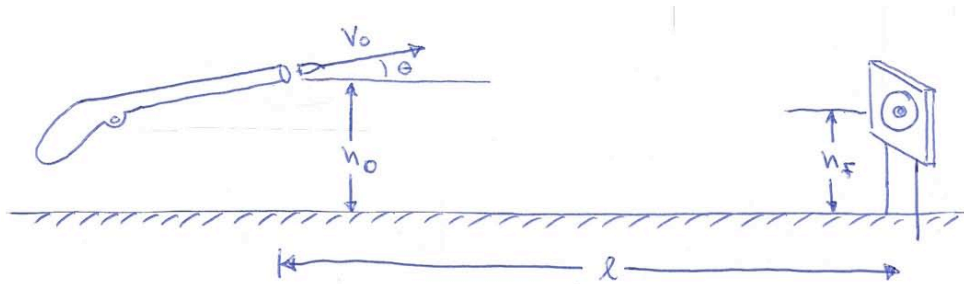
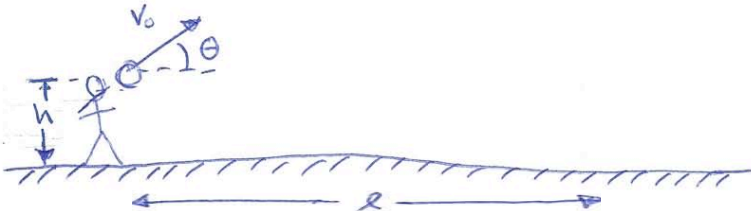


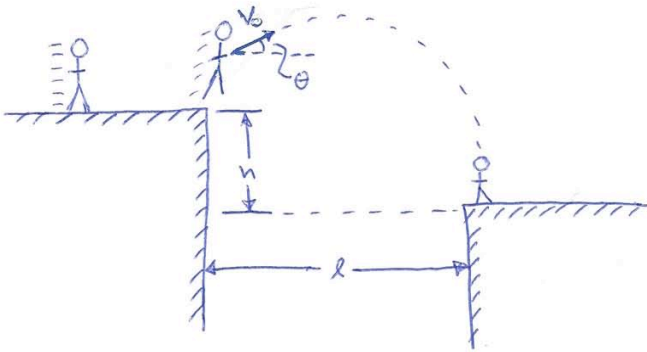
Determine the final height (h_f) that the bullet hits the target when being shot (l) 500m away with an initial bullet velocity (v_0) of 260 m/s at an angle (θ) of 2 degrees to the horizontal, which is a distance 1m off the ground (h_0).



Determine how far the ball goes (length) if thrown at an initial height of 2m, with an initial velocity of 20 m/s at an angle of 30 degrees in the horizontal.

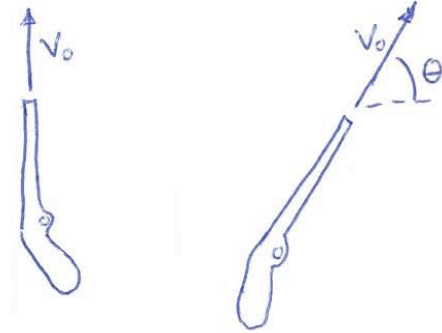


Determine how far you could jump to a build 3m below your building if you leap from with an initial velocity of 10 m/s at an angle of 25 degrees to the horizontal.

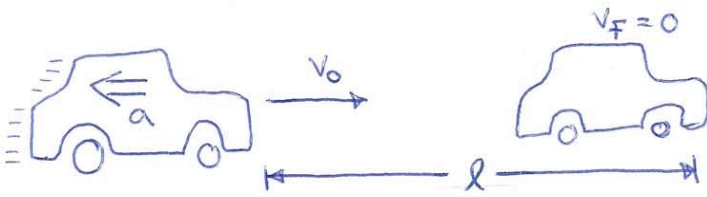


What would the time difference be if you shot a gun with a bullet velocity of 200 m/s straight up versus at an angle of 50 degrees to the horizontal of the bullet to:

- reach maximum height.
- hit the ground.

