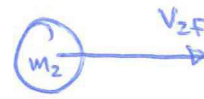
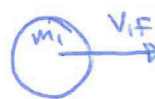


some loss of energy. = Q



Initial

$$P_0 = m_1 v_{10} + m_2 v_{20}$$

$$E_{KE} = \frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2$$

Conservation of momentum:

$$P_F - P_0 = \int \cancel{F_{NET}} dt = 0 \text{ (no external force)}$$

$$[m_1 v_{1f} + m_2 v_{2f}] - [m_1 v_{10} + m_2 v_{20}] = 0 \quad (1)$$

1st Law of Thermodynamics:

$$E_F - E_0 = Q - \cancel{W} = 0 \text{ (no work done on system)}$$

$$\left[\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right] - \left[\frac{1}{2} m_1 v_{10}^2 + \frac{1}{2} m_2 v_{20}^2 \right] = [-Q] \quad (2)$$

heat lost (-ive)
from the system

Solve eqs. (1) & (2) for v_{1f} & v_{2f}

Rearrange equ. (1):

$$m_1(v_{1f} - v_{10}) = m_2(v_{20} - v_{2f}) \quad (1a)$$

Rearrange equ. (2):

$$m_1(v_{1f}^2 - v_{10}^2) = m_2(v_{20}^2 - v_{2f}^2) - Q \quad (2a)$$

Divide equ. (2) by (1)

$$\frac{m_1(v_{1f}^2 - v_{10}^2)}{m_1(v_{1f} - v_{10})} = \frac{m_2(v_{20}^2 - v_{2f}^2)}{m_2(v_{20} - v_{2f})} - \frac{Q}{m_2(v_{20} - v_{2f})}$$

$$\frac{\cancel{m_1}(v_{1f} + v_{10})(\cancel{v_{1f} - v_{10}})}{\cancel{m_1}(\cancel{v_{1f} - v_{10}})} = \frac{\cancel{m_2}(v_{20} + v_{2f})(\cancel{v_{20} - v_{2f}})}{\cancel{m_2}(\cancel{v_{20} - v_{2f}})} - \frac{Q}{m_2(v_{20} - v_{2f})}$$

$$v_{1f} + v_{10} = v_{20} + v_{2f} - \frac{Q}{m_2(v_{20} - v_{2f})}$$

$$v_{1f} = v_{20} + v_{2f} - v_{10} - \frac{Q}{m_2(v_{20} - v_{2f})}$$

Sub v_{1f} into equ. (1) and solve for v_{2f} :

$$m_1 \left[v_{20} + v_{2f} - v_{10} - \frac{Q}{m_2(v_{20} - v_{2f})} \right] + m_2 v_{2f} = m_1 v_{10} + m_2 v_{20}$$

$$m_1 m_2 (v_{20} - v_{2f}) v_{20} + m_1 m_2 (v_{20} - v_{2f}) v_{2f} - \cancel{m_1 m_2 (v_{20} - v_{2f}) v_{10}} - Q + m_2^2 (v_{20} - v_{2f}) v_{2f} =$$

$$\cancel{m_1 m_2 (v_{20} - v_{2f}) v_{10}} + m_2^2 (v_{20} - v_{2f}) v_{20}$$

$$m_1 m_2 v_{20}^2 - \cancel{m_1 m_2 v_{20} v_{2f}} + \cancel{m_1 m_2 v_{20} v_{2f}} - m_1 m_2 v_{2f}^2 - Q + \cancel{m_2^2 v_{20} v_{2f}} - m_2^2 v_{2f}^2 =$$

$$2 m_1 m_2 v_{20} v_{10} - 2 m_1 m_2 v_{2f} v_{10} + m_2^2 v_{20}^2 - \cancel{m_2^2 v_{2f} v_{20}}$$

$$(m_2^2 + m_1 m_2) v_{2f}^2 - (2 m_2^2 v_{20} + 2 m_1 m_2 v_{10}) v_{2f} + (2 m_1 m_2 v_{20} v_{10} + m_2^2 v_{20}^2 - m_1 m_2 v_{20}^2 + Q) = 0$$

Quadratic form:

$$V_{2F} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} \sqrt{b^2 - 4ac} &= \sqrt{(2m_2^2 v_{20} + 2m_1 m_2 v_{10})^2 - 4(m_2^2 + m_1 m_2)(2m_1 m_2 v_{20} v_{10} + m_2^2 v_{20}^2 - m_1 m_2 v_{20}^2 + Q)} \\ &= \sqrt{4(\cancel{m_2^4 v_{20}^2} + \cancel{2m_1 m_2^3 v_{10} v_{20}} + m_1^2 m_2^2 v_{10}^2) - 4(\cancel{2m_1 m_2^3 v_{10} v_{20}} + 2m_1^2 m_2^2 v_{10} v_{20} + \cancel{m_2^4 v_{20}^2}} \\ &\quad + \cancel{m_1 m_2^3 v_{20}^2} - \cancel{m_1 m_2^3 v_{20}^2} - m_1^2 m_2^2 v_{20}^2 + m_2^2 Q + m_1 m_2 Q)} \\ &= \sqrt{4m_1^2 m_2^2 (v_{10}^2 - 2v_{10} v_{20} + v_{20}^2 - \frac{Q}{m_1^2} - \frac{Q}{m_1 m_2})} \\ &= \sqrt{4m_1^2 m_2^2 (v_{10} - v_{20})^2 - 4Q(m_2^2 + m_1 m_2)} \end{aligned}$$

sub into quadratic equation:

$$V_{2F} = \frac{(2m_2^2 v_{20} + 2m_1 m_2 v_{10}) \pm \sqrt{4m_1^2 m_2^2 (v_{10} - v_{20})^2 - 4Q(m_2^2 + m_1 m_2)}}{2(m_2^2 + m_1 m_2)}$$

and

$$V_{1F} = V_{20} + V_{2F} - V_{10} - \frac{Q}{m_2(v_{20} - V_{2F})}$$

Maximum Heat Loss

$$4m_1^2 m_2^2 (v_{10} - v_{20})^2 - 4Q(m_1^2 + m_1 m_2) > 0$$

$$Q < \frac{m_1^2 m_2^2 (v_{10} - v_{20})^2}{m_1^2 + m_1 m_2}$$

$$Q < \frac{m_1^2 m_2 (v_{10} - v_{20})^2}{m_1 + m_2}$$

Elastic Collision ($Q=0$)

then

$$v_{2F} = \frac{(\cancel{2m_2} v_{20} + \cancel{2m_1 m_2} v_{10}) \pm \cancel{2m_1 m_2} (v_{10} - v_{20})}{\cancel{2m_2} (m_1 + m_2)}$$

$$v_{2F} = \frac{(m_2 v_{20} + m_1 v_{10}) \pm (m_1 v_{10} - m_1 v_{20})}{m_1 + m_2} = \frac{2m_1 v_{10} + (m_2 - m_1) v_{20}}{m_1 + m_2}, v_{20}$$
$$v_{1F} = v_{20} + v_{2F} - v_{10} = \frac{2m_2 v_{20} + (m_1 - m_2) v_{10}}{m_1 + m_2}, v_{10}$$