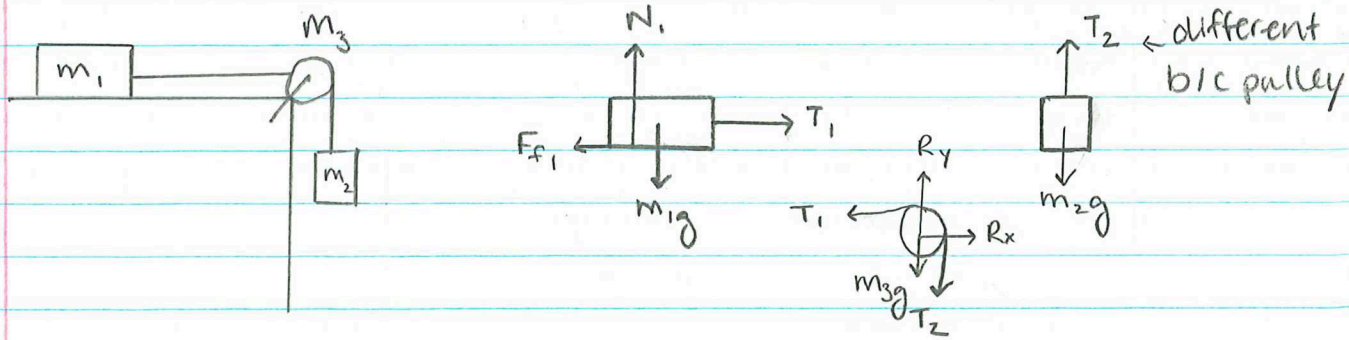
Jan 25, 23

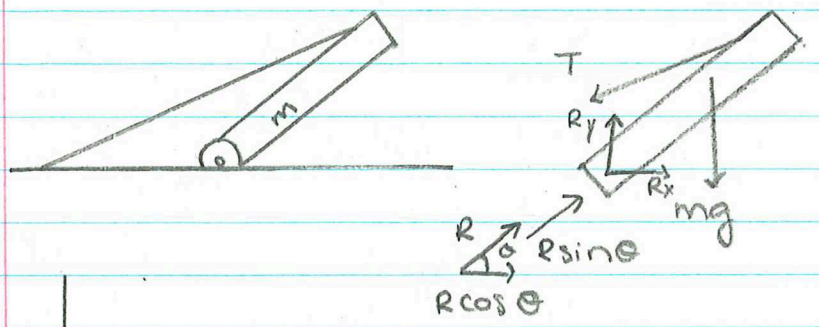


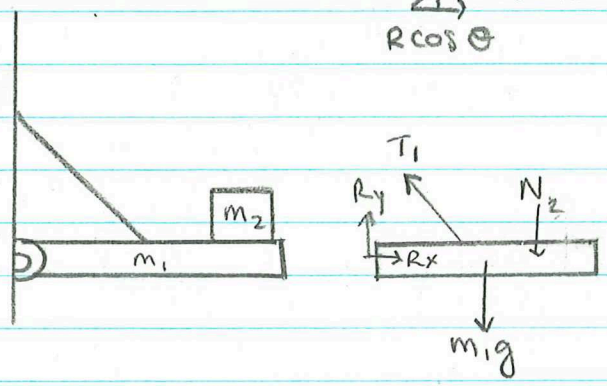


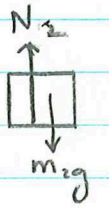
assuming you are pulling so there is friction.

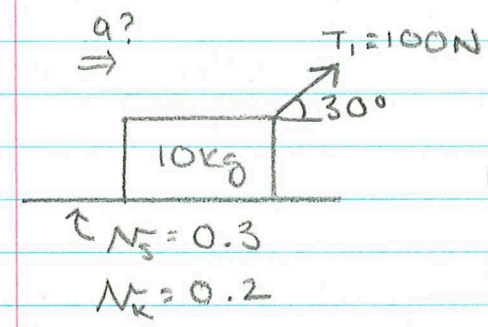




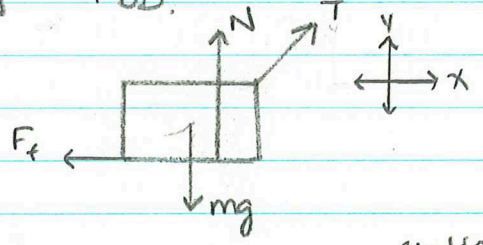


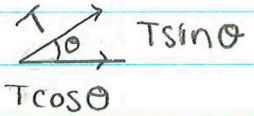


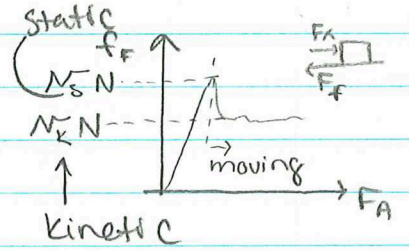




FBD:







Newton's 2nd Law $\Sigma \vec{F} = m\vec{a}$

(1) $\Sigma F_x = ma_x = T\cos\theta - F_f$

(2) $\Sigma F_y = \cancel{ma_y} = N + T\sin\theta - mg$

Does it move? set $F_f = \mu_s N$

$ma_x = T\cos\theta - \mu_s N$ (1)

$0 = N + T\sin\theta - mg$ (2)

Solve (2) for N:

$N = mg - T\sin\theta$ (2a)

Sub (2a) into 1

$ma_x = T\cos\theta = \mu_s [mg - T\sin\theta]$

$$a_x = \frac{1}{m}[T\cos\theta - \mu_s mg + \mu_s T\sin\theta]$$

$$a_x = \frac{1}{10}[100\cos(30) - (0.3)(10)(9.81) + (0.3)(100)\sin(30)]$$

in calc. $= (100\cos 30 - 0.3\times 10\times 9.81 + 0.3\times 100\times \sin 30)/10$

$= 7.22\ m/s^2$ b/c it is positive it means it moves.

= yes if it was neg → answer is $a_x = 0$

now solve using

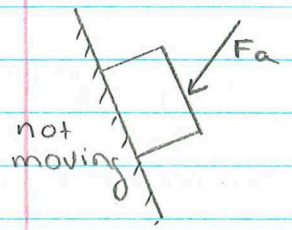
$$f_f = \mu_x N$$

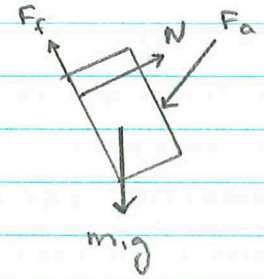
$$a_x = \frac{1}{m}[T\cos\theta - \mu_k mg + \mu_k T\sin\theta]$$

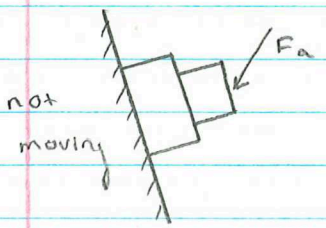
$$a_x = 7.69\ m/s^2$$

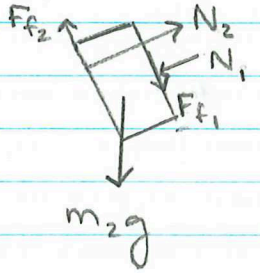
FBD

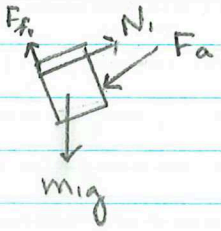
Jan. 27, 23. (1)

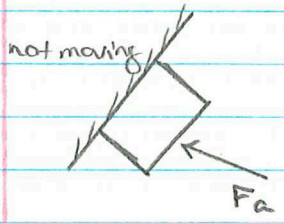


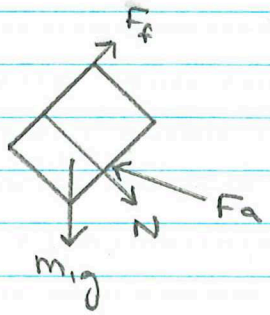


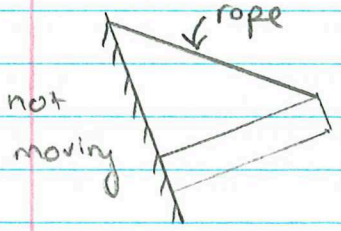


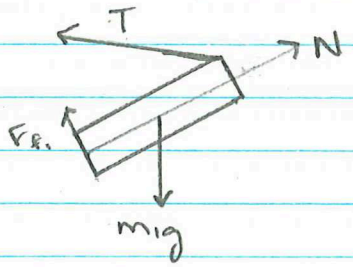






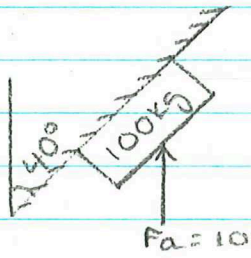






g = is always positive, rotate coordinate system to match.

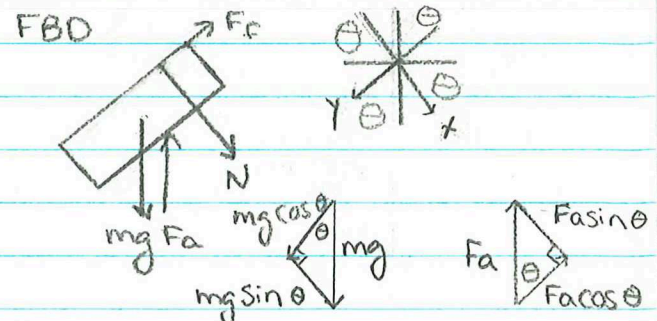
Jan. 27, 23 (2)



Does it fall? Does it slide? If so, what is $\vec{a}$

$\mu_s = 0.3$

$\mu_k = 0.2$



Newton's 2nd Law $\Sigma \vec{F} = m\vec{a}$

(1) $\Sigma F_x = ma_x^{\,0} = N - Fa\sin\theta + mg\sin\theta$

(2) $\Sigma F_y = ma_y = mg\cos\theta - Fa\cos\theta - F_f$

Solve (1) for N

$N = Fa\sin\theta - mg\sin\theta$

$= 1000\sin 40 - 100(9.81)\sin 40$

(1a) $= 12.21\ N$ ← Does not fall b/c positive.

if 0 or neg will have to solve for acceration of falling

$W = mg$

$12.21 = m(9.81)$

$m \cong 1\ kg$

assume $F_f = \mu N$ (3)

Sub (1a) into (3), (3) into (2)

$ma_y = mg\cos\theta - fa\cos\theta - \mu[Fa\sin\theta - mg\sin\theta]$

$\mu_s \rightarrow$ $a_y = g\cos\theta - \frac{fa}{m}\cos\theta - \mu\frac{Fa}{m}\sin\theta + \mu g\sin\theta$

$a_y =$

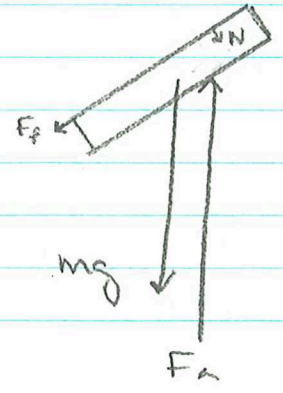
$a_y = -0.18$ ← the acceleration is 0 so it does not slide.

$ma_y^{\,0} = mg\cos\theta - Fa\cos\theta - f_f$

$F_f = mg\cos\theta - fa\cos\theta$

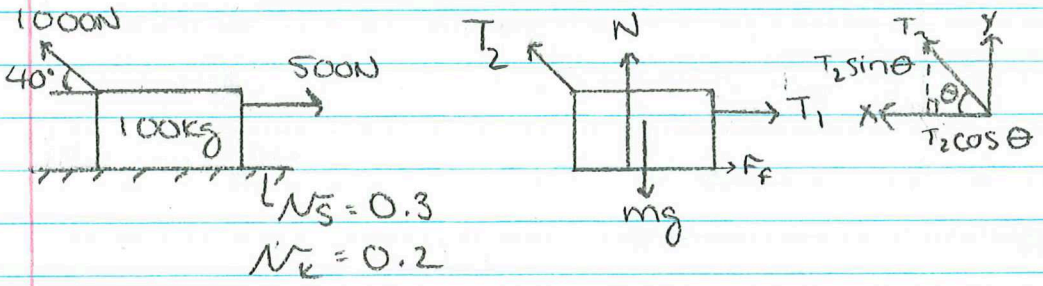
$= -14.5\ N$

Actual



Jan. 27. 23 (3)

FBD



does it move? Newton's 2nd Law $\Sigma \vec{F} = m\vec{a}$

no ↙ ↘ yes (1) $\Sigma F_x = ma_x = T_2\cos\theta - T_1 - F_f$

f_f a (2) $\Sigma F_y = m\cancel{a_y} = N - mg + T_2\sin\theta$

solve (2) for N to see if it is being lifted from the ground.

$N = mg - T_2 \sin\theta$

$= (100)(9.81) - 1000\sin(40)$

(2a) $= 338\ N$ positive so it is not lifting off the ground.

assume acceleration $F_f = \mu N$ (3)

sub (3) into (1)

$ma_x = T_2\cos\theta - T_1 - \mu N$

solve for a_x:

$a_x = \frac{1}{m}[T_2\cos\theta - T_1 - \mu_s N]$

$= \frac{1}{100}[1000\cos 40 - 500 - 0.3(338)]$

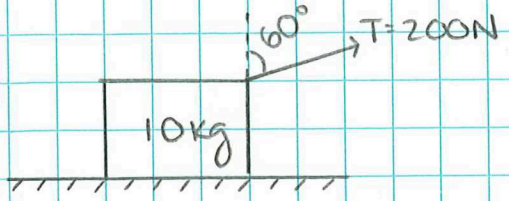
$= 1.64\ m/s^2$ - yes it does move. so resolve using μ_k

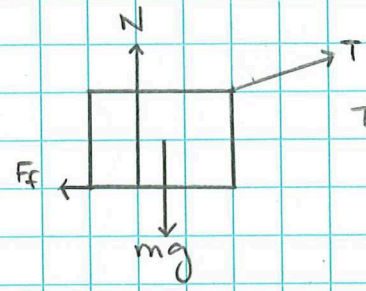
$a_y = \frac{1}{100}[1000\cos 40 - 500 - 0.2(338)]$

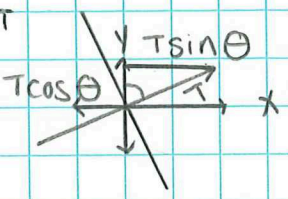
$a_y = 1.98\ m/s^2$

Does it move? what is the friction?

Feb 1 2023







$\mu_s = 0.2$

$\mu_k = 0.15$

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_x = T\sin\theta - F_f$

(2) $\Sigma F_y = ma_y^{\,0} = N - mg + T\cos\theta$

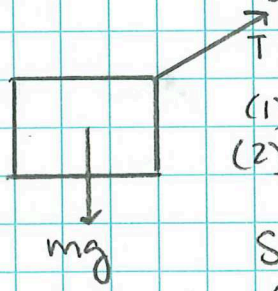
Calculate N to see if it is lifting off the ground

Solve (2) for N

$N = mg - T\cos\theta$

$= 10(9.81) - 200\cos 60$

$= -1.9$ Negative so it is lifting off ground



(1) $\Sigma F_x = ma_x = T\sin\theta$

(2) $\Sigma F_y = ma_y = T\cos\theta - mg$

Solve (1)

$$a_x = \frac{T}{m}\sin(60) = \frac{200\sin(60)}{10} = 17.3\ m/s^2$$

$$a_y = \frac{T}{m}\cos(60) - g = \frac{200\cos(60)}{10} - 9.81$$

$a_y = 0.19\ m/s^2$

$|a| = \sqrt{a_x^2 + a_y^2}$

$= \sqrt{(17.3)^2 + (0.19)^2}$

$= 17.3\ m/s^2$

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assume solution

↓

FBD

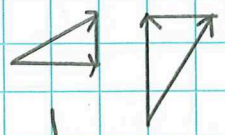
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inline ⇒ acceleration.

↓

vector components



↓

$\Sigma \vec{F} = m\vec{a}$

*

↓

check assumption

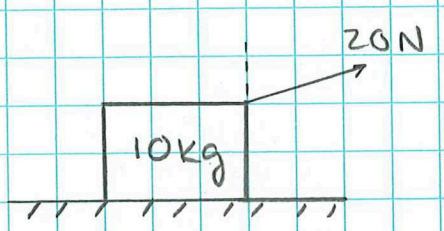
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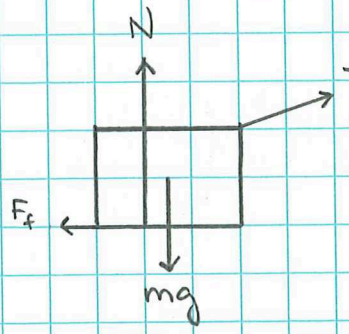
make more assumptions

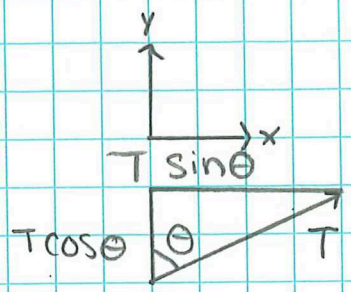
* Types of Assumptions

1 acc = 0

moving $\boxed{F_f = \mu N}$







$\mu_s = 0.2$
$\mu_k = 0.15$

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = T\sin(60) - F_f = ma_x$

(2) $\Sigma F_y = N - mg + T\cos(60) = ma_y^0$

Solve (2) for N

$N = mg - T\cos(60)$

$N = 10(9.81) - 20\cos(60)$

$= 88.1\ N$ positive so it stays on the ground.