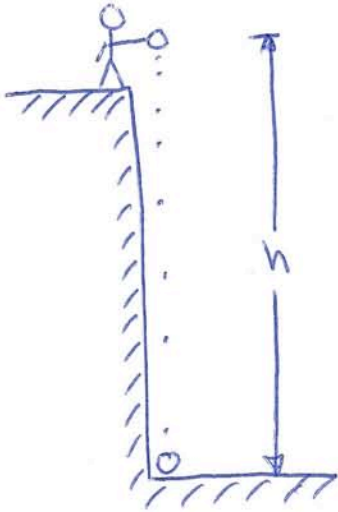


Determine the difference in breaking distance given the initial velocity and deceleration of two different cars.



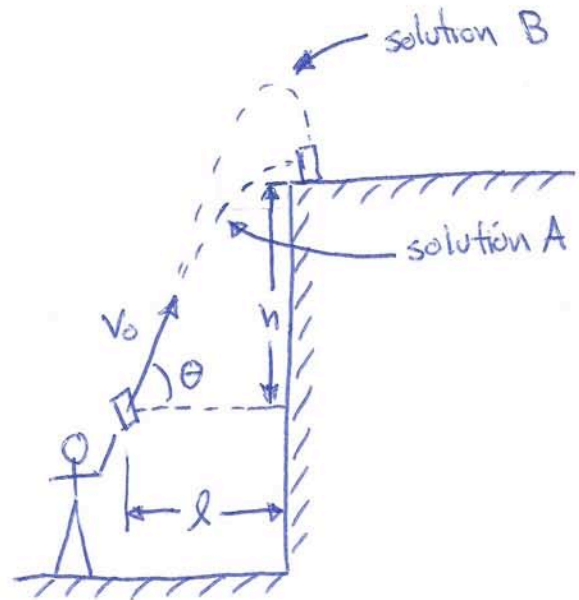
Determine how deep a cliff is if you drop a item and hear the sound 5 seconds later:

- without using the speed of sound
- with using the speed of sound.

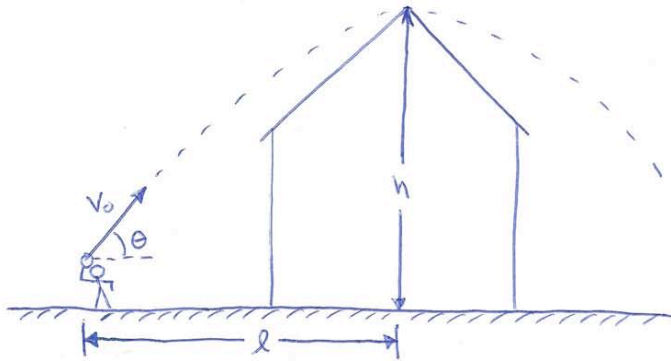


Given that $h=5\text{ m}$ and $L=2\text{ m}$.

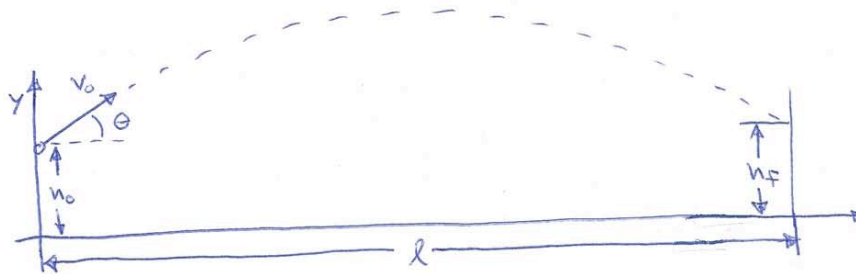
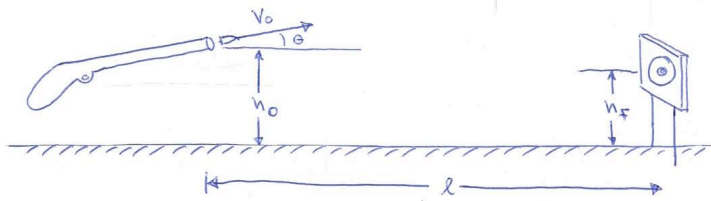
Determine the angle and velocity needed for the bottle to reach it's maximum at the edge of the ledge (solution A). Determine the angle of the bottle being thrown to reach the edge of the ledge with an initial velocity of 11 m/s (solution B).



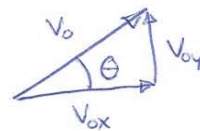
Determine the initial throw velocity (v_0) and angle (θ) to have the ball follow the drawn path, i.e. the maximum height will be at the top of the building. The height of the building (h) is 50m and you are 40m horizontally away from the centre (l).



