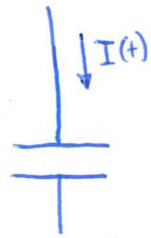
Capacitors

$$\boxed{V_c = \frac{1}{C}\int I\,dt}$$

equation



symbol

$C = [F] = [\text{Farad}]$

$C = \left[\frac{amp^2\ s^4}{Kg\ m^2}\right]$

units

---

if $I(t) = I_p \sin(\omega t - \phi_I)$

then: $V_c = \frac{1}{C}\int I_p \sin(\omega t - \phi_I)\,dt$

$= \frac{-1}{\omega C} I_p \cos(\omega t - \phi_I)$

$-\cos(x) = \sin(x - \frac{\pi}{2})$

$$\boxed{V_c = \frac{1}{\omega C} I_p \sin\left(\omega t - \phi_I - \frac{\pi}{2}\right)}$$

in time domain V_c leads the current by 90°

$e^{i\theta} = \cos\theta + i\sin\theta$

move to complex plane:

$V_c = \mathrm{Im}\left(\frac{1}{\omega C} I_p e^{i(\omega t - \phi_I - \frac{\pi}{2})}\right)$

$x^{m+n} = x^m x^n$

$= \mathrm{Im}\left(\frac{1}{\omega C} I_p e^{i\omega t} e^{-i\phi_I} e^{-i\frac{\pi}{2}}\right)$

$e^{-i\frac{\pi}{2}} = -i$

$= \mathrm{Im}\left(\frac{-i}{\omega C} I_p e^{-i\phi_I} e^{i\omega t}\right)$

phasors

move to phasor reference frame:

$$\mathrm{Im}\left(\tilde{V}_c \underbrace{e^{i\omega t}}\right) = \mathrm{Im}\left(\frac{-i}{\omega C}\tilde{I}\underbrace{e^{i\omega t}}\right)$$

rotating reference frame

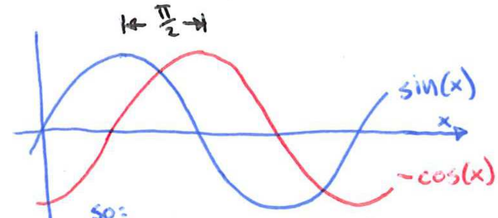
where: $\tilde{V}_c = V_{c,p} e^{-i\phi_V}$ & $\tilde{I} = I_p e^{-i\phi_I}$

seperate phasor part:

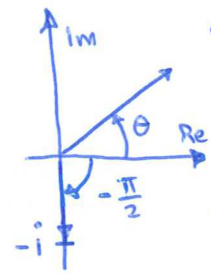
$$\tilde{V}_c = \frac{-i}{\omega C}\tilde{I}$$

$$\boxed{V_c = Z_c \tilde{I}} \quad \text{where} \quad \boxed{Z_c = \frac{-i}{\omega C}}$$

in phase domain Z_L is Imaginary and is $-90°$ to the real axis



so: $-\cos(x) = \sin(x - \frac{\pi}{2})$



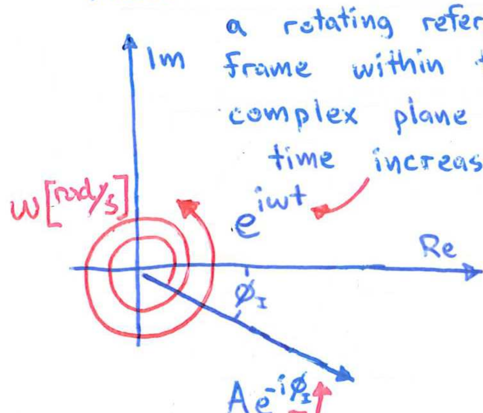
$e^{i\theta} = \cos\theta + i\sin\theta$

so: $\mathrm{Re}(e^{i\theta}) = \cos\theta$
$\mathrm{Im}(e^{i\theta}) = \sin\theta$

so: $e^{-i\frac{\pi}{2}} = -i$

Phasors:

a rotating reference frame within the complex plane as time increases



initial vector which rotates