Solve (1) for a_x

$a_x = \frac{1}{m}[T\sin\theta - F_f]$ (1a)

Sub $F_f = \mu_s N$ into (1a)

$a_x = \frac{1}{m}[T\sin\theta - \mu_s N]$

$= \frac{1}{10}[20\sin(60) - 0.2(88.1)]$

$= -0.0299$ Negative so it is not moving

Determined $a_x = 0$

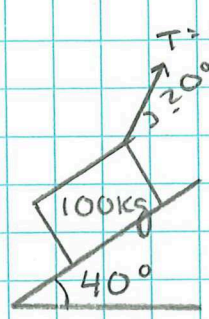
Solve (1) for F_f

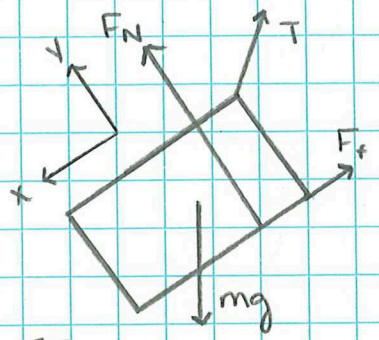
$0 = T\sin(60) - F_f$

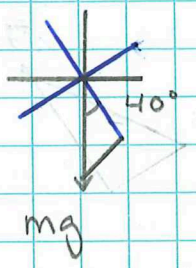
$F_f = T\sin(60)$

$= 20\sin(60)$

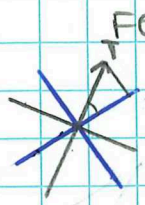
$= 17.3N$







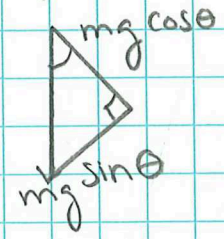
Feb. 1. 2023 (4)

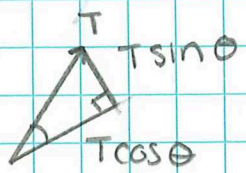


Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_x = mg\sin(40) - T\cos(20) - F_f$

(2) $\Sigma F_y = ma_y^{\nearrow 0} = F_N + T\sin(20) - mg\cos(40)$





assume $a_y = 0$, solve (2) for F_N:

$F_N = mg\cos(40) - T\sin(20)$

$= 520.6\,N$ it's on the ground

assume it's moving downwards so $F_f = \mu F_N$ (3)

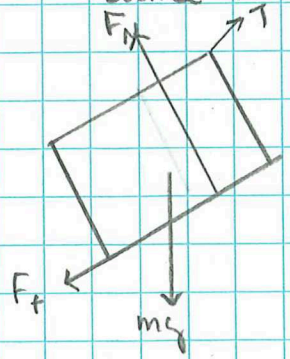
sub (3) into (1)

$ma_x = mg\sin(40) - T\cos(20) - \mu_s F_N$

$a_x = g\sin(40) - \frac{T}{m}\cos(20) - \frac{\mu_s F_N}{m}$

$a_x = -0.141\ m/s^2$ it is not moving downwards

assume it's moving upwards



$\Sigma F_x = ma_x = T\cos(20) - mg\sin(40) - F_f$ (4)

$\Sigma F_y = ma_y^{\nearrow 0} = F_N - mg\cos(40) + T\sin(20)$ (5)

assume moving, so $F_f = \mu_s F_N$ (3)

sub (3) into (4)

$ma_x = T\cos(20) - mg\sin(40) - \mu_s F_N$

$a_x = \frac{T}{m}\cos(20) - g\sin(40) - \frac{\mu_s F_N}{m}$

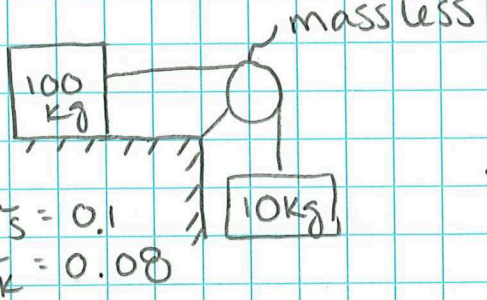
$= -0.067\ m/s^2$ not moving upwards

so $a_x = 0$ and $F_f \neq \mu_s F_N$

$ma_x^{\nearrow 0} = T\cos(20) - mg\sin(40) - F_f$

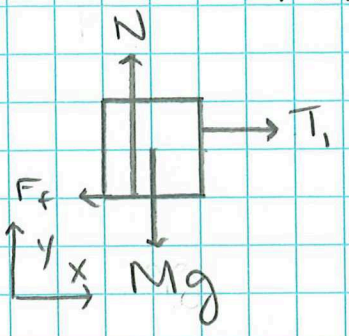
$f_f = T\cos(20) - mg\sin(40) = 3.71\,N$

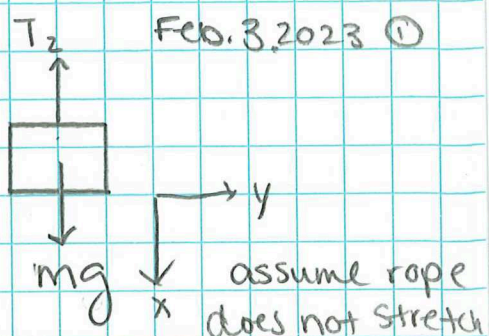
Tests are 25% each



$\mu_s = 0.1$
$\mu_k = 0.08$

FBD





Feb. 3. 2023 ①

assume rope does not stretch
$a_{x_1} = a_{x_2} = a_x$

What is the force of friction?

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_x^{\,0} = T_1 - F_f$

(2) $\Sigma F_y = ma_{y_1}^{\,0} = N - Mg$

(3) $\Sigma F_x = ma_{x_2}^{\,0} = mg - T_2$

(4) $\Sigma F_y = ma_{y_2}^{\,0} = 0$

$0 = N - Mg$
$N = 100(9.81)$
$N = 981\,N$ so it does not lift off ground

assume moving $\therefore F_f = \mu N$

$Ma_x = T_1 - \mu N$ (1)

Solve for T in (3)

$T = mg - ma_x$ (3a)

sub (3a) into (1), solve for a_x:

$$a_x = \frac{mg - \mu N}{M + m}$$

$Ma_x = [mg - ma_x] - \mu N$
$Ma_x + ma_x = mg - \mu N$
$a_x(M + m) = mg - \mu N$

use $\mu_s = 0.1$

$$a_x = \frac{(10)(9.81) - (0.1)(9.81)}{100 + 10} = 0, \quad a_x = 0$$

$\therefore F_f \neq \mu N$ so it does not move

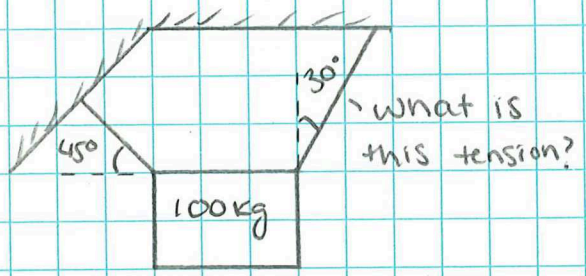
solve for F_f in equation (1)

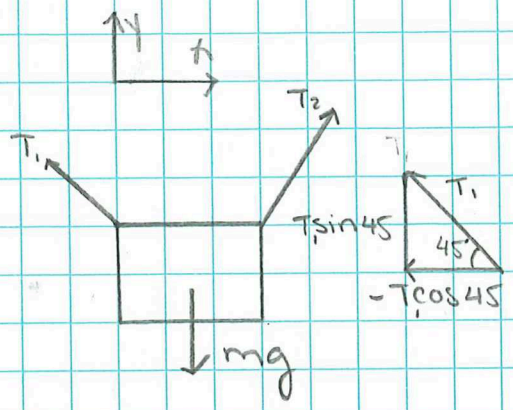
$F_f = T_1$ (1b)

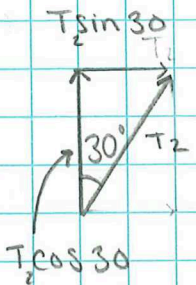
sub in (3) into (1b)

$F_f = mg = 98.1\,N$

Feb. 3. 2023 (2)







Newton's 2nd Law

(1) $\sum F_x = ma_x = T_2 \sin 30 - T_1 \cos 45$

(2) $\sum F_y = ma_y = T_2 \cos 30 + T_1 \sin 45 - mg$

Solve (1) for T_2

$$T_2 = T_1 \frac{\cos(45)}{\sin 30} \quad (1a)$$

Sub (1a) into (2):

$$0 = \left[T_1 \frac{\cos 45}{\sin 30}\right] \cos(30) + T_1 \sin(45) - mg$$

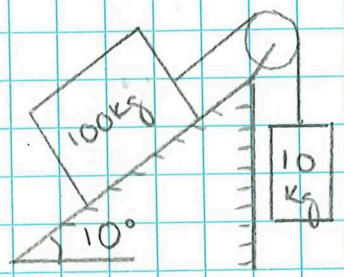
$$0 = T_1 \left(\frac{\cos(45)\cos(30)}{\sin(30)} + \sin(45)\right) - mg$$

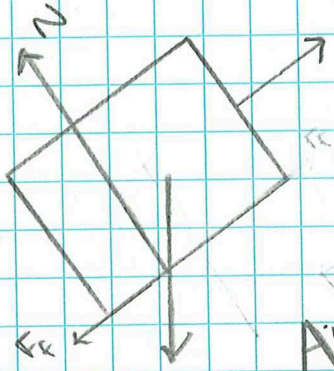
$$T_1 = \frac{mg}{\left(\frac{\cos(45)\cos(30)}{\sin(30)} + \sin(45)\right)} = 507.8 \quad (2a)$$

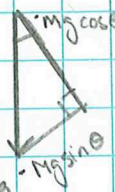
Sub (2a) into (1a)

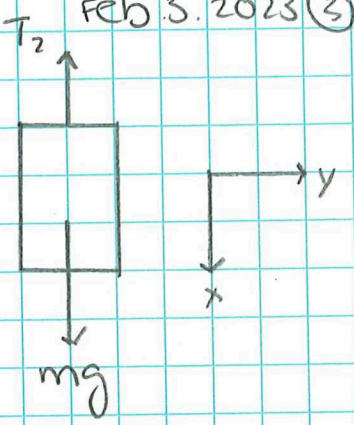
$$T_2 = 507.8 \frac{\cos(45)}{\sin(30)} = 718.1\,N$$

$\mu_s = 0.1$ $\mu_k = 0.08$









what is the force of friction?

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_{x_1} = T_1 - F_f - Mg\sin(10)$

(2) $\Sigma F_y = ma_{y_1} = N - Mg\cos(10)$

(3) $F_x = ma_{x_2} = mg - T$

(4) $F_y = ma_{y_2} = 0$

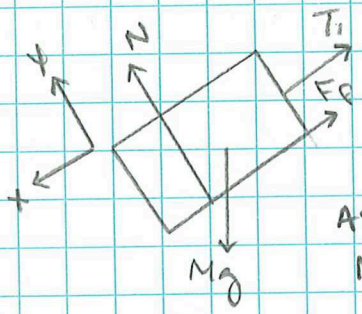
Assume not lift off ground

$N = Mg\cos(10) = (100)(9.81)\cos(10) = 966.1N$ ✓ does not lift off

Assume moving, $F_f = \mu_k N$ solve (1): ↗ up hill

$Ma_x = [mg - ma_x] - Mg\sin(10) - \mu[966]$

$a_x = \dfrac{mg - Mg\sin(10) - \mu 966}{M+m} = -1.53\ m/s^2$ does not move up



(1) $\Sigma F_x = Ma_{x_1} = Mg\sin(10) - T - F_f$

(2) $\Sigma F_y = Ma_{y_1} = N - Mg\cos(10)$

(3) $\Sigma F_x = ma_x = T - mg$

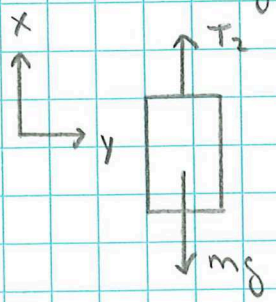
(4) $\Sigma F_y = m\vec{a}_y^{\,0} = 0$

Assume moving downhill, $F_f = \mu_k N$ solve (1): ↙

$Ma_x = Mg\sin(10) - [ma_x + mg] - \mu_k[9.66]$

$a_x = \dfrac{Mg\sin(10) - mg - 966\mu_k}{M+m} = -0.22 m/s^2$

does not move down



$F_f \neq \mu N$

Solve for F_f in (1)

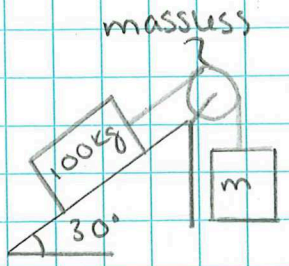
$F_f = Mg\sin(10) - T$ (1a)

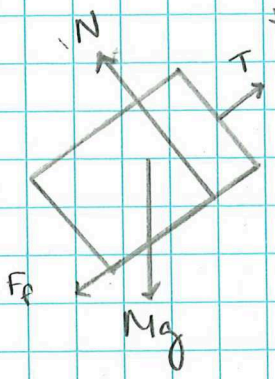
sub (3) into (1a)

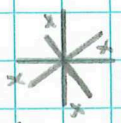
$F_f = Mg\sin(10) - [Mg] = 72N$

Minimum/maximum mass so system does not move

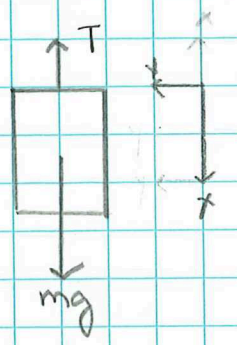
Feb 7, 2023 ①











$\mu_s = 0.2$

$\mu_k = 0.12$

Max weight

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = Ma_{x_1} = T - F_f - Mg\sin(30)$

(2) $\Sigma F_y = Ma_{y_1} = N - Mg\cos(30)$

(3) $\Sigma F_x = ma_{x_2} = mg - T$

(4) $\Sigma F_y = ma_{y_2} = 0$

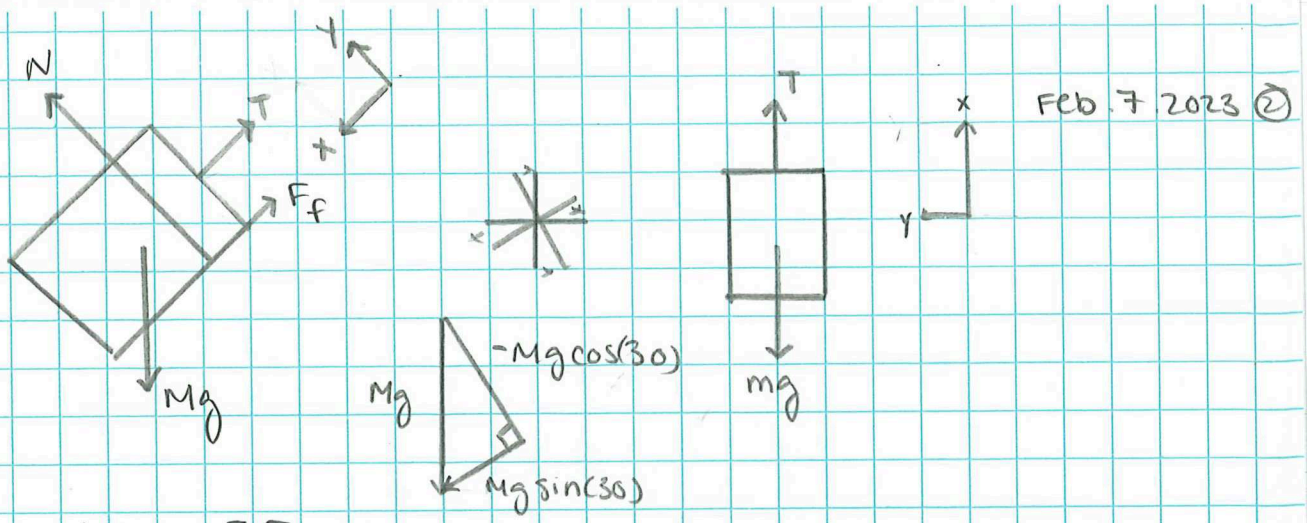
Set $a_{x_1} = a_{x_2} = 0$

Sub (2) and (3) into (1) with $F_f = \mu_s N$

$0 = [mg] - Mg\sin(30) - \mu_s[Mg\cos(30)]$

$m = M(\sin(30) + \mu_s\cos(30))$

$m = 67.32\ kg$



Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_{x_1} = Mg\sin(30) - T - F_f$ $\quad \Sigma F_x = ma_{x_2} = T - mg$

(2) $\Sigma F_y = ma_{y_1} = N - Mg\cos(30)$ $\quad \Sigma F_y = ma_{y_2} = 0$

set $a_{x_1} = a_{x_2} = 0$

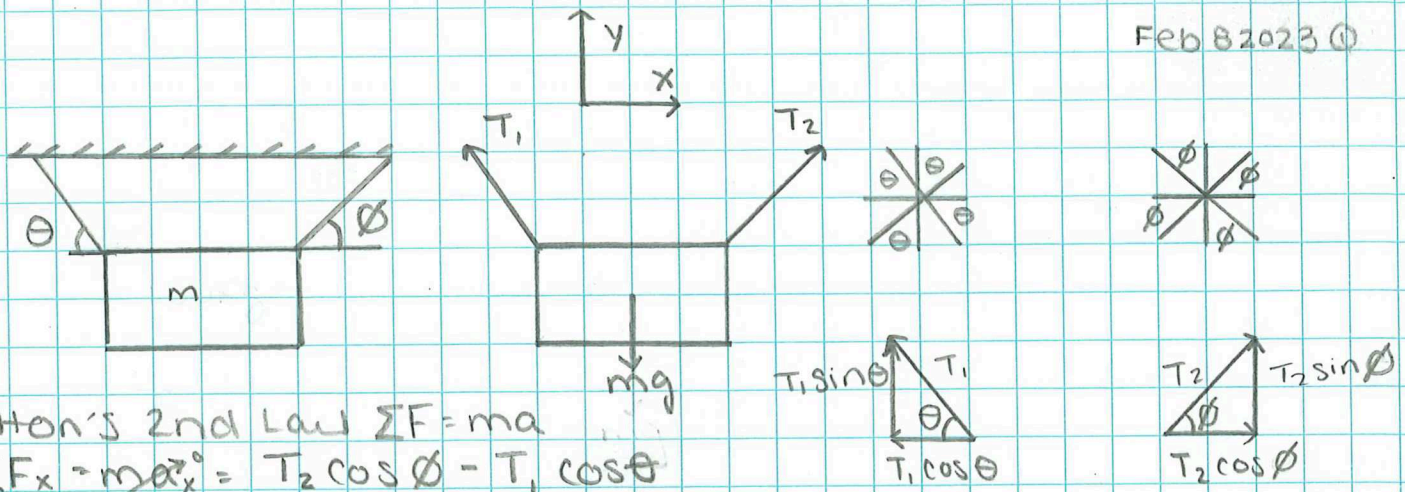
sub (2) and (3) into (1) with $F_f = \mu_s N$

$0 = Mg\sin(30) - [mg] - \mu_s [Mg\cos(20)]$

$m = M[\sin(30) - \mu_s \cos(30)]$

$m = 32.68$

Feb 8 2023



Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = m\cancel{a_x^{\circ}} = T_2 \cos\phi - T_1 \cos\theta$

(2) $\Sigma F_y = m\cancel{a_y^{\circ}} = T_1 \sin\theta + T_2 \sin\theta - mg$

Solve T_1 in (1)

$0 = T_2 \cos\phi - T_1 \cos\theta$

$T_1 \cos\theta = T_2 \cos\phi$

$T_1 = T_2 \dfrac{\cos\phi}{\cos\theta}$ (1a)

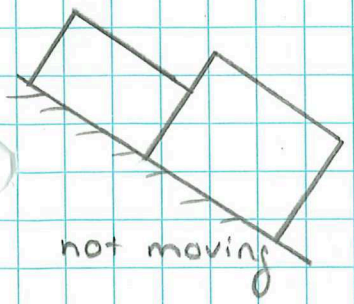
Sub (1a) into (2)

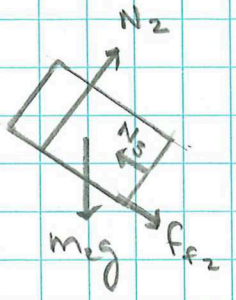
$0 = T_1 \sin\theta + T_2 \sin\theta - mg$

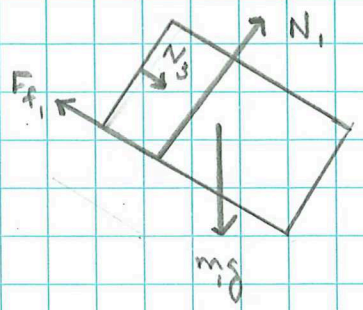
$0 = \left[T_2 \dfrac{\cos\phi}{\cos\theta}\right] \sin\theta + T_2 \sin\phi - mg$

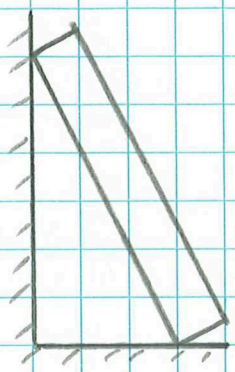
$mg = T_2 \left(\sin\phi + \dfrac{\cos\phi}{\cos\theta} \sin\theta\right)$

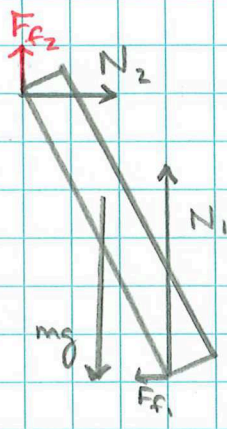
$T_2 = \dfrac{mg}{\sin\phi + \dfrac{\cos\phi}{\cos\theta} \sin\theta}$