Determine the final height (h_f) that the bullet hits the target when being shot (l) 500m away with an initial bullet velocity (v_0) of 260 m/s at an angle (θ) of 2 degrees to the horizontal, which is a distance 1m off the ground (h_0).



x-dir	y-dir	time
$x_0 = 0$	$y_0 = h_0$	$t =$
$x_f = l$	$y_f =$	
$v_{0x} = v_0 \cos \theta$	$v_{0y} = v_0 \sin \theta$	
$v_{fx} = v_0 \cos \theta$	$v_{fy} =$	
$a_x = 0$	$a_y = -9.81$	



$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta$$

Kinematic equations

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$x_f = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$l = v_0 \cos \theta t$$

$$t = \frac{l}{v_0 \cos \theta}$$

$$t = \frac{500}{260 \cos(2)}$$

$$t = 1.92 \text{ sec}$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$y_f = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

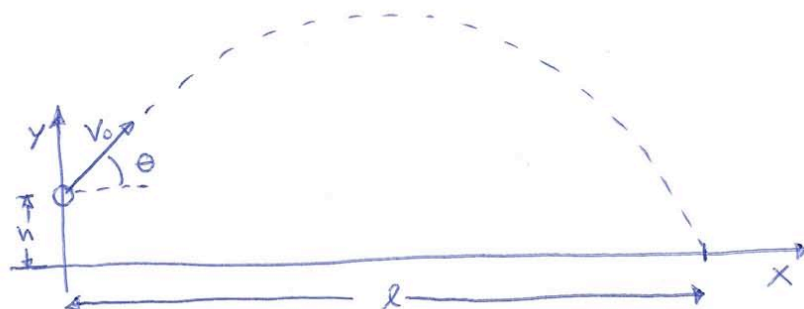
$$h_f = h_0 + v_0 \sin \theta t + \frac{1}{2} a_y t^2$$

$$h_f = 1 + 260 \sin(2)(1.92) + \frac{1}{2}(-9.81)(1.92)^2$$

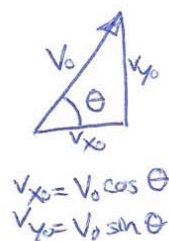
$$h_f = 0.298$$

$$h_f = 0.3 \text{ m}$$

Determine how far the ball goes (length) if thrown at an initial height of 2m, with an initial velocity of 20 m/s at an angle of 30 degrees in the horizontal.



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_0 = 0$	$y_0 = h$	$t =$
$x_F =$	$y_F = 0$	
$v_{x0} = v_0 \cos \theta$	$v_{y0} = v_0 \sin \theta$	
$v_{xF} = v_0 \cos \theta$	$v_{yF} =$	
$a_x = 0$	$a_y = -9.81$	



$$v_{x0} = v_0 \cos \theta$$

$$v_{y0} = v_0 \sin \theta$$

Kinematics equations:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{Fy}^2 = v_{0y}^2 + 2a_y(y_F - y_0)$$

$$v_{Fy} = \pm \sqrt{v_0^2 \sin^2 \theta - 2a_y h}$$

$$v_{Fy} = \pm \sqrt{(20 \sin 30)^2 - 2(-9.81)(2)}$$

$$v_{Fy} = -11.8 \text{ m/s}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{Fy} = v_{0y} + a_y t$$

$$t = \frac{v_{Fy} - v_{0y}}{a}$$

$$t = \frac{v_{Fy} - v_0 \sin \theta}{a}$$

$$t = \frac{-7.02 - (20 \sin 30)}{-9.81}$$

$$t = 1.73 \text{ s}$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$x_F = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

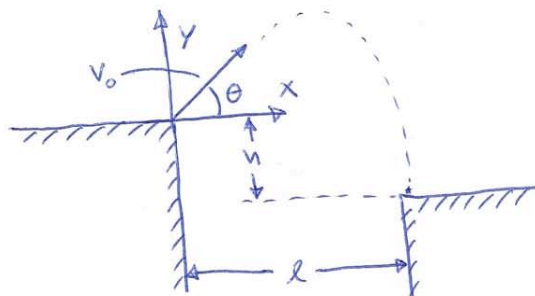
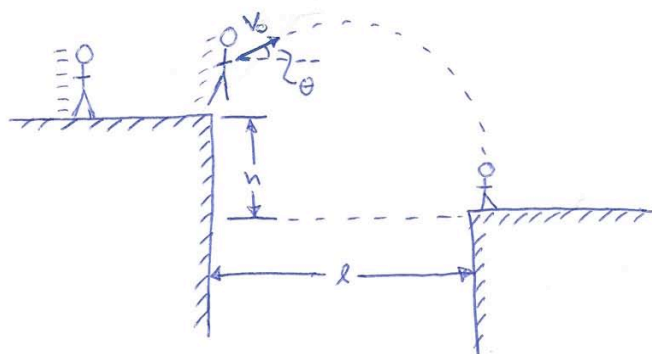
$$x_F = (v_0 \cos \theta) t$$