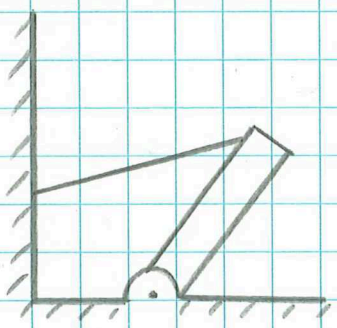


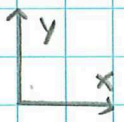


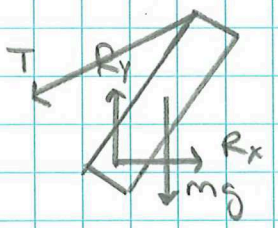




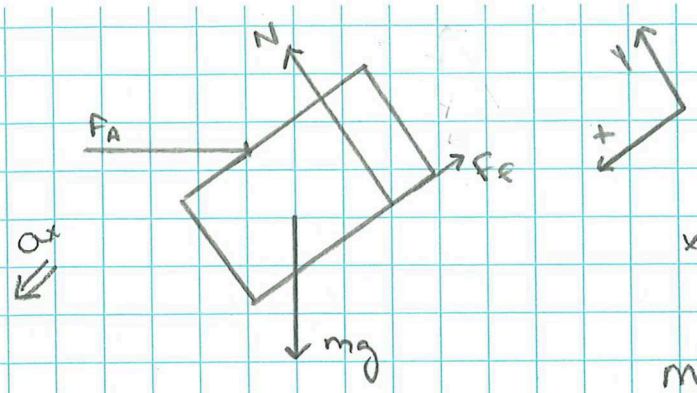


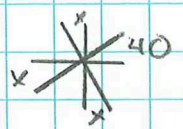


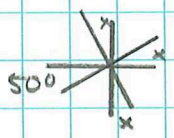




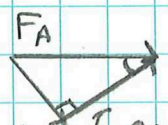
R = reaction force of the pin











$F_A \sin 50° = F_A \cos 50°$

Newton's 2nd Law $\Sigma F = ma$

(1) $\Sigma F_x = ma_x = mg\sin(40) - F_a\cos(50) - f_f$

(2) $\Sigma F_y = \cancel{ma_y^0} = N - mg\cos(40) - F_A\sin(50)$

Set $a_x = 0$ and $F_f = \mu_s N$

Solve (2) for N

$N = mg\cos(40) + F_A\sin(50)$ (2a)

Sub (2a) into (1)

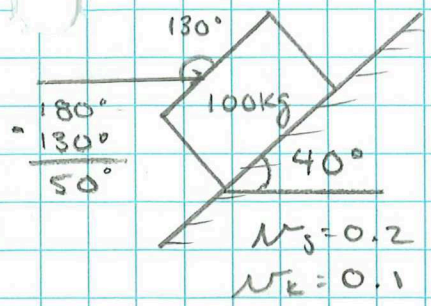
$0 = mg\sin(40) - F_a\cos(50) - \mu_s[mg\cos(40) + F_a\sin(50)]$

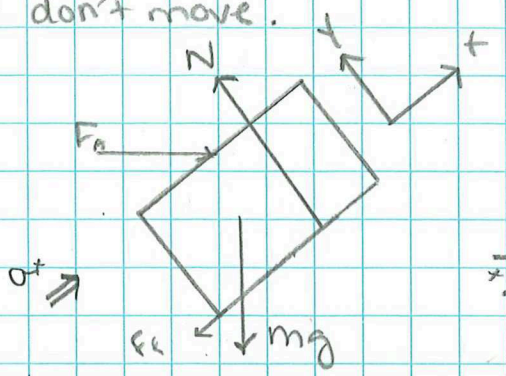
$F_a(\cos(50) + \mu_s\sin(50)) = mg\sin(40) - \mu_s mg\cos(40)$

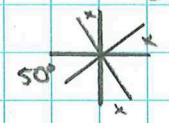
$F_A = \dfrac{mg\sin(40) - \mu_s mg\cos(40)}{\cos(50) + \mu_s\sin(50)} = 603.4\,N$

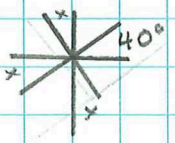
$F_a\cos(50) = mg\sin(40) - \mu_s mg\cos(40) - F_a\mu_s\sin(50)$

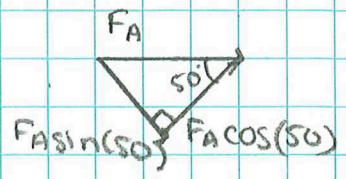
what is the min/max of F_a so it don't move.

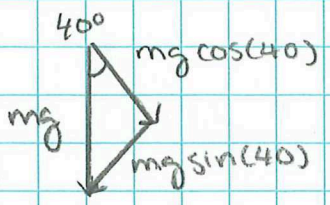












Newton's 2nd Law $\Sigma F = ma$

$\Sigma F_x = ma_x = F_a \cos(50) - mg \sin(40) - F_f$

$\Sigma F_y = ma_y^{\,0} = N - F_a \sin(50) - mg \cos(40)$

Set $a_x = 0$ & $F_f = \mu_s N$

Solve (2) for N

$N = mg \cos(40) + F_a \sin(50)$

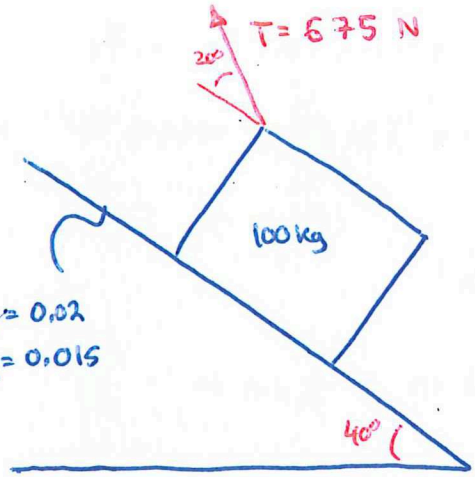
Sub (2a) into (1)

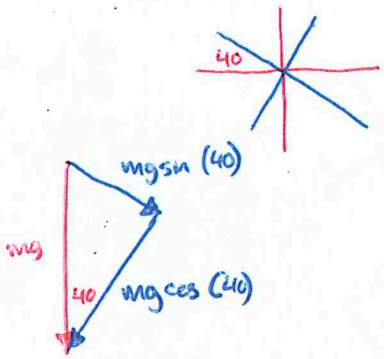
$0 = F_a \cos(50) - mg \sin(40) - \mu_s [mg \cos(40) + F_a \sin(50)]$

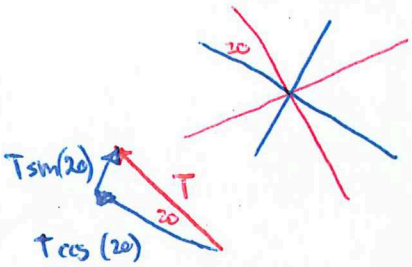
$0 = F_a \cos(50) - \mu_s F_a \sin(50) - mg \sin(40) - \mu_s mg \cos(40)$

$F_a (\cos(50) - \mu_s \sin(50)) = mg \sin(40) + \mu_s mg \cos(40)$

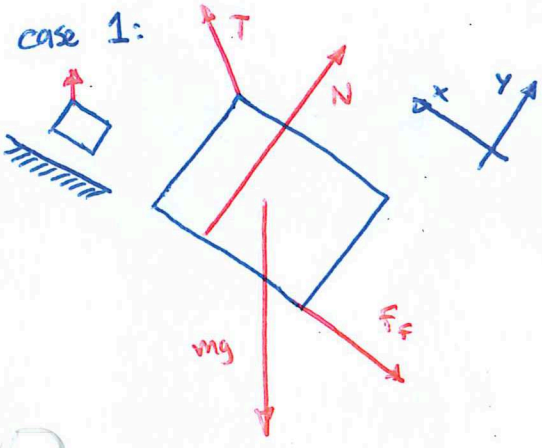
$$F_a = \frac{mg \sin(40) + \mu_s mg \cos(40)}{\cos(50) - \mu_s \sin(50)} = 1594.99\ N$$







case 1:



$$\sum F_x = ma_x = T\cos 20 - mg\sin 40 - f_f \quad (1)$$

$$\sum F_y = ma_y = N + T\sin 20 - mg\cos 40 \quad (2)$$

assume $a_y = 0$

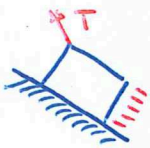
solve (2):

$$ma_y = N + T\sin(20) - mg\cos(40)$$

$$N = mg\cos 40 - T\sin 20 = 521\ N \checkmark$$

not lift.

case 2:



solve (1) for a_x:

$$a_x = \frac{T}{m}\cos(20) - g\sin 40 - \frac{f_f}{m} \quad (1a)$$

assume $f_f = \mu N$ (3)

sub (3) into (1a):

$$a_x = \frac{T}{m}\cos(20) - g\sin 40 - \frac{\mu N}{m}$$

use $\mu_s = 0.02 \Rightarrow a_x = -0.1\ m/s^2$

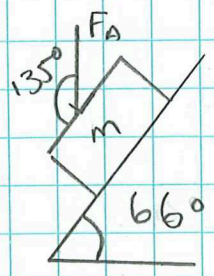
not moving up

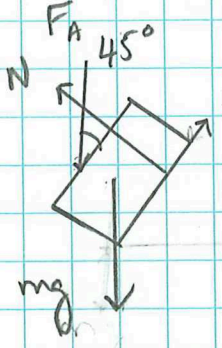
What is the normal force?

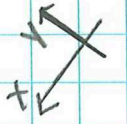
① $f_a = 1400\ N$ $m = 100\ kg$ $\phi = 135°$ $\theta = 66°$

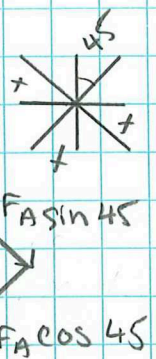
$\mu_s = 0.3$

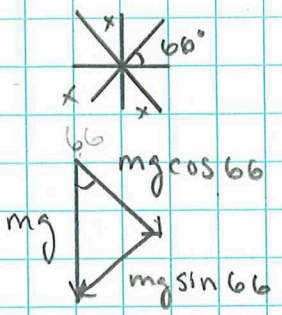
$\mu_k = 0.2$











Newton's 2nd Law

$\Sigma F = ma_y^{\,0} = N - F_A \sin 45 - mg \cos 66$

$\Sigma F = ma_x = F_A \cos 45 + mg \sin 66 - F_f$

Does it lift up?

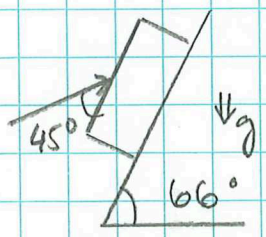
$0 = N - 1400 \sin 45 - 100(9.81) \cos 66$

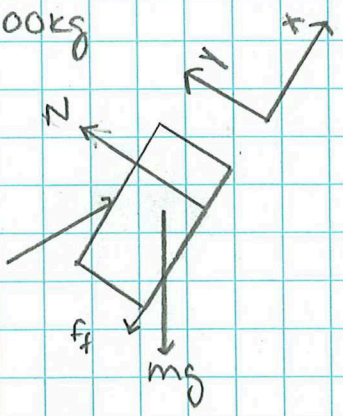
$N = 989.949 + 399.001$

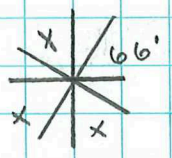
$N = 1388.96\ N$ ✓

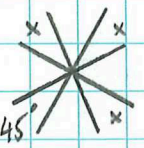
Box 2 #2

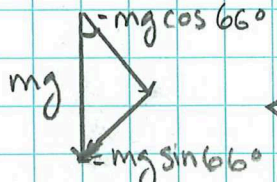
① $F_A = 800\,N$ $M = 100\,kg$

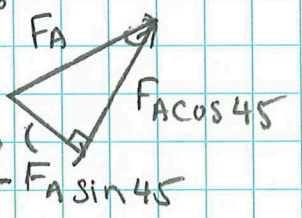












$\mu_s = 0.3$

$\mu_K = 0.2$

Newton's 2nd Law

(1) $\sum F_x = ma_x = F_A\cos 45 - F_f - mg\sin 66°$

(2) $\sum F_x = m\vec{a}_y^{\,0} = N - mg\cos 66° - F_A\cos 45°$

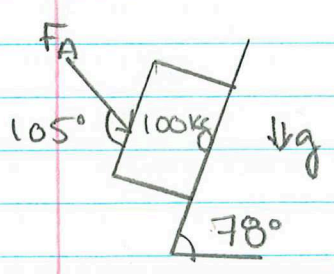
$0 = N - 100(9.81)\cos 66 + 800\cos 45°$

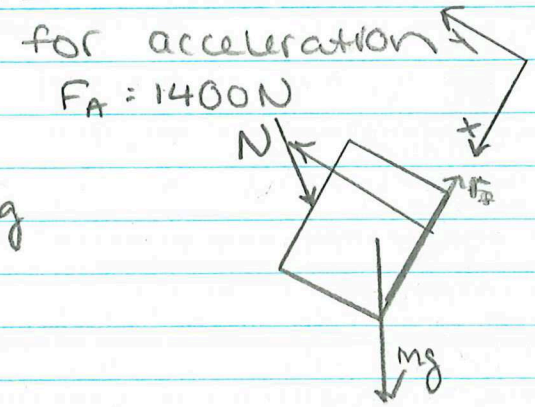
$N = 399.009 + 565.685$

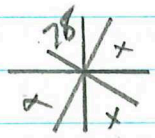
$N = 964.694\,N$ ✓

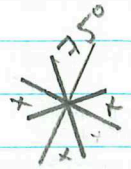
Box 2 #2

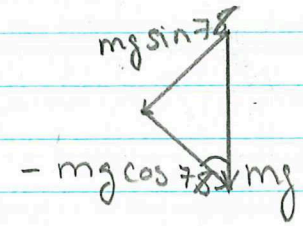
② Solve for acceleration

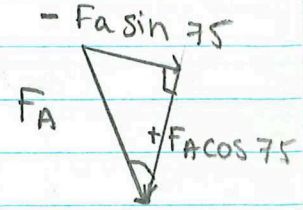












$\mu_s = 0.3$

$\mu_k = 0.2$

Newton's 2nd Law

(1) $\sum F_x = ma_x = mg \sin 78 + Fa \cos 75 - ff$

(2) $\sum F_y = ma_y^{\,0} = N - mg \cos 78 - Fa \sin 75$



Does it float?

$0 = N - 100(9.81)\cos 78 - 1400 \sin 75$

$N = 203.961 + 1352.296$

$N = 1556.257\ N$

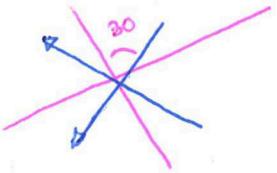
Does it accelerate? $\mu_s N = f_f$ $0.3(1556.257)$

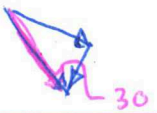
$100 a_x = 100(9.81)\sin 78 + 1400 \cos 75 - 0.3(1556.257)$

$\boxed{a_x = 8.55\ m/s^2}$

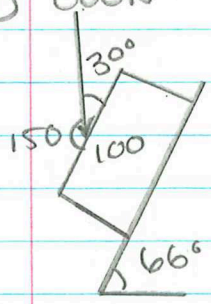
needed to resolve ∴ $\mu_k = 0.2$ A: 10.11

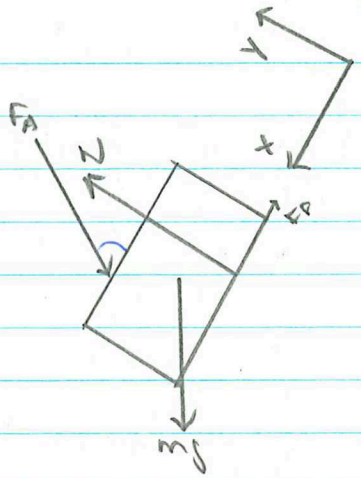
Box 2 #3

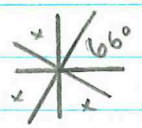


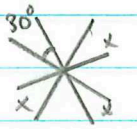


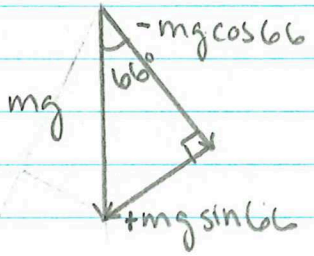
① 800N

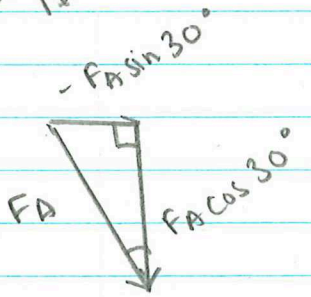












$\Sigma F_x = ma_x = mg\sin 66 + Fa\cos 30 - F_f$

$\Sigma F_y = ma_y^{\,0} = N - mg\cos 66 - f_A\sin 30$



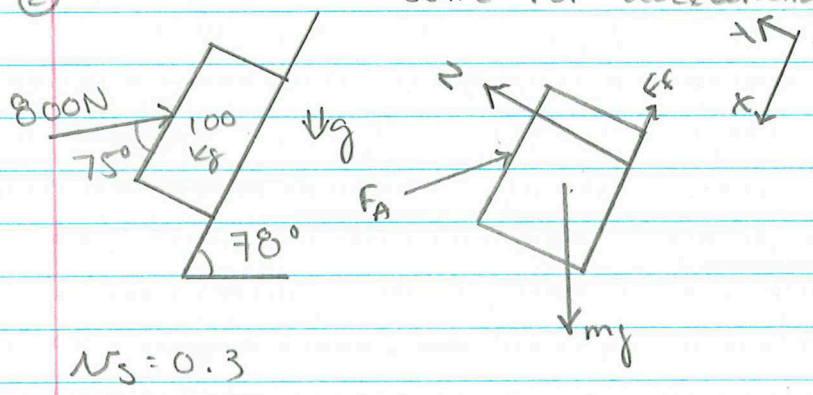
$0 = N - 100(9.81)\cos 66 - 800\sin 30$

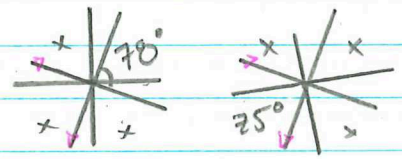
$N = 399.009 + 692.820$ → 400

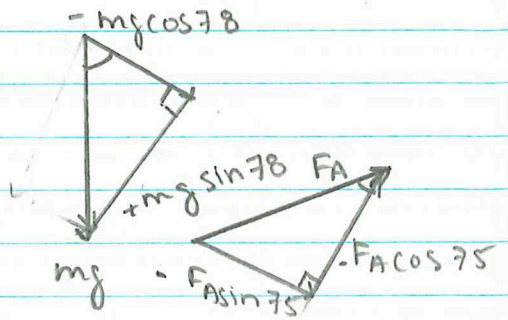
$N = 1091.83\,N$ $\quad A = 799.01$

Box 2 #3

② Solve for acceleration.