$$x_F = v_0 t \cos \theta$$

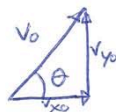
$$x_F = (20)(1.73)(\cos 30)$$

$$x_F = 30 \text{ m}$$

Determine how far you could jump to a build 3m below your building if you leap from with an initial velocity of 10 m/s at an angle of 25 degrees to the horizontal.



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_0 = 0$	$y_0 = 0$	$t =$
$x_F = l$	$y_F = -h$	
$v_{x0} = v_0 \cos \theta$	$v_{y0} = v_0 \sin \theta$	
$v_{xF} = v_0 \cos \theta$	$v_{yF} =$	
$a_x = 0$	$a_y = -9.81$	



$$v_{x0} = v_0 \cos \theta$$

$$v_{y0} = v_0 \sin \theta$$

Kinematic equations

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{yF}^2 = v_{y0}^2 + 2a_y(y_F - y_0)$$

$$v_{yF} = \pm \sqrt{v_0^2 \sin^2 \theta - 2a_y h}$$

$$v_{yF} = \pm \sqrt{(10 \sin 25)^2 - 2(-9.81)(3)}$$

$$v_{yF} = -8.76 \text{ m/s}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{yF} = v_{y0} + a_y t$$

$$t = \frac{v_{yF} - v_{y0}}{a_y}$$

$$t = \frac{v_{yF} - v_0 \sin \theta}{a_y}$$

$$t = \frac{-8.76 - (10 \sin 25)}{-9.81}$$

$$t = 1.32 \text{ s}$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$x_F = \cancel{x_0} + v_{0x} t + \cancel{\frac{1}{2} a_x t^2} = 0$$

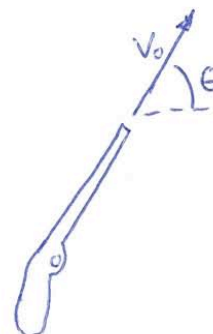
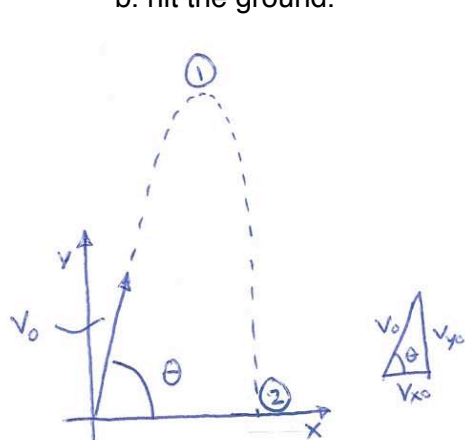
$$x_F = v_0 t \cos \theta$$

$$x_F = (10)(1.32) \cos 25$$

$$x_F = 12 \text{ m}$$

What would the time difference be if you shot a gun with a bullet velocity of 200 m/s straight up versus at an angle of 50 degrees to the horizontal of the bullet to:

- reach maximum height.
- hit the ground.



x-dir	y-dir	time
$x_0 = 0$	$y_0 = 0$	$t =$
$x_1 =$	$y_1 =$	
$x_2 =$	$y_2 = 0$	
$v_{0x} = v_0 \cos \theta$	$v_{0y} = v_0 \sin \theta$	
$v_{1x} = v_0 \cos \theta$	$v_{1y} = 0$	
$v_{2x} = v_0 \cos \theta$	$v_{2y} =$	
$a_x = 0$	$a_y = -9.81$	