$\mu_S = 0.3$
$\mu_K = 0.2$

$\Sigma F_x = ma_x = mg\sin 78 - F_A\cos 75 - f_f$
$\Sigma F_y = ma_y = N - mg\cos 78 - F_A\sin 75$

$0 = N - (100)(9.81)\cos 78 - 800\sin 75$
$N = 203.961 + \cancel{207.055}$ → 772
$N = \cancel{411.016}$ N
976.70

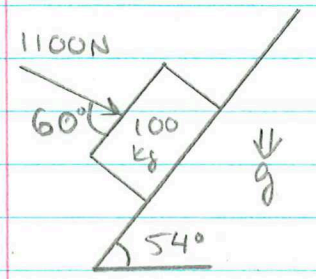
$\mu_S N = f_f$

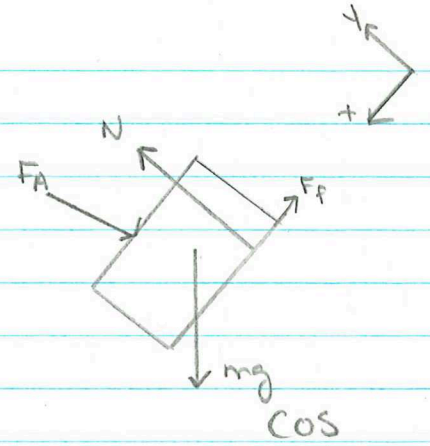
$100\,a_x = 100(9.81)\sin 78 - 800\cos 75 - 0.3(411.016)$
$a_x = 6.292\ m/s^2$ ✓ yes it accelerates.

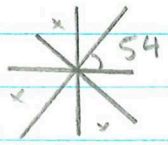
replace $\mu_S \rightarrow \mu_K$ 0.2

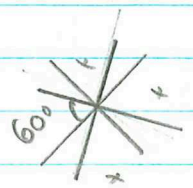
$a_x = 6.703\ m/s^2$ A: 5.57

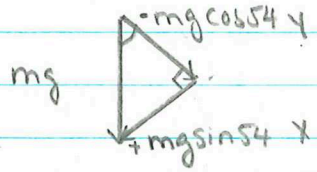
Box 2 #4

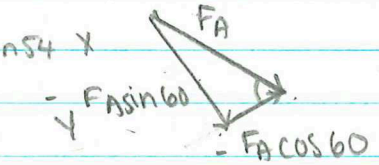
① 











$\Sigma F_x = ma_x = mg\sin 54 - F_A(\sin)60$

$\Sigma F_y = ma_y^0 = N - mg\cos 54 - Fa(\cos)60$
sin

$0 = N - (100)(9.81)\cos 54 - 1100(\cos 60)$
sin

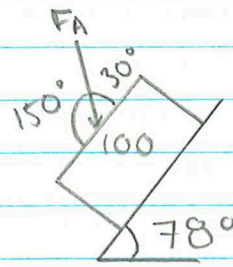
$N = 576.617 + 550 \leftarrow 952.627$

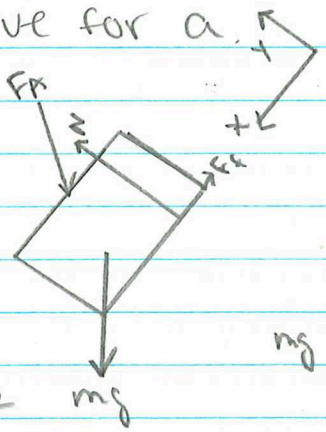
$N = 1126.617$ $\quad A = 1529.25$

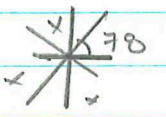
1,529.24 N

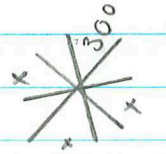
Box 2 #4 Solve for a.

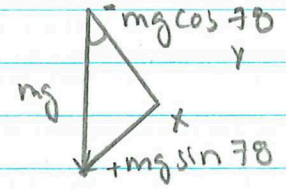
②

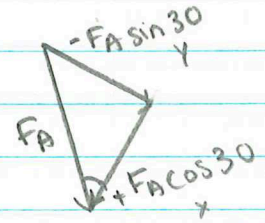












$\mu_S = 0.3, \mu_k = 0.2$

$\Sigma F_x = ma_x = mg \sin 78 + F_A \cos 30 - f_f$

$\Sigma F_y = ma_y^0 = N - mg \cos 78 - F_A \sin 30$

$0 = N - (100)(9.81)\cos 78 - 800 \sin 30$

$N = 203.961 + 400$

$N = 603.961\ N$

$F_f = \mu_S N$

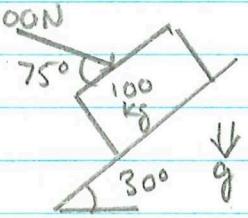
$100(a_x) = 100(9.81)\sin 78 + 800(\cos)30 - 0.3(603.961)$

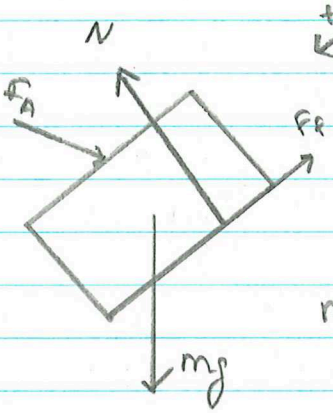
$a_x = 14.712$

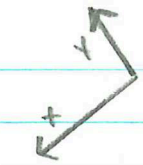
$\mu_S \to \mu_k = 15.316$ ✓

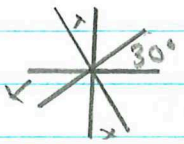
Box 2 #5

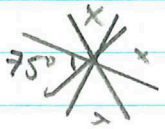
① 1400N

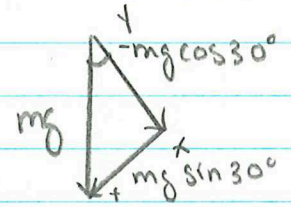


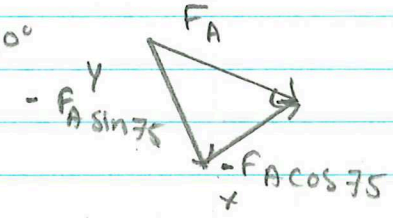












$\Sigma F_x = ma_x =$

$\Sigma F_y = ma_y = N - mg\cos 30° - F_A \sin 75°$

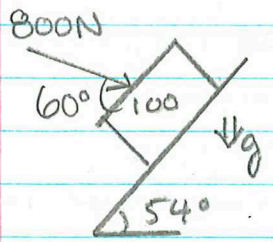
$0 = N - (100)(9.81)\cos(30) - 1400\sin(75)$

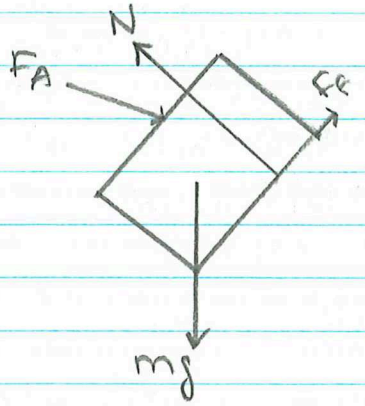
$N = 849.571 + 1352.296$

$N = 2201.87$

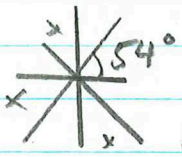
Box 2 #5

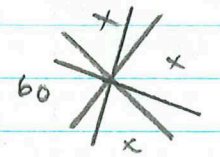
②

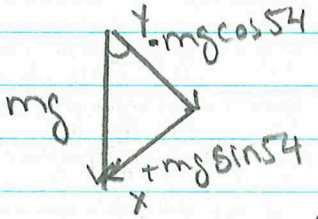


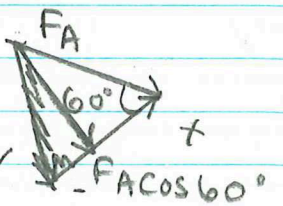












$\mu_s = 0.3$

$\mu_k = 0.2$

$\Sigma F_x = ma_x = mg\sin 54 - F_A\cos 60 - f_f$ $\left\{ - \text{Y } f_A \sin 60° \right.$

$\Sigma F_y = ma_y = N - mg\cos 54 - fa\sin 60°$ $\left\{ \right.$

$0 = N - (100)(9.81)\cos 54 - 800\sin(60)$

$N = 576.617 + 692.820$

$N = 1269.437\ N$

$\mu_s N = f_f \quad 0.3$

$100(a_x) = 100(9.81)\sin 54 - 800\cos(60) - 0.3(1269.437)$

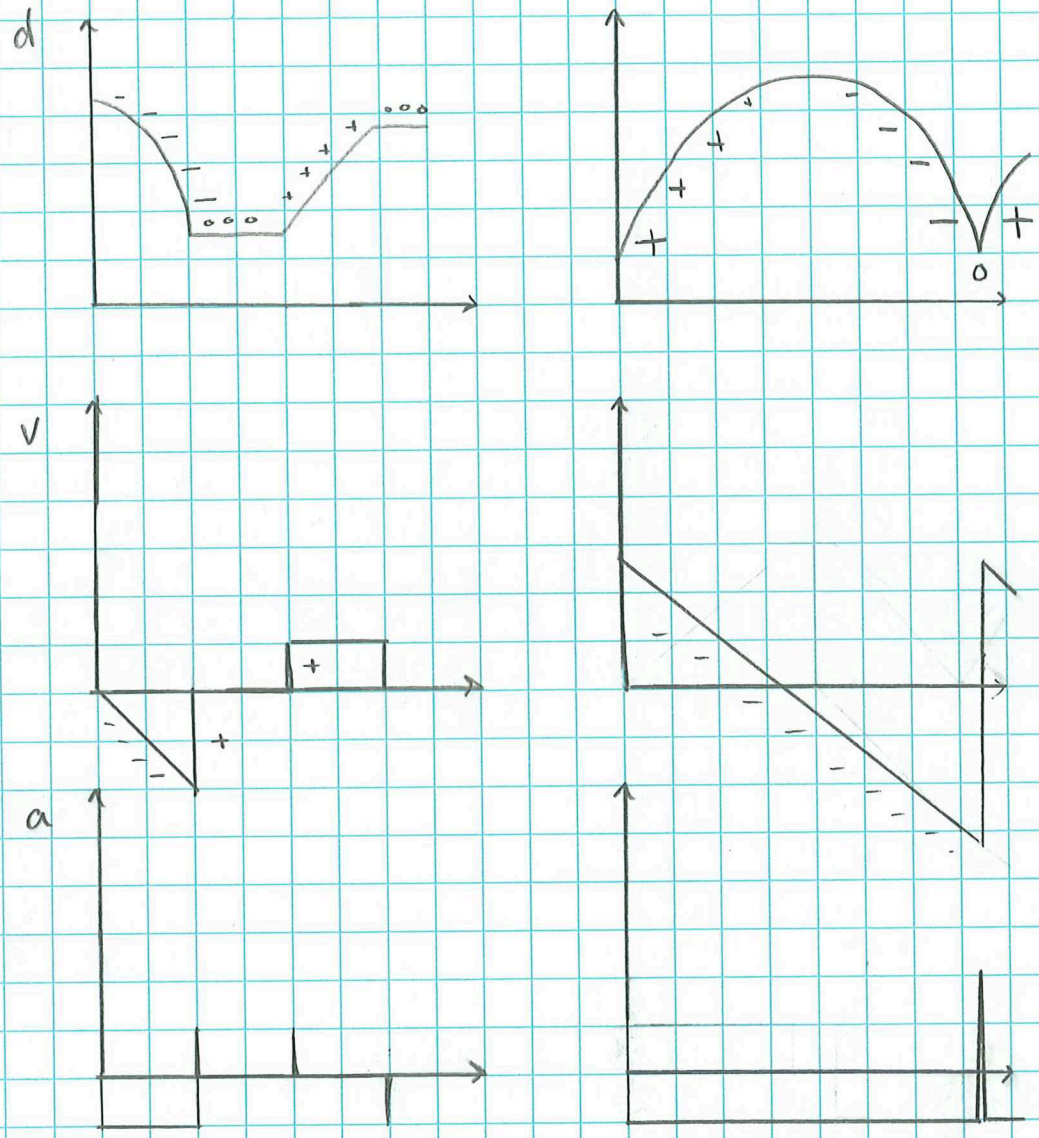
$a_x = 0.128$

$\mu_k N = f_f \quad 0.2$

$a_x = 1.39758$

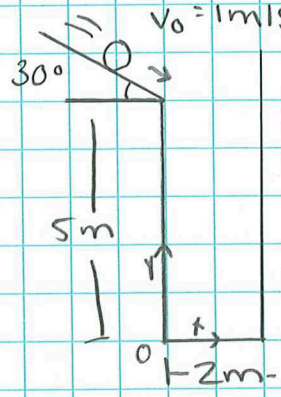
Test 1 Review

Feb 14 23 ①



Test 1 Review

Foo. 14.23 (2)



where does the ball hit?

$\sin 30 = \frac{V_{oy}}{1}$

$V_{oy} = 0.5$

$\cos 30 = \frac{V_{ox}}{1}$

$V_{ox} = 0.866$

x	y	t
$x_0 = 0$	$y_0 = 5$	$t =$
$x_1 = 2?$	$y_1 = 0?$	
$V_{0x} = 0.866$	$V_{oy} = -0.5$	
$V_{1x} = 0.866$	$V_{1y} = -9.917$	
$a_x = 0$	$a_y = -9.81$	

Solve V_{1y}

$V_{1y}^2 = V_{0y}^2 + 2a_y(y_1 - y_0)$

$V_{1y}^2 = (-0.5)^2 + 2(-9.81)(0-5)$

$V_{1y}^2 = 0.25 + 98.1$

$V_{1y} = -9.917$ m/s

Solve x_1

$x_1 = \cancel{x_0}^{\,0} + V_{ox}t + \frac{1}{2}a_x t^2$ (the $\frac{1}{2}a_x t^2$ term also $= 0$)

$2 = 0.866(t)$

$t = \frac{2}{0.866} = 2.309$s

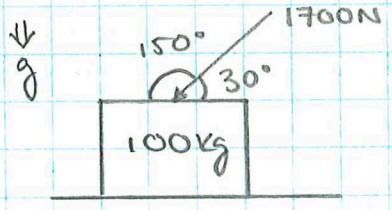
$y_1 = y_0 + V_{oy}t + \frac{1}{2}a_y t^2$

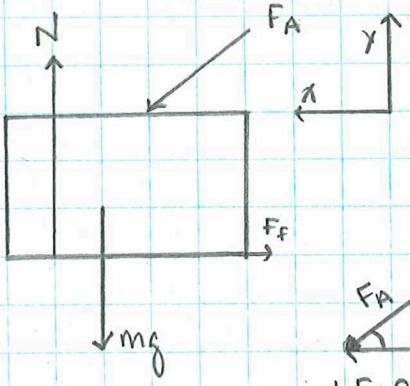
$y = 5 + (-0.5)(2.3) + \frac{1}{2}(-9.81)(2.3)^2$

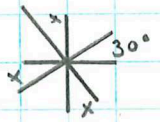
$= -25.0$ = wrong assumption

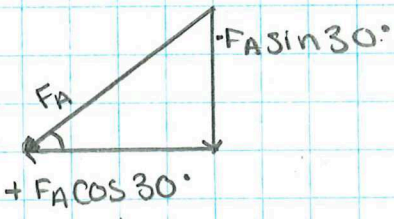
Box 1 Attempt 1

Feb 15 23 ①









$\mu_s = 0.3$

$\mu_k = 0.2$

$\Sigma F_x = ma_x = F_A \cos 30 - F_f$

$\Sigma F_y = m\cancel{a_y}^{0} = N - mg - F_A \sin 30$

Solve for Normal

$0 = N - (100)(9.81) - (1700)\sin 30$

$N = 981 + 850 = 1831\ N$

Solve for acceleration $F_f = \mu_s N$ does it move ←?

$ma_x = F_A \cos(30) - \mu_s N$

$100(a_x) = (1700)\cos(30) - 0.3(1831)$

$a_x = \dfrac{1472.24 - 549.3}{100} = 9.2294$ ✓ Does accelerate ←

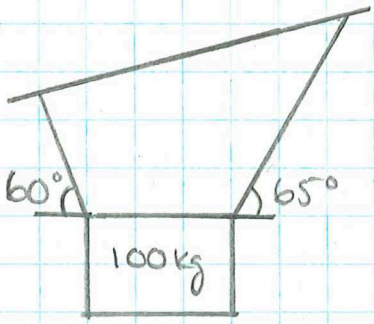
Resolve with $\mu_k = 0.2$

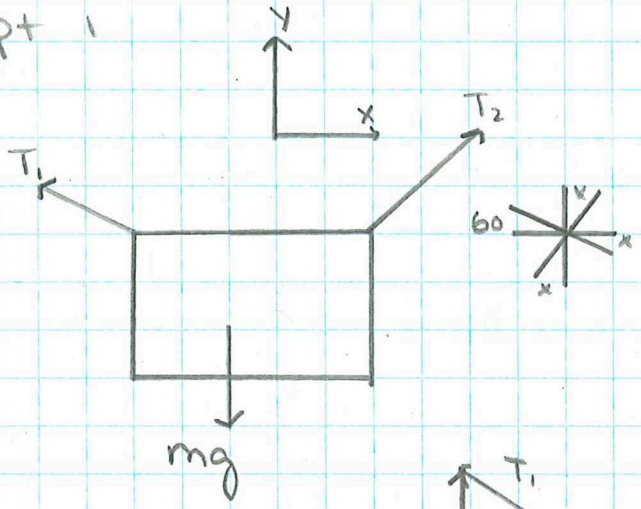
$100(a_x) = 1700\cos 30 - 0.2(1831)$

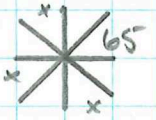
$a_x = \dfrac{1472.24 - 366.2}{100} = 11.06\ m/s^2$ ✓

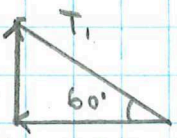
hanging Attempt 1

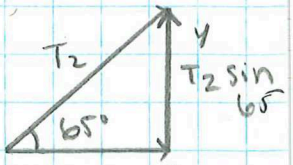
Feb. 15. 23 (2)