Kinematic equations

State 1

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{y1} = v_{y0} + a_y t$$

$$t = \frac{v_{y1} - v_{y0}}{a_y}$$

$$t_1 = \frac{-v_0 \sin \theta}{a_y}$$

State 2

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{y2}^2 = v_{y0}^2 + 2a_y(y_2 - y_0)$$

$$v_{y2} = \pm \sqrt{v_{y0}^2}$$

$$v_{y2} = -v_{y0}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{y2} = v_{y0} + a_y t$$

$$t = \frac{v_{y2} - v_{y0}}{a_y}$$

$$t = \frac{(-v_{y0}) - v_{y0}}{a_y}$$

$$t_2 = \frac{-2v_0 \sin \theta}{a_y}$$

$$\theta = 90^\circ (\text{straight up})$$

$$v_0 = 200 \text{ m/s}$$

$$a. \quad t_1 = \frac{-v_0 \sin \theta}{a_y} = \frac{-(200 \sin 90)}{-9.81}$$

$$t_1 = 20 \text{ sec}$$

$$b. \quad t_2 = \frac{-2v_0 \sin \theta}{a_y} = \frac{-2(200) \sin 90}{-9.81}$$

$$t_2 = 40 \text{ sec}$$

$$\theta = 50^\circ$$

$$v_0 = 200 \text{ m/s}$$

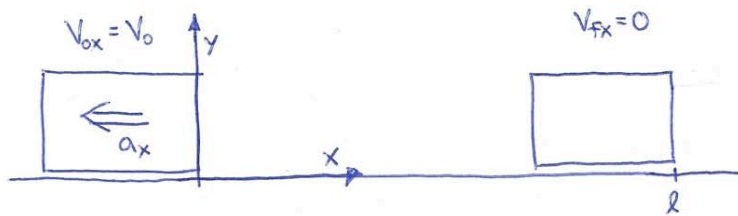
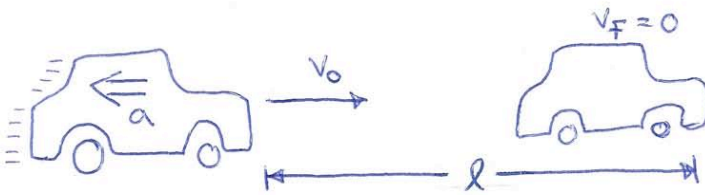
$$a. \quad t_1 = \frac{-(200 \sin 50)}{-9.81}$$

$$t_1 = 16 \text{ sec}$$

$$b. \quad t_2 = \frac{-2(200) \sin 50}{-9.81}$$

$$t_2 = 32 \text{ sec}$$

Determine the difference in breaking distance given the initial velocity and deceleration of two different cars.



x-direction

$$x_0 = 0$$

$$x_F = l$$

$$v_{0x} = v_0$$

$$v_{Fx} = 0$$

$$a_x = a_x$$

For example:

$$\text{let } a_x = -5 \text{ m/s}^2$$

so

v_0	l
5 m/s	3 m
10 m/s	10 m
15 m/s	23 m
20 m/s	40 m

Kinematic equations:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

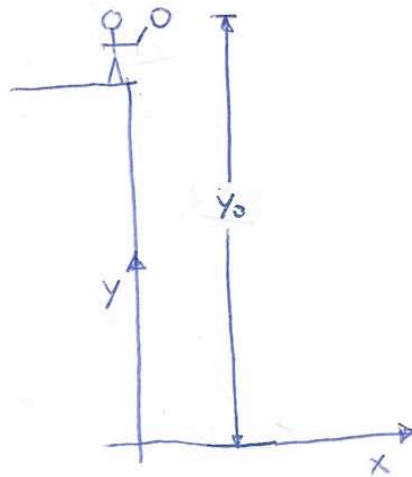
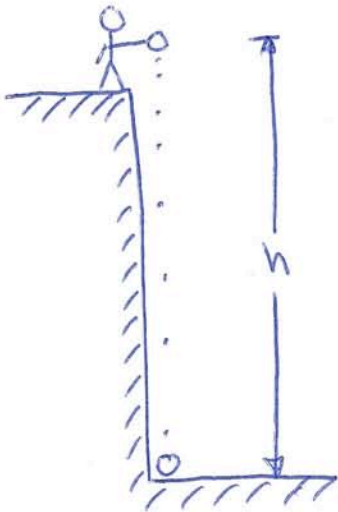
$$v_{Fx}^2 = v_{0x}^2 + 2a_x(x_F - x_0)$$

$$0^2 = v_0^2 + 2a_x(l - 0)$$

$$0 = v_0^2 + 2a_x l$$

$$l = -\frac{v_0^2}{2a}$$

- Determine how deep a cliff is if you drop a item and hear the sound 5 seconds later:
- without using the speed of sound
 - with using the speed of sound.



y-dir

time

$$y_0 =$$

$$t = 5$$

$$y_f = 0$$

$$v_{0y} = 0$$

$$v_{fy} =$$

$$a_y = -9.81$$

Kinematic equations:

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\cancel{y_f} = y_0 + \cancel{v_{0y} t} + \frac{1}{2} a_y t^2$$

$$0 = y_0 + \frac{1}{2} a_y t^2$$

$$y_0 = -\frac{1}{2} a_y t^2$$

$$y_0 = -\frac{1}{2} a_y t^2$$

$$= -\frac{1}{2} (-9.81) (5)^2$$

$$y_0 = 123 \text{ m}$$

with the speed of sound.

breakup time into two parts: ball falling
sound travelling back to ear.

So

$$t_{\text{total}} = t_{\text{falling}} + t_{\text{sound}}$$

use kinematic equations

Falling:

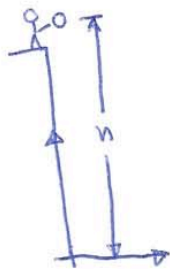
$$y_0 = h$$

$$y_f = 0$$

$$v_{0y} = 0$$

$$v_{fy} =$$

$$a_y = -9.81$$



$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$y_f = y_0 + v_{0y} t_f + \frac{1}{2} a_y t_f^2$$

$$0 = h + \frac{1}{2} a_y t_f^2 \quad (1)$$

sound:

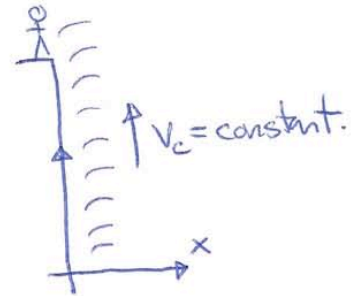
$$y_0 = 0$$

$$y_f = h$$

$$v_{0y} = v_c$$

$$v_{fy} = v_c$$

$$a_{y_{\text{sound}}} = 0$$



$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$y_f = y_0 + v_{0y} t_s + \frac{1}{2} a_{y_{\text{sound}}} t_s^2$$

$$h = v_c t_s \quad (2)$$

solve for t_f & t_s in eqs (1) & (2), then substitute into $t = t_f + t_s$

$$0 = h + \frac{1}{2} a_y t_f^2 \quad (1)$$

$$h = v_c t_s$$

$$\frac{1}{2} a_y t_f^2 = -h$$

$$t_s = \frac{h}{v_c}$$

$$t_f^2 = -\frac{2h}{a_y}$$

$$t_f = \sqrt{-\frac{2h}{a_y}}$$

$$t = t_f + t_s$$

$$t = \sqrt{-\frac{2h}{a_y}} + \frac{h}{v_c}$$

solve for h .

$$t = \sqrt{-\frac{2h}{a_y}} + \frac{h}{v_c}$$

solve for h:

$$\sqrt{-\frac{2h}{a_y}} = t - \frac{h}{v_c}$$

square both sides

$$\left(\sqrt{-\frac{2h}{a_y}}\right)^2 = \left(t - \frac{h}{v_c}\right)^2$$

$$-\frac{2h}{a_y} = \left(t^2 - \frac{2ht}{v_c} + \frac{h^2}{v_c^2}\right)$$

$$0 = \frac{h^2}{v_c^2} + \frac{2h}{a_y} - \frac{2ht}{v_c} + t^2$$

$$0 = h^2 + \left(\frac{2v_c^2}{a_y} - \frac{2v_c t}{v_c}\right)h + v_c^2 t^2$$

use quadratic formula:

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where

$$a = 1$$

$$b = \left(\frac{2v_c^2}{a_y} - 2v_c t\right)$$

$$c = v_c^2 t^2$$

using: $v_c = 330 \text{ m/s}$, $a_y = -9.81 \text{ m/s}^2$ & $t = 5 \text{ sec}$