(1) $\sum F_x = m\cancel{a_x}^{0} = T_2\cos 65 - T_1\cos 60$

(2) $\sum F_y = m\cancel{a_y}^{0} = T_2\sin 65 + T_1\sin 60 - mg$

$$0 = T_2\cos 65 - T_1\cos 60$$

$$T_1\cos 60 = T_2\cos 65$$

$$T_1 = \frac{T_2\cos 65}{\cos 60} \quad (1a)$$

Sub (1a) into (2)

$$0 = T_2\sin 65 + \left[\frac{T_2\cos 65}{\cos 60}\right]\sin 60 - (100)(9.81)$$

$$981 = T_2\sin 65 + \left[\frac{T_2\cos 65}{\cos 60}\right]\sin 60$$

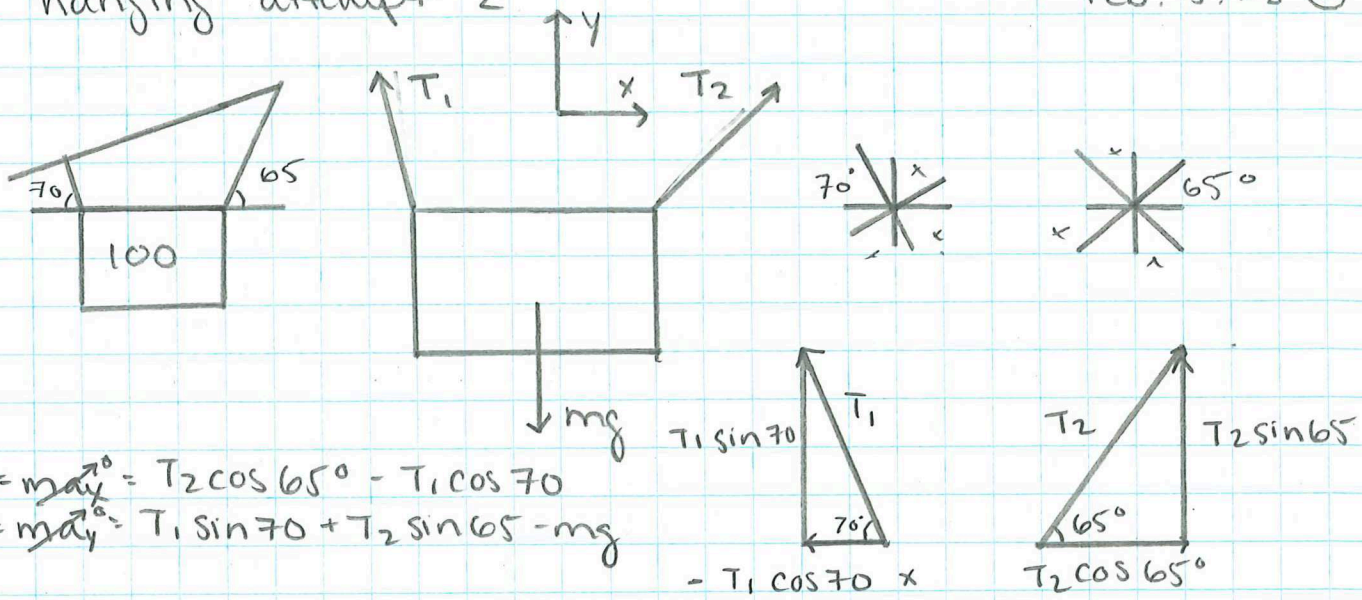
$$981 = T_2\left(\sin 65 + \frac{\cos 65(\sin 60)}{\cos 60}\right)$$

$$T_2 = \frac{981}{\left(\sin 65 + \frac{\cos 65(\sin 60)}{\cos 60}\right)} = \frac{981}{\left(\frac{1.27}{\cos 60}\right)}$$

$$T_2 = \frac{981}{2.54} = 385.52\,N \times A = 598.79\,N$$

$$T_1 = \frac{385.52\cos 65}{\cos 60} = 325.86\,N \times A = 506.11\,N$$

hanging attempt 2



(1) $\Sigma F_x = m\vec{a}_x^{\,0} = T_2 \cos 65° - T_1 \cos 70$

(2) $\Sigma F_y = m\vec{a}_y^{\,0} = T_1 \sin 70 + T_2 \sin 65 - mg$

$$0 = T_2 \cos 65 - T_1 \cos 70$$

$$T_1 \cos 70 = T_2 \cos 65$$

$$T_1 = \frac{T_2 \cos 65}{\cos 70} \quad (1a)$$

Sub (1a) into (2)

$$0 = \left[\frac{T_2 \cos 65}{\cos 70}\right] \sin 70 + T_2 \sin 65 - mg$$

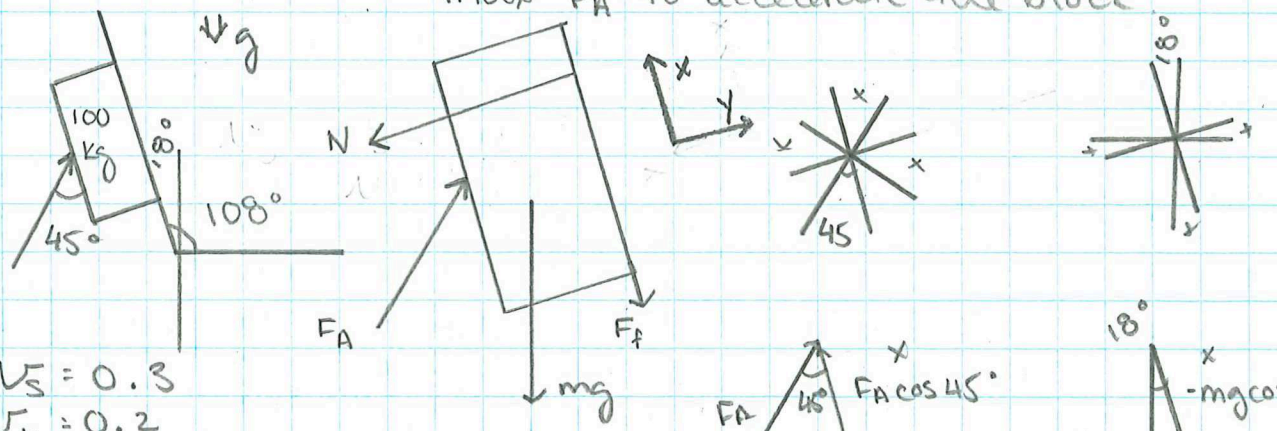
$$981 = T_2 \left(\frac{\cos 65 (\sin 70)}{\cos 70} + \sin 65\right)$$

$$\frac{981}{\left(\frac{\cos 65 (\sin 70)}{\cos 70} + \sin 65\right)} = T_2$$

$$T_2 = \frac{981}{2.067} = 474.50\,N$$

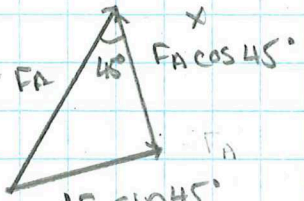
Box 3 Attempt 1 (min max) Feb. 15. 23 (2)

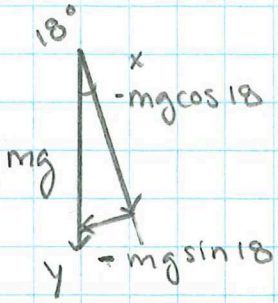
max F_A to accelerate the block



$\mu_s = 0.3$

$\mu_k = 0.2$





(1) $\Sigma F_x = ma_x^{\,0} = F_A \cos 45 - mg\cos 18 - f_f$

(2) $\Sigma F_y = ma_y^{\,0} = -mg\sin 18 + F_A \sin 45 - N$

Set $a_x = 0$ $F_f = \mu_s N$ to solve for F_A

$0 = F_A \cos 45 - mg\cos 18 - F_f$

$0 = F_A \cos 45 - (100)(9.81)\cos 18 - (0.3)(N)$

$0 = -mg\sin 18 + F_A \sin 45 - N$

$N = F_A \sin(45) - mg\sin(18)$

sub into the other equation.

$0 = F_A \cos 45 - (100)(9.81)\cos 18 - (0.3)[(F_A \sin 45) - (100)(9.81)\sin 18]$

$0 = F_A(\cos 45 - 0.3\sin 45) - 932.986 - 303.146$

$1236.132 = F_A(\cos 45 - 0.3\sin 45)$

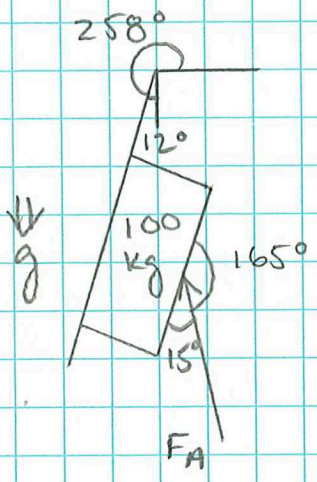
$F_A = \dfrac{1236.132}{\cos 45 - 0.3\sin 45} = \dfrac{1236.132}{0.4950} = 2497.36\ N$

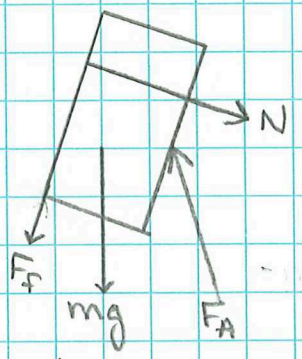
$A = 1701.16$

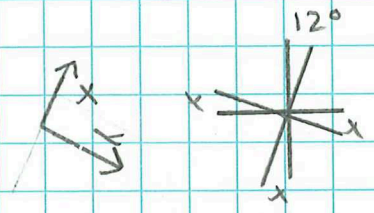
$= 1113.87.$

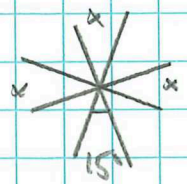
Box 3 Attempt 2. Feb. 19. 23 ①

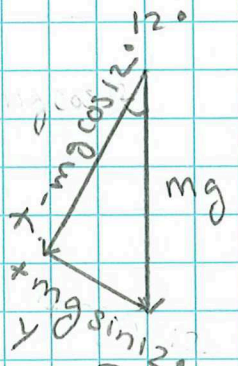
Minimum applied force to accelerate.

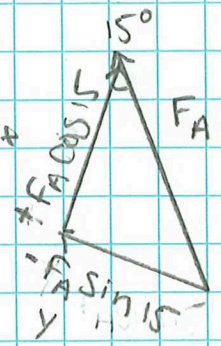












$\mu_s = 0.3$

$\mu_K = 0.2$

(1) $\Sigma F_x = ma_x^{\to 0} = F_A \cos 15 - mg \cos 12 - F_f$

(2) $\Sigma F_y = ma_y^{\to 0} = N + mg \sin 12 - F_A \sin 15$

$F_f = \mu_s N$ set $a_x = 0$ solve for N

$0 = F_A \cos 15 - (100)(9.81)\cos 12 - (0.3)(N)$

$(0.3)(N) = F_A \cos 15 - (100)9.81(\cos 12)$

$N = \dfrac{F_A \cos 15 - 100(9.81)\cos 12}{0.3}$

sub into (2) to find F_A set $a_y = 0$

$0 = \dfrac{F_A \cos 15 - 100(9.81)\cos 12}{0.3} + (100)(9.81)\sin 12 - F_A \sin 15$

$0 = F_A\left(\dfrac{\cos 15 - [959.56]}{0.3} - \sin 15\right) + 203.96$

$-\dfrac{959.56}{0.3}$ seperate entity

$0 = F_A(-3195.31) + 203.96$

$\dfrac{-203.96}{-3195.31} = F_A = 0.0638\,N$

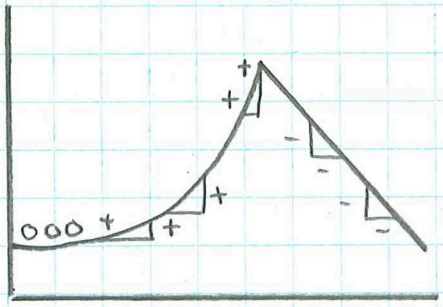
resolve using μ_K

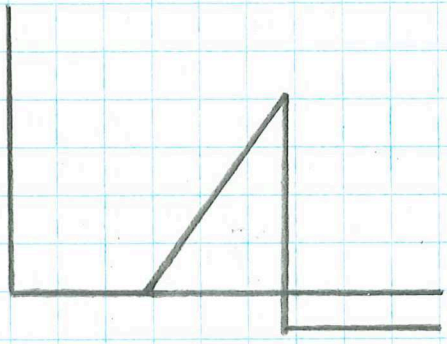
$0 = F_A\left(\dfrac{\cos 15 - 959.56}{0.2}\right) + 203.96$

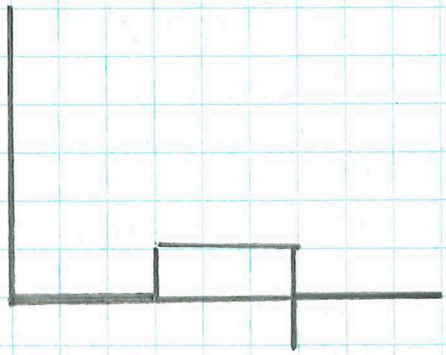
$\dfrac{-203.96}{-4,792.97} = F_A = 0.0425\,N$ A-978.11, 1011.34

Test Review Q1

slope = $\frac{\Delta y}{\Delta x}$







Instructor:	Frank Secretain
Course:	Civl 11
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers. This sheet can be ripped off and used as formula sheet. Write your name at the top of the second sheet (question 1).
Marks allocated:	4 questions worth 40 marks + bonus worth 2 marks **(on back)**
Percentage of final grade:	20% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

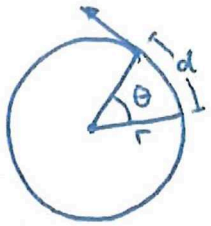
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x_o} + \vec{v_o}t + \frac{1}{2}\vec{a}t^2 \qquad \theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\vec{v}^{\,2} = \vec{v_o}^{\,2} + 2\vec{a}(\vec{x} - \vec{x_o}) \qquad \omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\vec{v} = \vec{v_o} + \vec{a}t \qquad \omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \qquad \vec{F_g} = \frac{Gm_1m_2}{\vec{r}^{\,2}} = mg \qquad \vec{F_f} = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M_p} = \frac{d(I_p\vec{\omega_p})}{dt} = I_p\vec{\alpha_p} \qquad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

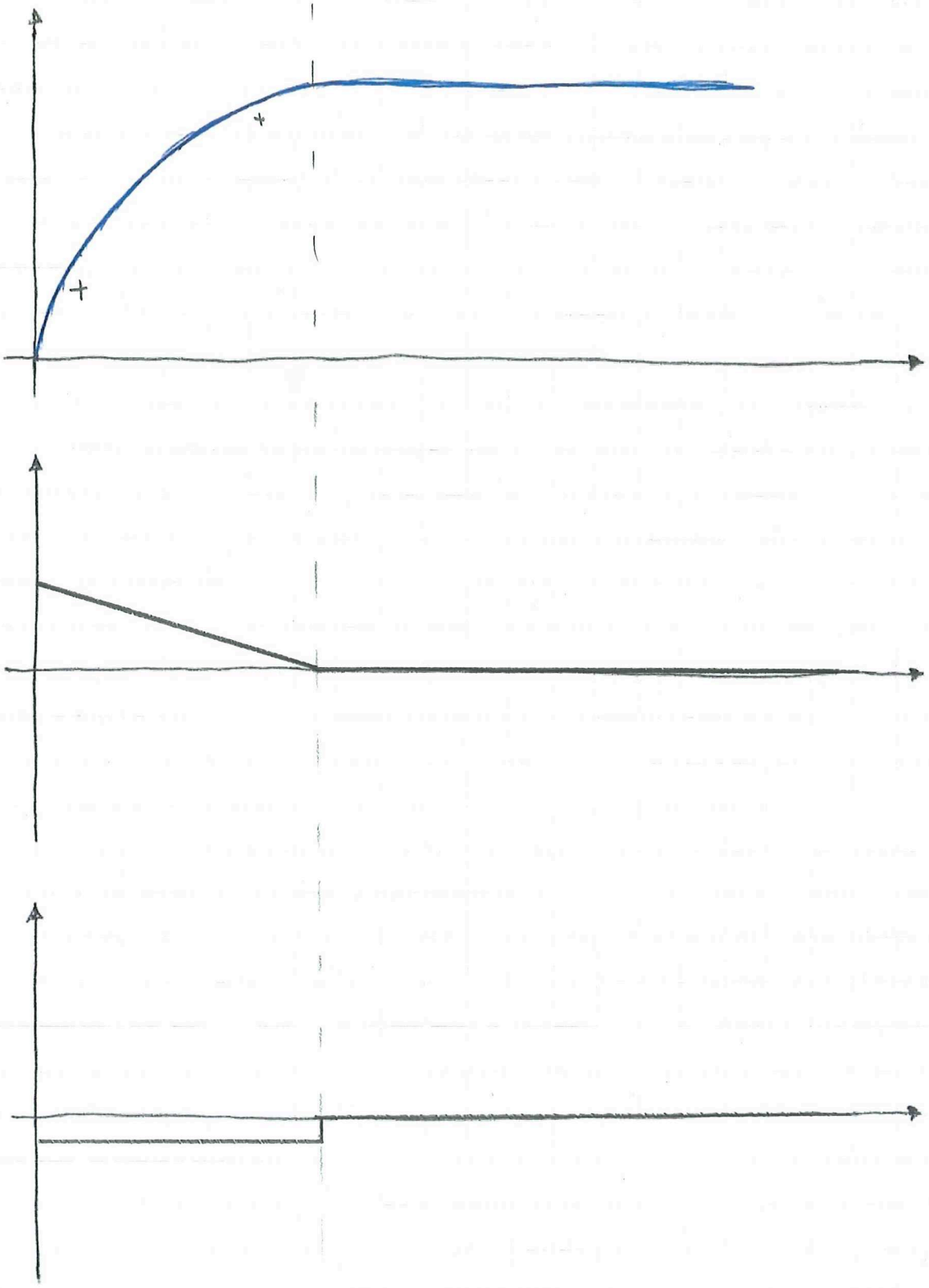
$$\vec{p} = m\vec{v} \qquad \vec{p} - \vec{p_o} = \int \vec{F}dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

$$E_f - E_o = Q - W \qquad E_{PE} = mg(y - y_o) \qquad E_{KE} = \frac{1}{2}mv^2$$

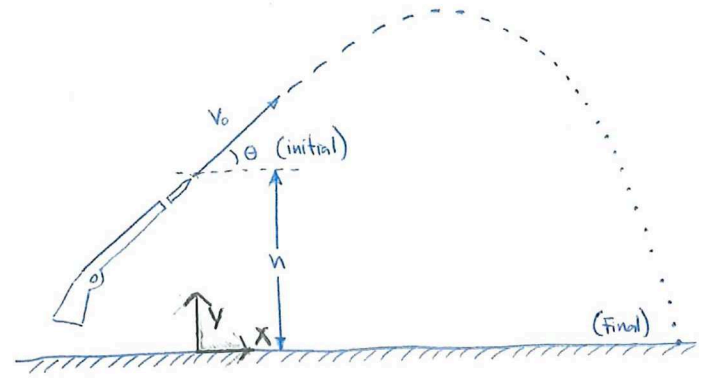
$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

(5 marks) Complete the distance-velocity-acceleration graph.