$$a = 1$$

$$b = \left(\frac{2(330)^2}{-9.81} - 2(330)(5)\right) = -25502$$

$$c = (330)^2(5)^2 = 2722500$$

$$h = \frac{+25502 \pm \sqrt{(25502)^2 - 4(2722500)}}{2} = 12751 \pm 12644$$

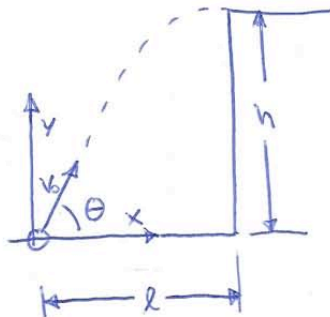
$$= 25395, \underline{107}$$

$$h = 107 \text{ m}$$

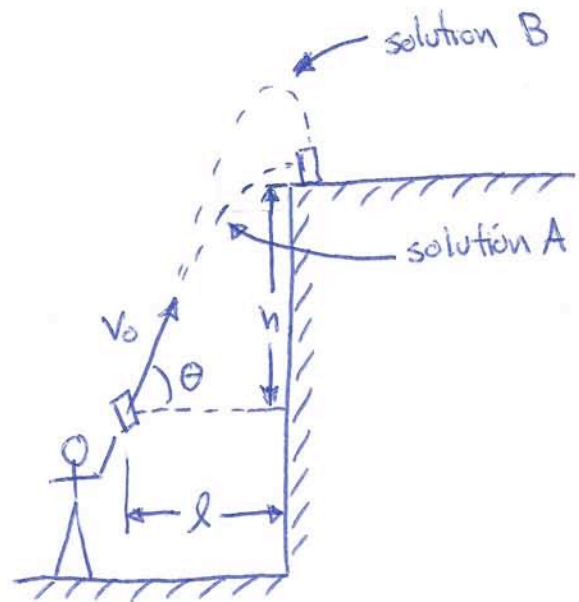
Given that $h=5$ m and $L=2$ m.

Determine the angle and velocity needed for the bottle to reach it's maximum at the edge of the ledge (solution A). Determine the angle of the bottle being thrown to reach the edge of the ledge with an initial velocity of 10 m/s (solution B).

Solution A



$$\begin{aligned} V_{0x} &= V_0 \cos \theta \\ V_{0y} &= V_0 \sin \theta \end{aligned}$$



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_0 = 0$	$y_0 = 0$	$t =$
$x_F = L = 2$	$y_F = h = 5$	
$v_{0x} = V_0 \cos \theta$	$v_{0y} = V_0 \sin \theta$	
$v_{Fx} = V_0 \cos \theta$	$v_{Fy} = 0$	
$a_x = 0$	$a_y = -9.81$	

Kinematic equations:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{Fy}^2 = v_{0y}^2 + 2a_y(y_F - y_0)$$

$$v_{0y} = \pm \sqrt{-2a_y y_F}$$

$$V_0 \sin \theta = + \sqrt{-2a_y y_F}$$

$$\boxed{V_0 \sin \theta = \sqrt{-2a_y y_F}} \quad (1)$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$x_F = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_{0x} = \frac{x_F}{t}$$

$$\begin{aligned} V_0 \cos \theta &= \frac{x_F}{t} \\ &= \frac{L}{t} \end{aligned}$$

$$\boxed{V_0 \cos \theta = \frac{L}{t}} \quad (2)$$

$$\vec{V} = \vec{V}_0 + \vec{a}t$$

$$V_{Fy} = V_{0y} + a_y t$$

$$V_{0y} = -a_y t$$

$$\boxed{V_0 \sin \theta = -a_y t} \quad (3)$$

solve 3 eqn. with 3 unknowns (V_0, θ & t)

sub (1) into (3)

$$\left[\sqrt{-2a_y y_F} \right] = -a_y t$$

$$t = \frac{\sqrt{-2a_y y_F}}{-a_y}$$

$$t = \frac{\sqrt{-2(-9.81)(5)}}{-(-9.81)}$$

$$\boxed{t = 1.0 \text{ s}}$$

divide eqn. (3) by eqn. (2)

$$\frac{V_0 \sin \theta}{V_0 \cos \theta} = \frac{-a_y t}{l/t}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{-a_y t^2}{l}$$