(10 marks) Determine the time of flight for the given initial conditions:

a. v_o = 330 m/s, h = 2 m, theta = 90 degrees
b. v_o = 330 m/s, h = 2 m, theta = 30 degrees

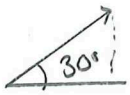


A. x: $x_0 = 0$, $x_1 =$; y: $y_0 = 2$, $y_1 = 0$; t: $t =$

$V_{ox} = 285.79$ $V_{oy} = 165$

$V_{1x} = 285.79$ $V_{1y} =$

$a_x = 0$ $a_y = -9.81$



$\cos 30 = \frac{V_{ox}}{330}$ $V_{ox} = 285.788$

$\sin 30 = \frac{V_{oy}}{330}$ $V_{oy} = 165$

① $V_{1y}^2 = V_{oy}^2 + 2(a_y)(y_1 - y_0)$

$V_{1y}^2 = (165)^2 + (2)(-9.81)(0-2)$

$V_{1y}^2 = 27225 + 39.24$

$V_{1y} = \pm\sqrt{27264.24}$

$V_{1y} = -165.12$ m/s

② $V_{1y} = V_{oy} + a_y(t)$

$-165.12 = 165 + (-9.81)(t)$

$\frac{-330.12}{-9.81} = t$

$\boxed{t = 33.655}$ ✓

B. x: $x_0 = 0$, $x_1 =$; y: $y_0 = 2$, $y_1 = 0$; t: $t =$

$V_{ox} = 0$ $V_{oy} = 330$

$V_{1x} = 0$ $V_{1y} =$

$a_x = 0$ $a_y = -9.81$

$V_{1y}^2 = V_{oy}^2 + 2(a_y)(y_1 - y_0)$

$V_{1y}^2 = (330)^2 + (2)(-9.81)(-2)$

$V_{1y}^2 = 108900 + 39.24$

$V_{1y} = \pm\sqrt{108939.24}$

$V_{1y} = -330.059$

↳ $\cos 90 = \frac{V_{ox}}{330}$ $V_{ox} = 0$

$\sin 90 = \frac{V_{oy}}{330}$ $V_{oy} = 330$

$V_{1y} = V_{oy} + a_y(t)$

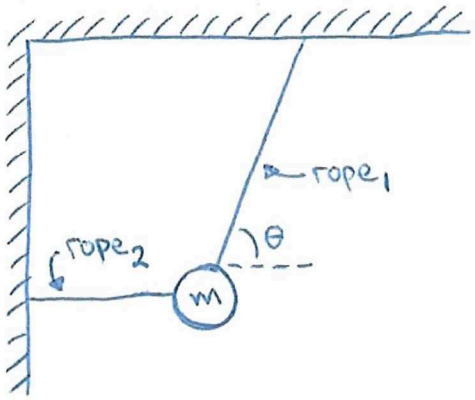
$-330.059 = 330 + (-9.81)t$

$\frac{-660.059}{-9.81} = t$

$\boxed{t = 67.285}$ ✓

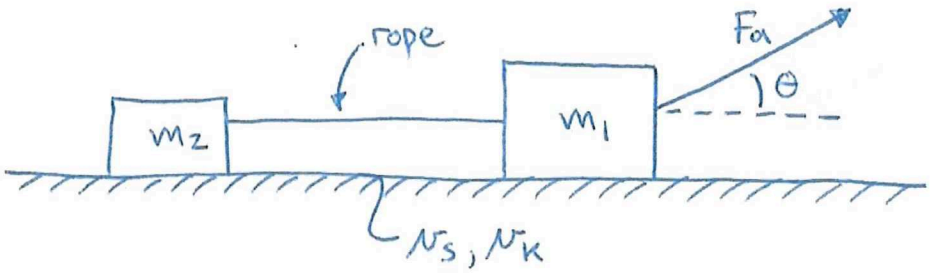
(10 marks) Determine the tension in each rope given initial conditions:

m = 100 kg
theta = 75 degrees



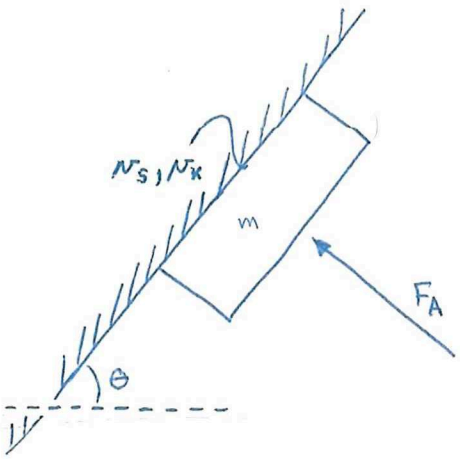
(15 marks) Determine the tension in the rope and the acceleration given initial conditions:

F_a = 500 N
theta = 35 degrees
m_1 = 100 kg
m_2 = 50 kg
u_s = 0.1
u_k = 0.05

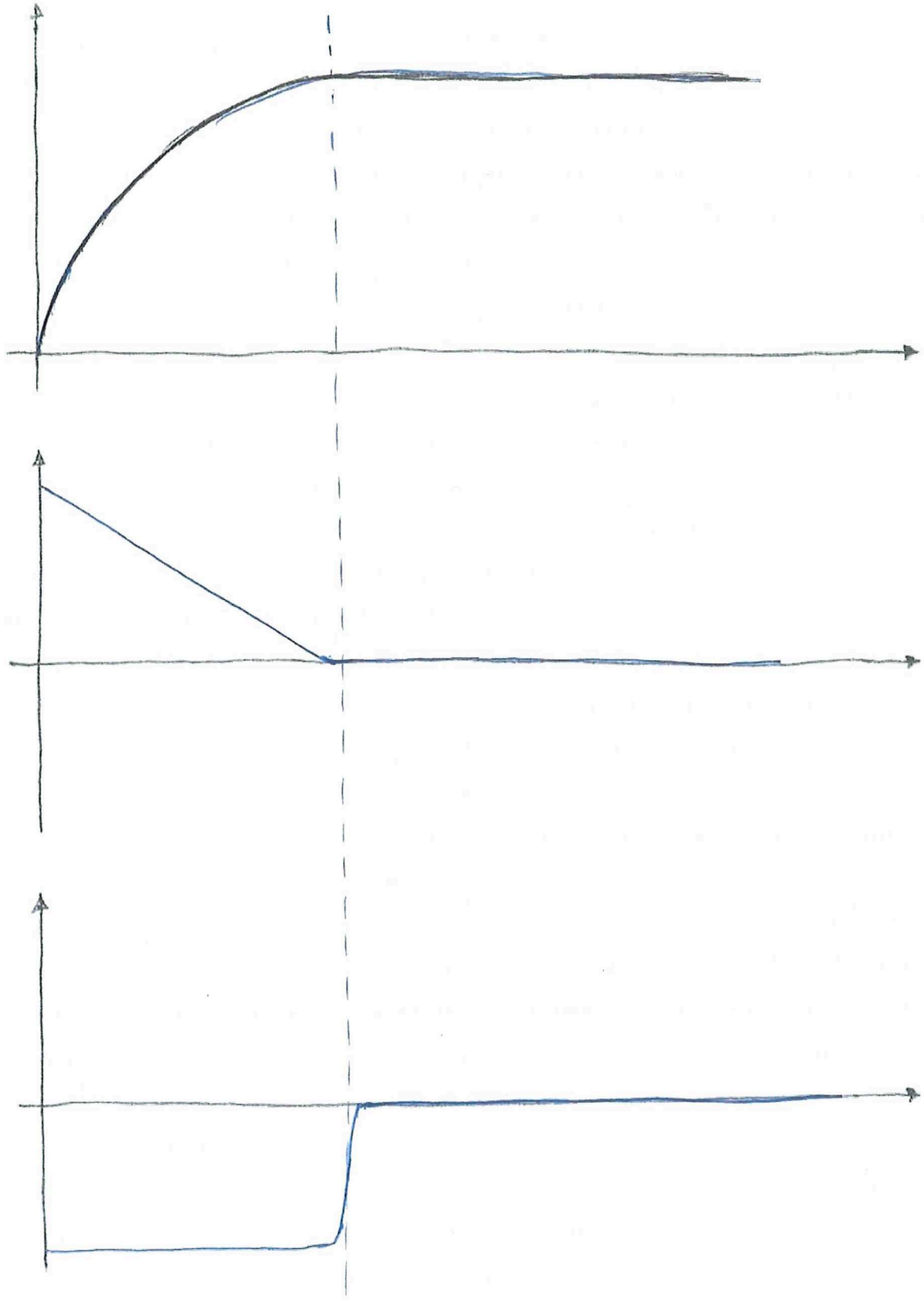


BONUS (2 marks)

Determine the minimum force need to keep the mass from moving.
Does the mass fall or does it slip first?

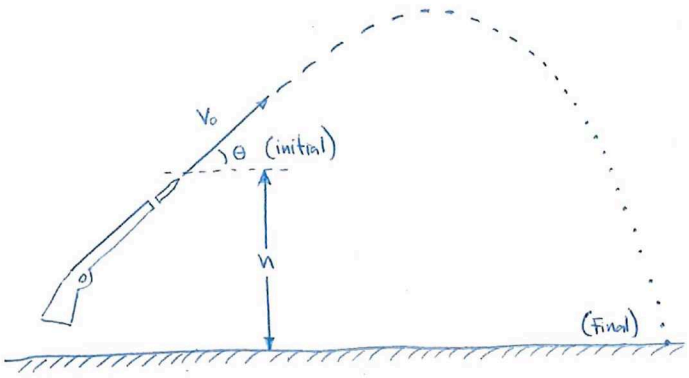


(5 marks) Complete the distance-velocity-acceleration graph.

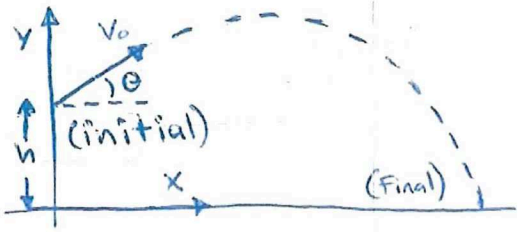


(10 marks) Determine the time of flight for the given initial conditions:

a. v_o = 330 m/s, h = 2 m, theta = 90 degrees
b. v_o = 330 m/s, h = 2 m, theta = 30 degrees



x-dir	y-dir	time
$x_o = 0$	$y_o = h$	$t =$
$x_f =$	$y_f = 0$	
$V_{ox} = V_o \cos\theta$	$V_{oy} = V_o \sin\theta$	
$V_{fx} =$	$V_{fy} =$	
$a_x = 0$	$a_y = -9.81\ m/s^2$	





Kinematic equations:

$$\vec{V}^2 = \vec{V}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o) \qquad \vec{V} = \vec{V}_o + at$$

$$V_{fy}^2 = V_{oy}^2 + 2a_y(y_f - y_o) \qquad V_{fy} = V_{oy} + a_y t$$

$$V_{fy} = \pm\sqrt{V_{oy}^2 - 2a_y h} \qquad t = \frac{V_{fy} - V_{oy}}{a_y}$$

a)

$$V_{fy} = -\sqrt{(330\sin 90)^2 - 2(-9.81)(2)}$$

$$= -330.1\ m/s$$

$$t = \frac{-330.1 - 330\sin 90}{-9.81}$$

$$\boxed{t = 67\ sec}$$

b)

$$V_{fy} = -\sqrt{(330\sin 30)^2 - 2(-9.81)(2)}$$

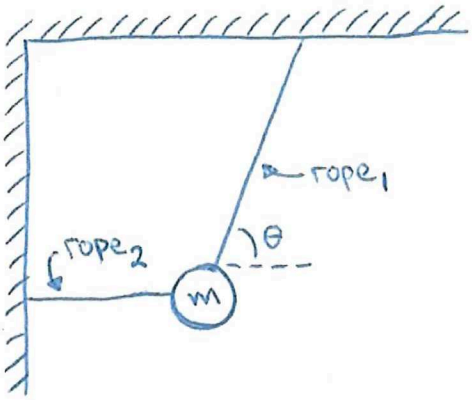
$$= -165.1\ m/s$$

$$t = \frac{-165.1 - 330\sin 30}{-9.81}$$

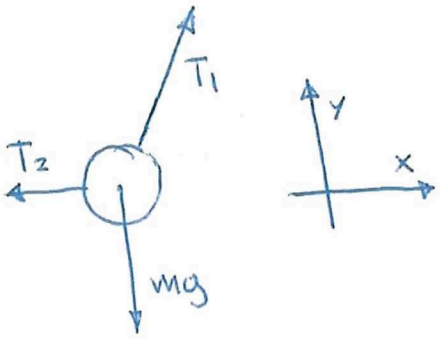
$$\boxed{t = 34\ sec}$$

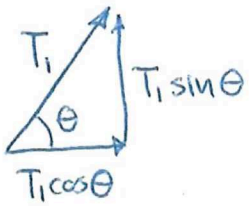
(10 marks) Determine the tension in each rope given initial conditions:

m = 100 kg
theta = 75 degrees



FBD:





Newton's 2nd Law:

$$\sum F_x = \cancel{ma_x}^{\,0} = T_1 \cos\theta - T_2 \qquad (1)$$

$$\sum F_y = \cancel{ma_y}^{\,0} = T_1 \sin\theta - mg \qquad (2)$$

Solve eqn. (2) for T_1:

$$T_1 = \frac{mg}{\sin\theta} = \frac{(100)(9.81)}{\sin(75)} = 1016 \text{ N} \qquad (2a)$$

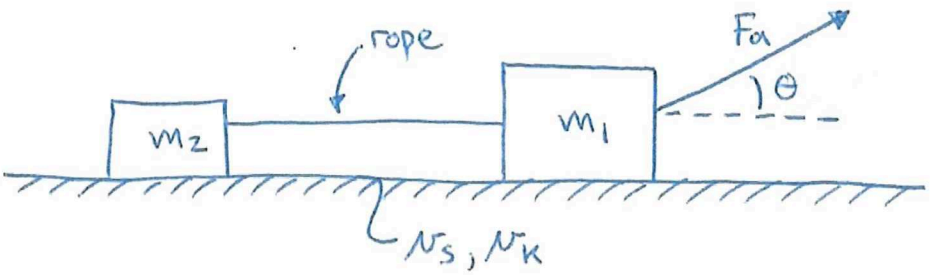
sub (2a) into (1) and solve for T_2:

$$0 = \left[\frac{mg}{\sin\theta}\right]\cos\theta - T_2$$

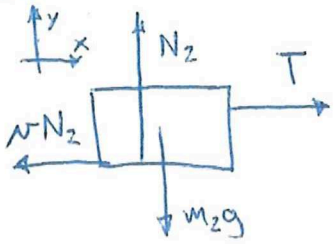
$$T_2 = mg\frac{\cos\theta}{\sin\theta} = (100)(9.81)\frac{\cos(75)}{\sin(75)} = 263 \text{ N}$$

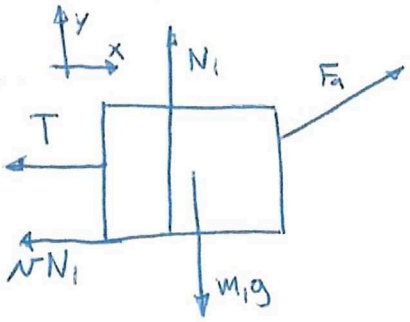
(15 marks) Determine the tension in the rope and the acceleration given initial conditions:

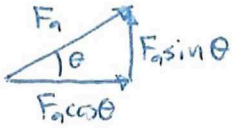
F_a = 500 N
theta = 35 degrees
m_1 = 100 kg
m_2 = 50 kg
u_s = 0.1
u_k = 0.05



FBD:







Newton's 2nd Law:

$$\sum F_x = m_2 a_{x_2} = T - \mu N_2 \quad (1)$$

$$\sum F_y = m_2 a_{y_2} = N_2 - m_2 g \quad (2)$$

$$\sum F_x = m_1 a_{x_1} = F_a \cos\theta - T - \mu N_1 \quad (3)$$

$$\sum F_x = m_1 a_{y_1} = F_a \sin\theta + N_1 - m_1 g \quad (4)$$

assume mass-less, stretch-less rope so $a_{x_1} = a_{x_2} = a_x$

solve for N_1 in equ (4) and N_2 in equ. (2):

$$N_1 = m_1 g - F_a \sin\theta \quad (4a)$$

$$N_2 = m_2 g \quad (2a)$$

sub (2a) into (1) and solve for T:

$$m_2 a_x = T - \mu [m_2 g]$$

$$T = m_2 a_x + \mu m_2 g \quad (1a)$$

Sub (1a) and (4a) into (3) and solve for a_x:

$$m_1 a_x = F_a\cos\theta - \left[m_2 a_x + \mu m_2 g\right] - \mu\left[m_1 g - F_a \sin\theta\right]$$

$$m_1 a_x = F_a\cos\theta - m_2 a_x - \mu m_2 g - \mu m_1 g + \mu F_a \sin\theta$$

$$(m_1 + m_2)\, a_x = F_a(\cos\theta + \mu\sin\theta) - \mu g(m_1 + m_2)$$

$$\boxed{a_x = \frac{F_a(\cos\theta + \mu\sin\theta)}{m_1 + m_2} - \mu g}$$

check if it moves:

$$a_x = \frac{500(\cos 35 + (0.1)\sin 35)}{100 + 50} - (0.1)(9.81)$$

$$= 1.94 \text{ m/s}^2 \quad \text{(it moves!)}$$

calculate acceleration:

$$a_x = \frac{500(\cos 35 + (0.05)\sin 35)}{100 + 50} - (0.05)(9.81)$$

$$= 2.34 \text{ m/s}^2$$

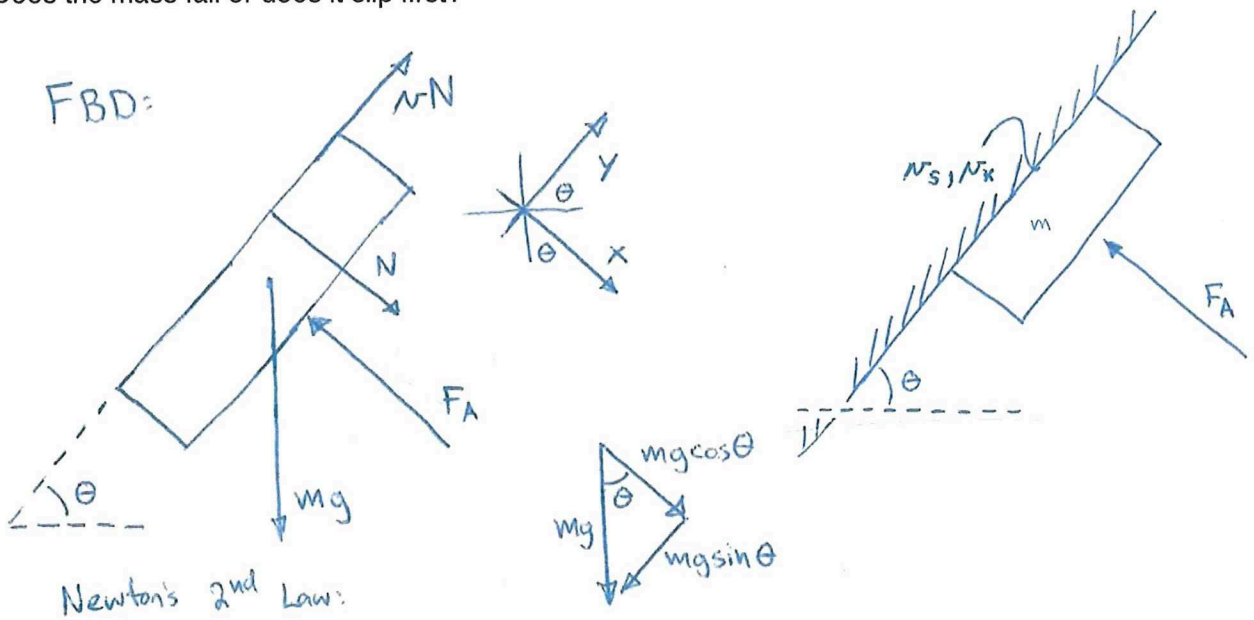
substitute into (1a):

$$T = m_2 a_x + \mu m_2 g$$
$$= (50)(2.3) + (0.05)(50)(9.81)$$
$$= 141.5$$

$$\boxed{a_x = 2.3 \text{ m/s}^2 \;, \quad T = 142 \text{ N}}$$

BONUS (2 marks)

Determine the minimum force need to keep the mass from moving.
Does the mass fall or does it slip first?



Newton's 2nd Law:

$$\sum F_x = ma_x = 0 \quad N + mg\cos\theta - F_A \qquad (1)$$

$$\sum F_y = ma_y = 0 \quad \nu N - mg\sin\theta \qquad (2)$$

solve for N in equ. (2):

$$N = \frac{mg\sin\theta}{\nu} \qquad (2a)$$

sub (2a) into (1) and solve for F_A =

$$0 = \left[\frac{mg\sin\theta}{\nu}\right] + mg\cos\theta - F_A$$

$$F_A = mg\left(\frac{\sin\theta}{\nu} + \cos\theta\right)$$

minimum force needed:

$$\boxed{F_A = mg\left(\frac{\sin\theta}{\nu_s} + \cos\theta\right)}$$

Falls when N goes to 0, equ. (2a)

$$\boxed{N = \frac{mg\sin\theta}{\nu}}$$

only @ $\sin\theta = 0$

$\theta = 0$ (slips first).

Instructor:	Frank Secretain
Course:	CIVL 11
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 30 marks
Percentage of final grade:	25% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

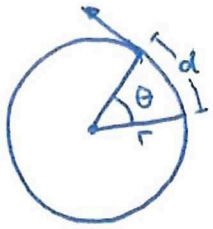
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x_o} + \vec{v_o}t + \frac{1}{2}\vec{a}t^2 \qquad \theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\vec{v}^{\,2} = \vec{v_o}^{\,2} + 2\vec{a}(\vec{x} - \vec{x_o}) \qquad \omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\vec{v} = \vec{v_o} + \vec{a}t \qquad \omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \qquad \vec{F_g} = \frac{Gm_1m_2}{\vec{r}^{\,2}} = mg \qquad \vec{F_f} = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p\vec{\omega_p})}{dt} = I_p\vec{\alpha_p} \qquad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

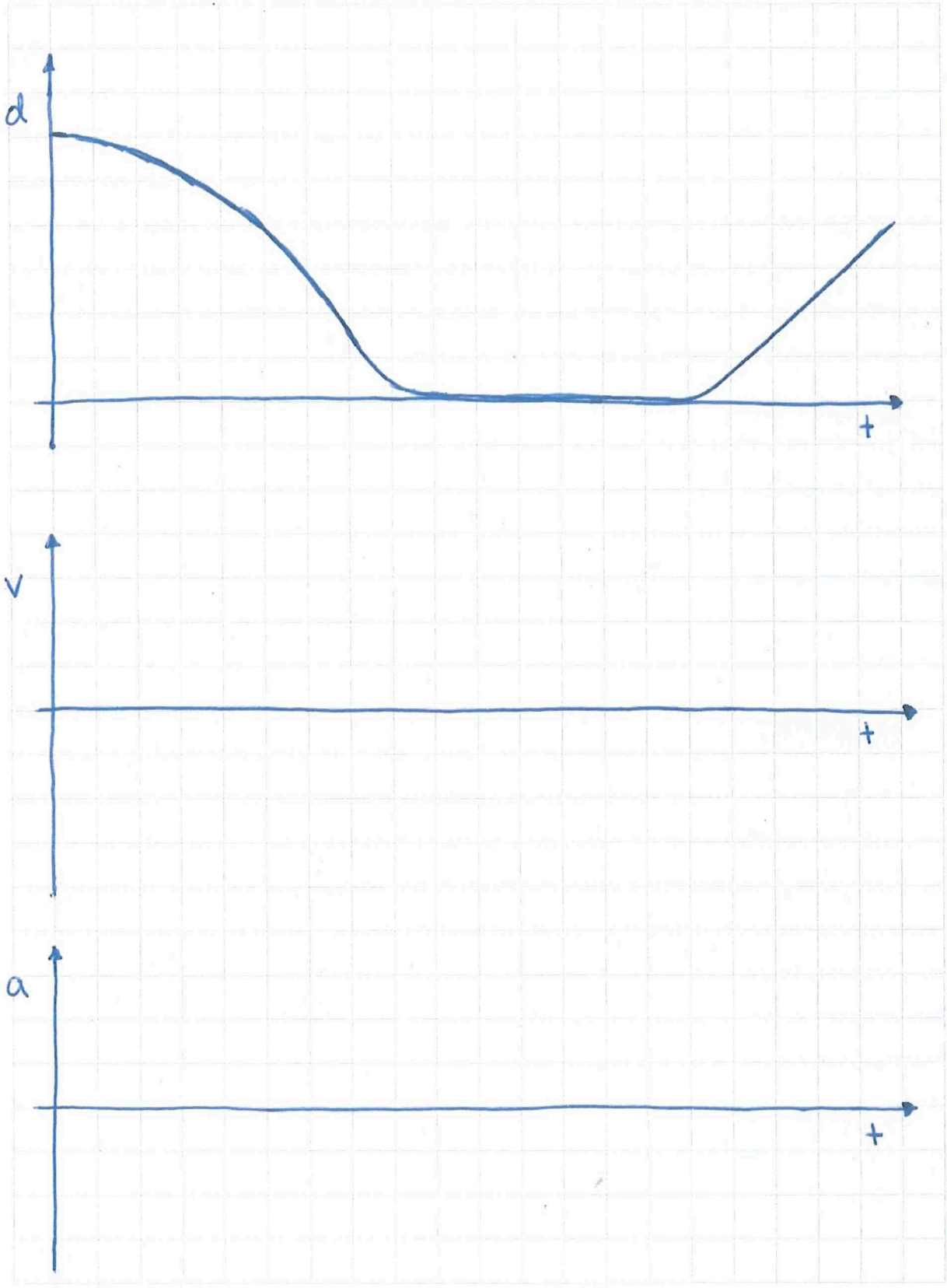
$$\vec{p} = m\vec{v} \qquad \vec{p} - \vec{p_o} = \int \vec{F}dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

$$E_f - E_o = Q - W \qquad E_{PE} = mg(y - y_o) \qquad E_{KE} = \frac{1}{2}mv^2$$

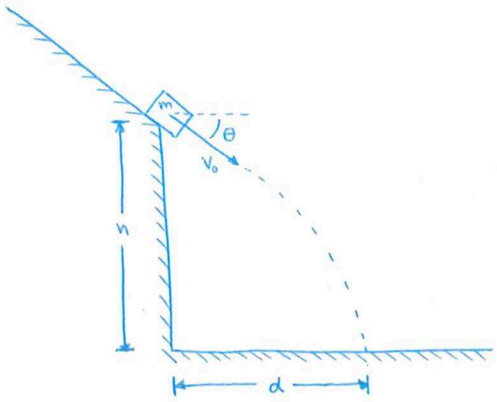
$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

(5 marks) Complete the distance-velocity-acceleration graph.



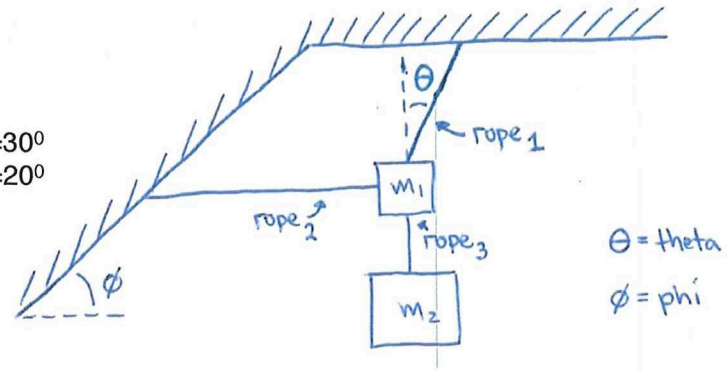
(10 marks) Determine the horizontal distance (d) travelled by the block given initial conditions:

a. v_o=10m/s, h= 2m, m=2kg, theta=30^0
b. v_o=10m/s, h= 9m, m=2kg, theta=60^0



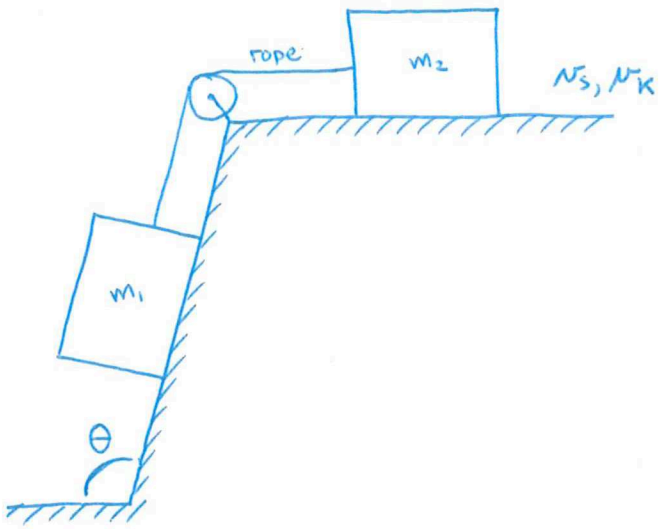
(5 marks) Determine the tension in each rope given:

a. m1=100kg, m2=200kg, theta=30^0, phi=30^0
b. m1=200kg, m2=100kg, theta=30^0, phi=20^0



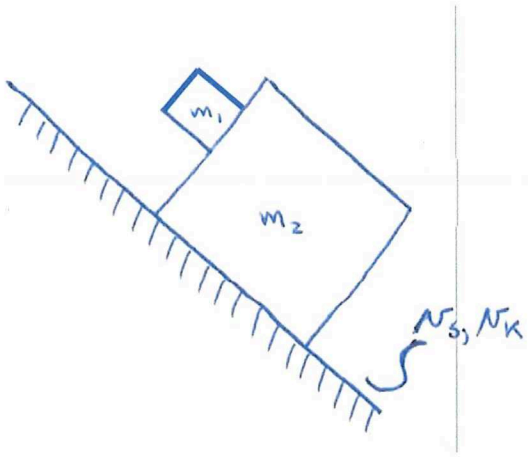
(10 marks) Determine the minimum mass (m1) needed to accelerated the system (i.e. move m2), given:

theta = 120 degrees
m_2 = 100 kg
u_s = 0.2
u_k = 0.1

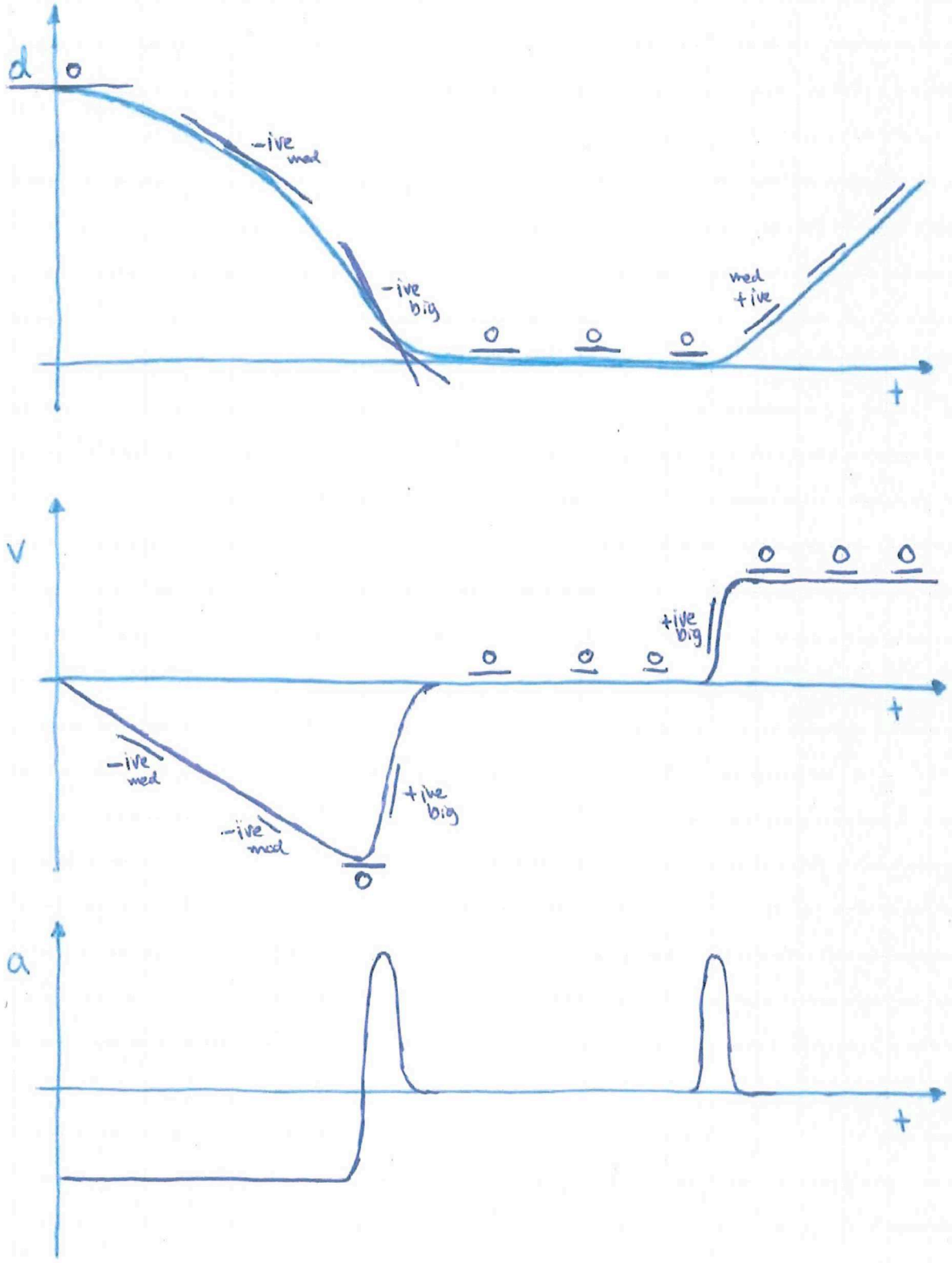


BONUS (2 marks)

Draw the free body diagram for each of the blocks. Assume the blocks are static (not moving).

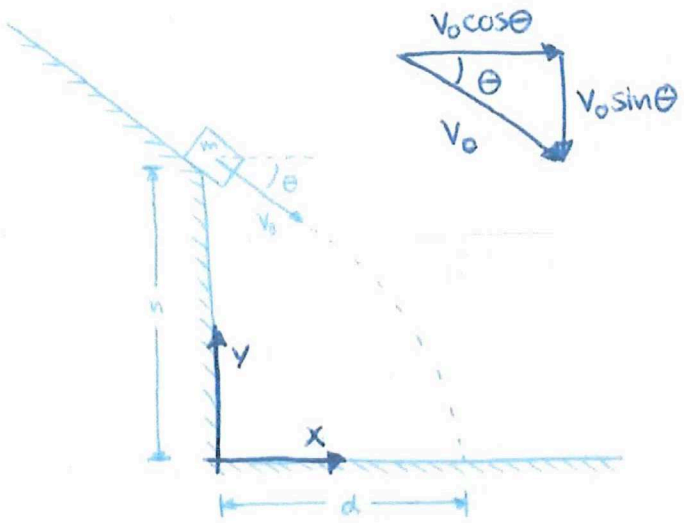


(5 marks) Complete the distance-velocity-acceleration graph.



(10 marks) Determine the horizontal distance (d) travelled by the block given initial conditions:

a. v_o=10m/s, h= 2m, m=2kg, theta=30°
b. v_o=10m/s, h= 9m, m=2kg, theta=60°



x-dir	y-dir	time
$x_o = 0$	$y_o = h$	$t =$
$x_f =$	$y_f = 0$	
$v_{xo} = v_o\cos\theta$	$v_{yo} = -v_o\sin\theta$	
$v_{xf} = v_o\cos\theta$	$v_{yf} =$	
$a_x = 0$	$a_y = -g$	

Kinematic equations

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o) \qquad \vec{v} = \vec{v}_o + \vec{a}t \qquad \vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2}\vec{a}t^2$$

$$v_{fy}^2 = v_{oy}^2 + 2a_y(y_f - y_o) \qquad v_{fy} = v_{oy} + a_y t \qquad x_f = x_o + v_{ox}t + \frac{1}{2}a_x t^2$$

$$v_{fy} = \pm\sqrt{v_{oy}^2 - 2a_y y_o} \qquad t = \frac{v_{fy} - v_{oy}}{a} \quad (2) \qquad x_f = v_{ox}t \quad (3)$$

$$v_{fy} = -\sqrt{v_o^2\sin^2\theta + 2gh} \quad (1)$$

Solve

sub (1) into (2)

$$t = \frac{\left[-\sqrt{v_o^2\sin^2\theta + 2gh}\right] - \left[-v_o\sin\theta\right]}{-g} = \frac{\sqrt{v_o^2\sin^2\theta + 2gh} - v_o\sin\theta}{g} \quad (2a)$$

sub (2a) into (3)

$$x_f = \left[v_o\cos\theta\right]\left[\frac{\sqrt{v_o^2\sin^2\theta + 2gh} - v_o\sin\theta}{g}\right]$$

a) $$x_f = \left[(10)\cos(30)\right]\left[\frac{\sqrt{(10\sin 30)^2 + 2(9.81)(2)} - (10\sin 30)}{9.81}\right] = 2.66$$

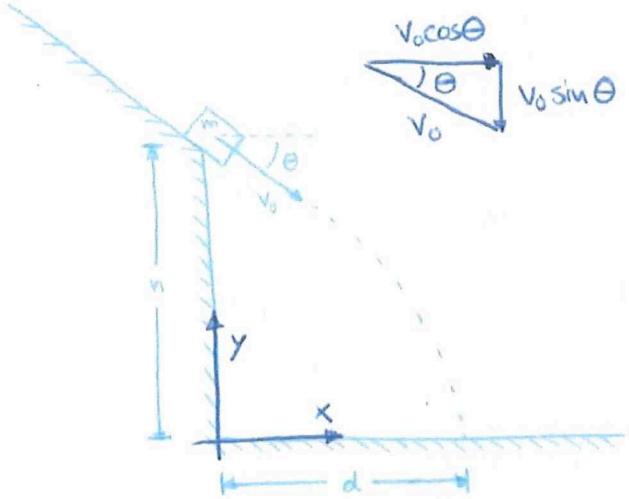
b) $$x_f = \left[(10)\cos(60)\right]\left[\frac{\sqrt{(10\sin 60)^2 + 2(9.81)(9)} - (10\sin 60)}{9.81}\right] = 3.67$$

$d_a = 2.7$ m

$d_b = 3.7$ m

(10 marks) Determine the horizontal distance (d) travelled by the block given initial conditions:

a. v_o=10m/s, h= 2m, m=2kg, theta=30°
b. v_o=10m/s, h= 9m, m=2kg, theta=60°



x-dir	y-dir	time
$x_0 = 0$	$y_0 = h$	$t =$
$x_f =$	$y_f = 0$	
$v_{ox} = v_0\cos\theta$	$v_{oy} = -v_0\sin\theta$	
$v_{fx} = v_0\cos\theta$	$v_{fy} =$	
$a_x = 0$	$a_y = -g$	

Kinematic equations

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$y_f = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$$

$$0 = h - v_0\sin\theta\, t - \frac{1}{2}gt^2$$

use quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{v_0\sin\theta \pm \sqrt{(v_0\sin\theta)^2 - 4(h)(-\frac{1}{2}g)}}{2(-\frac{1}{2}g)}$$

$$= \frac{v_0\sin\theta \pm \sqrt{(v_0\sin\theta)^2 + 2gh}}{-g} \quad (1)$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

$$x_f = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$$

$$x_f = v_{ox}t \quad (2)$$

solve

sub (1) into (2)

$$x_f = [v_0\cos\theta]\left[\frac{v_0\sin\theta \pm \sqrt{(v_0\sin\theta)^2 + 2gh}}{-g}\right]$$

a) $$x_f = [(10)\cos(30)]\left[\frac{(10)\sin(30) \pm \sqrt{(10\sin(30))^2 + 2(9.81)(2)}}{-9.81}\right] = \begin{matrix}2.66\\-11.5\end{matrix}$$

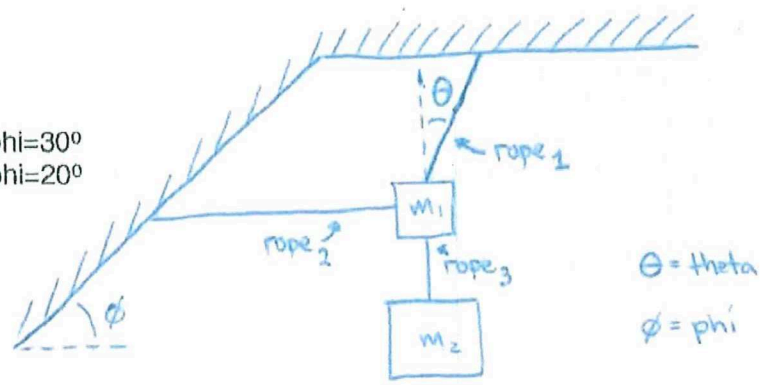
$d_a = 2.7\ m$

b) $$x_f = [(10)\cos(60)]\left[\frac{(10)\sin(60) \pm \sqrt{(10\sin(60))^2 + 2(9.81)(9)}}{-9.81}\right] = \begin{matrix}3.67\\-12.5\end{matrix}$$

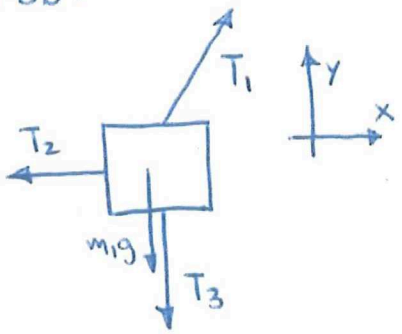
$d_b = 3.7\ m$

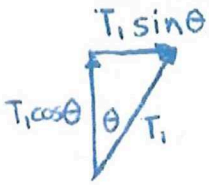
(5 marks) Determine the tension in each rope given:

a. m1=100kg, m2=200kg, theta=30°, phi=30°
b. m1=200kg, m2=100kg, theta=30°, phi=20°



FBD:

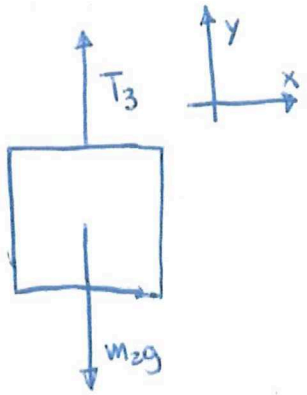




Newton's 2nd Law

$\Sigma F_x = m_1 a_x = T_1 \sin\theta - T_2$ (1)

$\Sigma F_y = m_1 a_y = T_1 \cos\theta - m_1 g - T_3$ (2)



$\Sigma F_x = m_2 a_x = 0$ (3)

$\Sigma F_y = m_2 a_y = T_3 - m_2 g$ (4)

solve (4) for T_3

$T_3 = m_2 g$ (4a)

solve (1) for T_2

$T_2 = T_1 \sin\theta$ (1a)

sub (4a) in (2) and solve for T_1

$0 = T_1 \cos\theta - m_1 g - [m_2 g]$

$T_1 = \frac{1}{\cos\theta}(m_1 g + m_2 g)$ (2a)

a. $T_1 = \frac{1}{\cos 30}[(9.81)(100+200)] = 3398$ N

$T_2 = (3398)\sin 30 = 1699$ N

$T_3 = (200)(9.81) = 1962$ N

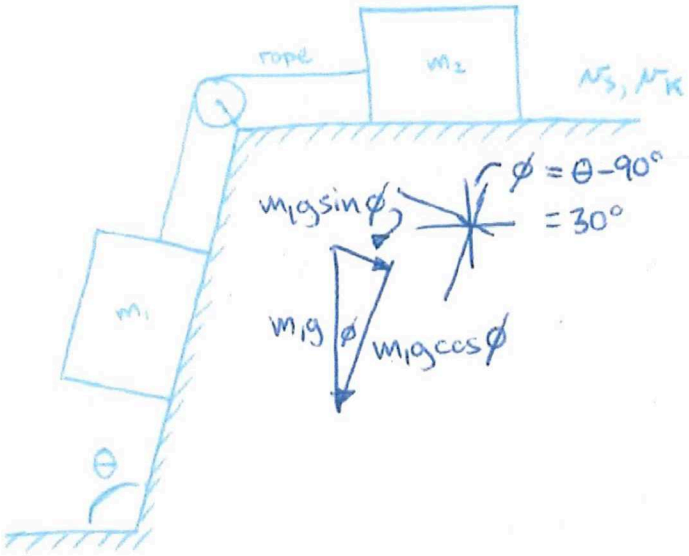
b. $T_1 = \frac{1}{\cos 30}[(9.81)(200+100)] = 3398$ N

$T_2 = (3398)\sin 30 = 1699$ N

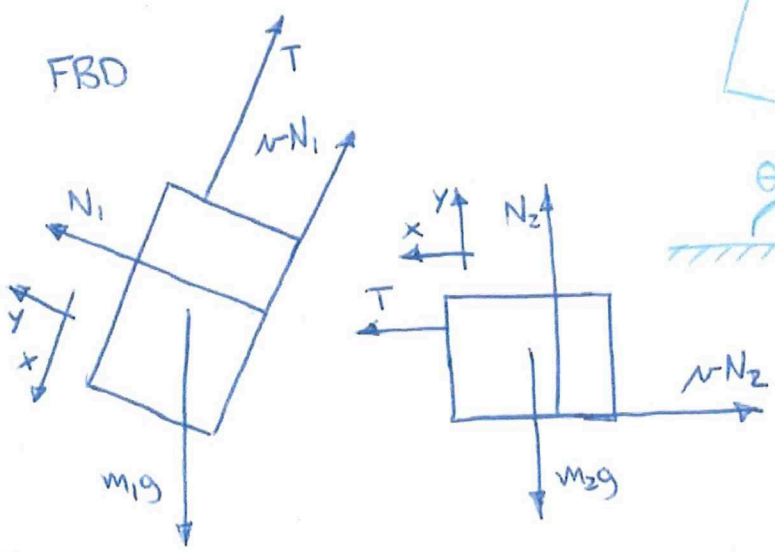
$T_3 = (100)(9.81) = 981$ N

(10 marks) Determine the minimum mass (m1) needed to accelerated the system (i.e. move m2), given:

theta = 120 degrees
m_2 = 100 kg
u_s = 0.2
u_k = 0.1



FBD



Newton's 2nd Law

$\sum F_x = m_1 a_{x_1} = m_1 g\cos\phi - T - \nu N_1$ (1)

$\sum F_y = \cancel{m_1 a_{y_1}}^{0} = N_1 - m_1 g\sin\phi$ (2)

$\sum F_x = m_2 a_{x_2} = T - \nu N_2$ (3)

$\sum F_y = \cancel{m_2 a_{y_2}}^{0} = N_2 - m_2 g$ (4)

massless, stretchless rope: $a_{x_1} = a_{x_2} = a_x$ (5)

solve for N_1 and N_2 in (2) & (4)

$N_1 = m_1 g\sin\phi$ (2a)

$N_2 = m_2 g$ (4a)

sub (4a) & (5) into (3)

$m_2[a_x] = T - \nu[m_2 g]$

$T = m_2 a_x + \nu m_2 g$ (3a)

sub (3a) and (5) into (1)

$m_1[a_x] = m_1 g\cos\phi - [m_2 a_x + \nu m_2 g] - \nu[m_1 g\sin\phi]$

$m_1(a_x - g\cos\phi + \nu g\sin\phi) = -m_2 a_x - \nu m_2 g$

$$m_1 = m_2 \frac{\nu g + a_x}{g\cos\phi - \nu g\sin\phi - a_x}$$

set $a_x = 0$ (just moves)

use ν_s:

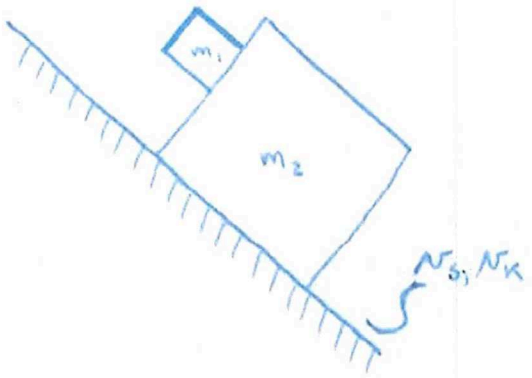
$$m_1 = (100)\frac{(0.2)(\cancel{9.81})}{(\cancel{9.81})\cos(30) - (0.2)(\cancel{9.81})\sin(30)}$$

$= 26.1$

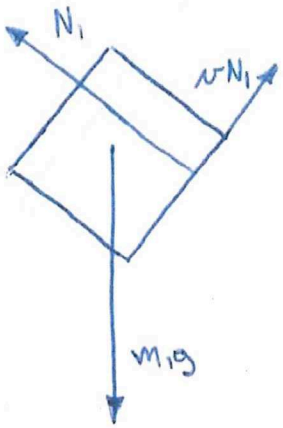
$m_{min} = 26$ kg

BONUS (2 marks)

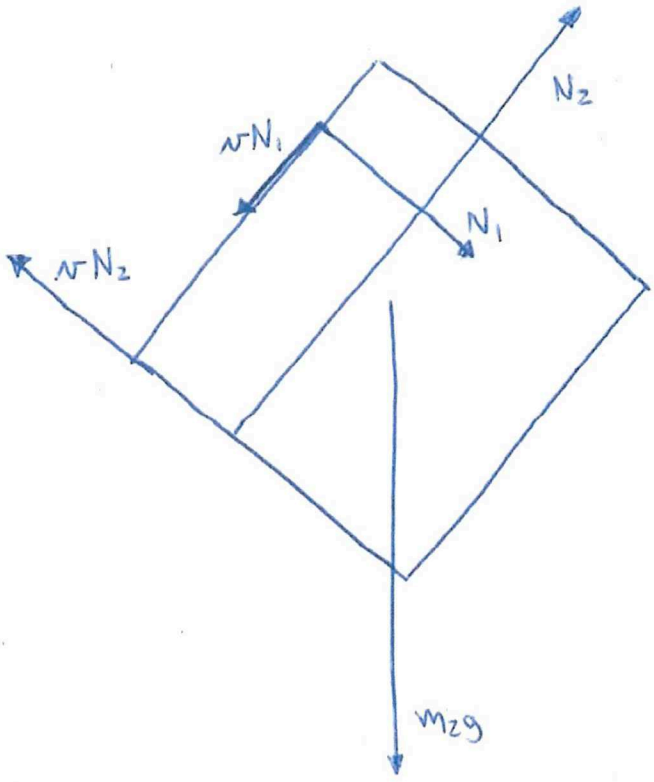
Draw the free body diagram for each of the blocks. Assume the blocks are static (not moving).



FBD_1:

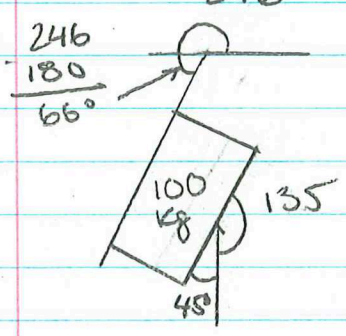


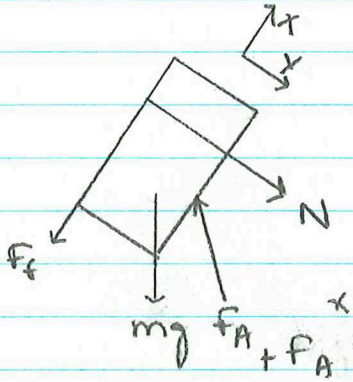
FBD_2:



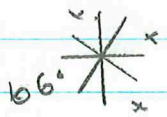
1568.74.

246° Solve for the minimum force



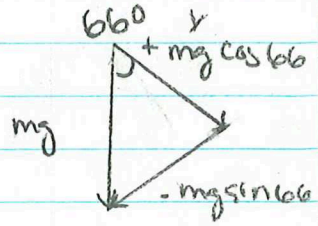






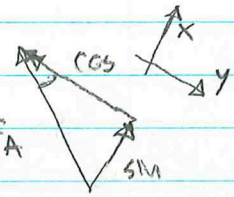
$\mu_s = 0.3$

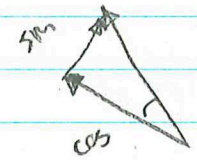




(1) $\sum F_x = ma_x = F_A \cos 45 - mg \sin 66 - F_f$

(2) $\sum F_y = ma_y^{\,0} = N - F_A \sin 45 + mg \cos 66$





$0 = N - F_A \sin 45 + mg \cos 66$

$N = F_A \sin 45 - mg \cos 66$

set $a_y = 0$ $F_f = \mu N$

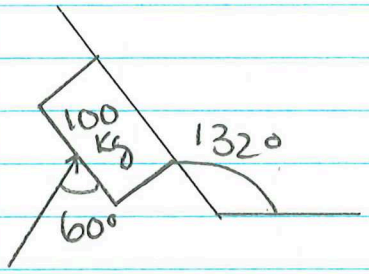
$0 = F_A \cos 45 - (100)(9.81)(\sin 66) - 0.3[F_A \sin 45 - 981 \cos 66]$

$0 = F_A \cos 45 - (981) \sin 66 - 0.3 F_A \sin 45 - 0.3(981) \cos 66$

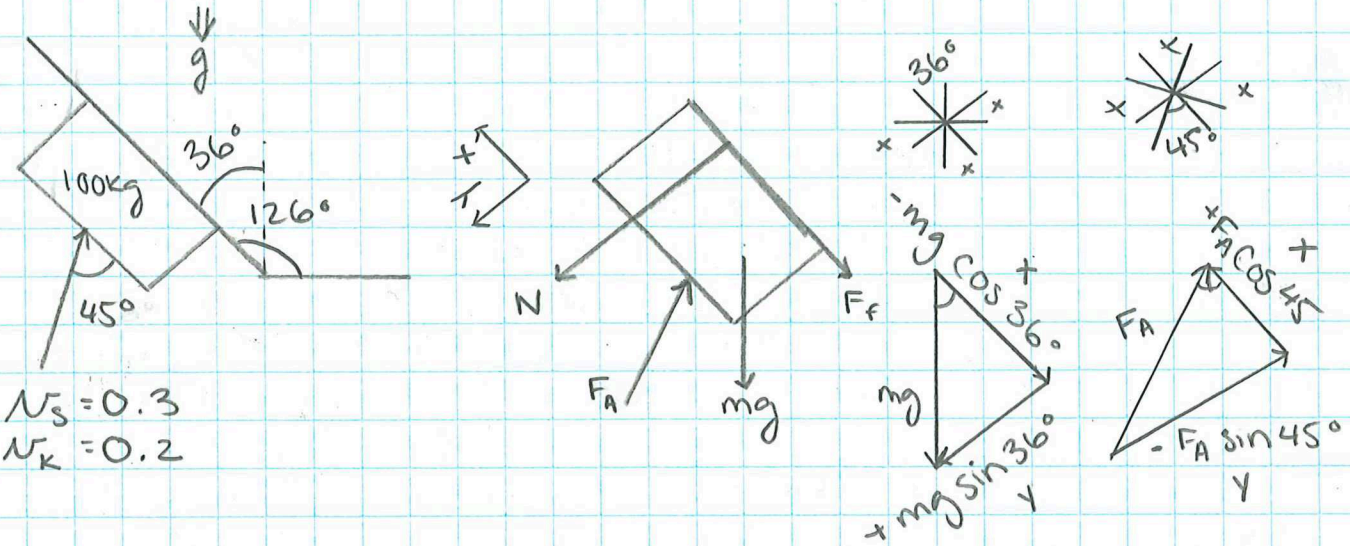
$0 = F_A [\cos 45 - 0.3(\sin 45)] - 981 \sin 66 + 0.3(981) \cos 66$

$F_A [\cos 45 - 0.3(\sin 45)] = 981 \sin 66 - 0.3(981) \cos 66$

$F_A = \dfrac{896.19 - 119.70}{0.495} = 1706.57$


100 kg
132°
60°

box 3



$\mu_s = 0.3$
$\mu_k = 0.2$

(1) $\Sigma F_x = m\vec{a}_x^{\,0} = F_A\cos 45 - mg\cos 36 - F_f$

(2) $\Sigma F_y = m\vec{a}_y^{\,0} = N + mg\sin 36 - F_A\sin 45$

set $a_y = 0$ solve for N

$0 = N + mg\sin 36 - F_A\sin 45$

(2a) $N = F_A\sin 45 - mg\sin 36$

Sub (2a) into (1) set $a_x = 0$ $F_f = \mu_s N$ to solve for F_A

$0 = F_A\cos 45 - mg\cos 36 - \mu_s[F_A\sin 45 - mg\sin 36]$

$0 = F_A\cos 45 - mg\cos 36 - \mu_s F_A\sin 45 + \mu_s mg\sin 36$

$0 = F_A[\cos 45 - \mu_s\sin 45] - mg\cos 36 + \mu_s mg\sin 36$

$F_A[\cos 45 - \mu_s\sin 45] = mg\cos 36 - \mu_s mg\sin 36$

$$F_A = \frac{mg\cos 36 - \mu_s mg\sin 36}{\cos 45 - \mu_s\sin 45}$$

$$F_A = \frac{100(9.81)\cos 36 - 0.3(100)(9.81)\sin 36}{\cos 45 - 0.3(\sin 45)}$$

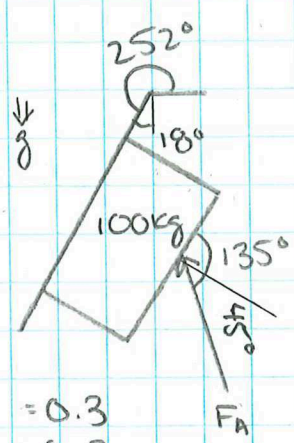
$$F_A = \frac{793.646 - 172.985}{0.495}$$

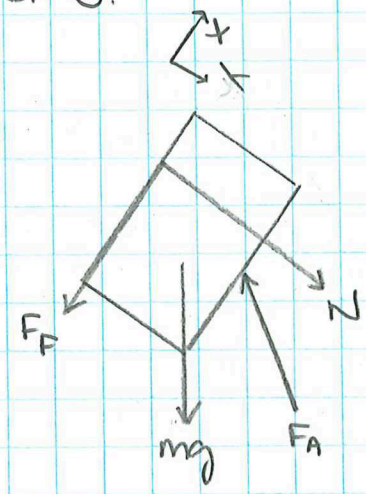
$F_A = 1253.86\ N$

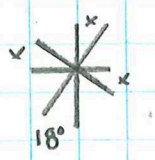
A: 1253.90

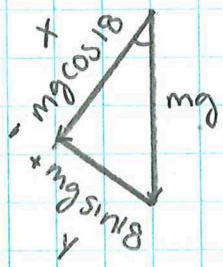
rounding error

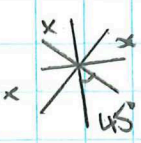
box 3.

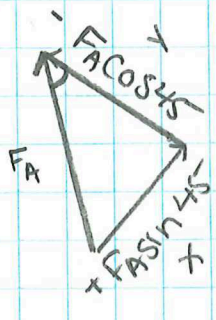












$\mu_s = 0.3$
$\mu_k = 0.2$

$\Sigma F_x = m\vec{a}_x^{\,0} = F_A \sin 45 - mg\cos 18 - F_F$
$\Sigma F_y = m\vec{a}_y^{\,0} = N + mg\sin 18 - F_A \cos 45$

$0 = N + mg\sin 18 - F_A \cos 45$
$N = F_A \cos 45 - mg\sin 18$

$0 = F_A \sin 45 - mg\cos 18 - \mu_s [F_A \cos 45 - mg\sin 18]$
$0 = F_A \sin 45 - mg\cos 18 - \mu_s F_A \cos 45 + \mu_s mg\sin 18$
$0 = F_A [\sin 45 - \mu_s \cos 45] - mg\cos 18 + \mu_s mg\sin 18$

$$F_A = \frac{mg\cos 18 - \mu_s mg\sin 18}{\sin 45 - \mu_s \cos 45}$$

$$F_A = \frac{100(9.81)\cos 18 - 0.3(100)9.8(\sin 18)}{\sin 45 - 0.3\cos 45} = 1701.18\ N$$