$$\tan \theta = \frac{-a_y t^2}{l}$$

$$\theta = \tan^{-1} \left(\frac{-a_y t^2}{l} \right)$$

$$= \tan^{-1} \left(\frac{-(-9.81)(1.0)^2}{2} \right)$$

$$\boxed{\theta = 78.5^\circ}$$

solve eqn. (2) for V_0

$$V_0 \cos \theta = \frac{l}{t}$$

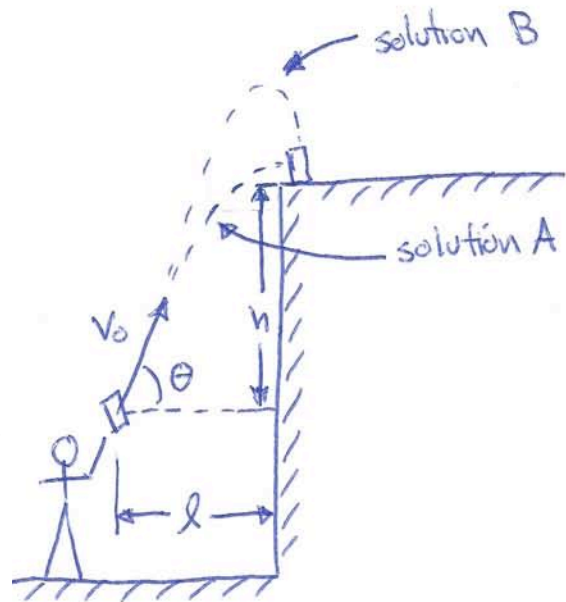
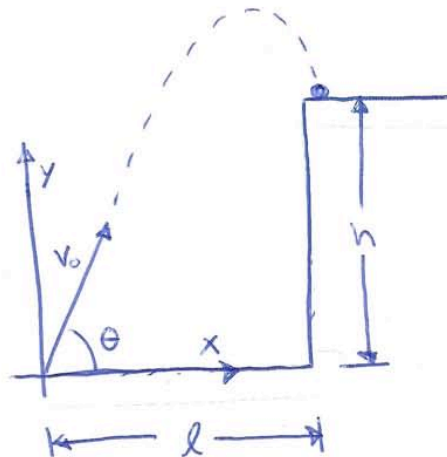
$$V_0 = \frac{l}{t \cos \theta} = \frac{2}{(1.0) \cos(78.5)}$$

$$\boxed{V_0 = 10 \text{ m/s}}$$

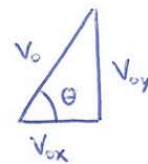
Given that $h=5$ m and $L=2$ m.

Determine the angle and velocity needed for the bottle to reach it's maximum at the edge of the ledge (solution A). Determine the angle of the bottle being thrown to reach the edge of the ledge with an initial velocity of 10 m/s (solution B).

Solution B



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_0 = 0$	$y_0 = 0$	$t =$
$x_f = l = 2$	$y_f = h = 5$	
$v_{0x} = v_0 \cos \theta$	$v_{0y} = v_0 \sin \theta$	
$v_{fx} = v_0 \cos \theta$	$v_{fy} =$	
$a_x = 0$	$a_y = -9.81$	



$$v_{0x} = v_0 \cos \theta$$

$$v_{0y} = v_0 \sin \theta$$

kinematic equations:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{fy}^2 = v_{0y}^2 + 2a_y(y_f - y_0)$$

$$v_{fy}^2 = v_0^2 \sin^2 \theta + 2a_y(h - 0)$$

$$\boxed{v_{fy}^2 = v_0^2 \sin^2 \theta + 2a_y h} \quad (1)$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$v_{fy} = v_{0y} + a_y t$$

$$\boxed{v_{fy} = v_0 \sin \theta + a_y t} \quad (2)$$

$$\vec{X} = \vec{X}_0 + \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

$$Y_F = \cancel{Y_0} + V_{0y} t + \frac{1}{2} a_y t^2$$

$$h = V_0 \sin \theta + \frac{1}{2} a_y t^2$$

$$\boxed{h = V_0 t \sin \theta + \frac{1}{2} a_y t^2} \quad (3)$$

$$\vec{X} = \vec{X}_0 + \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

$$X = \cancel{X_0} + V_{0x} t + \frac{1}{2} a_x t^2$$

$$l = V_0 \cos \theta +$$

$$\boxed{l = V_0 t \cos \theta} \quad (4)$$

solve 4 equations with 3 unknowns (θ, t, V_{Fy})

From observation ... use equ. (3) & (4):

rearrange equ. (3):

$$h = V_0 t \sin \theta + \frac{1}{2} a_y t^2$$

$$V_0 t \sin \theta = h - \frac{1}{2} a_y t^2$$

$$\boxed{\sin \theta = \frac{h - \frac{1}{2} a_y t^2}{V_0 t}}$$

equ. (4)

$$l = V_0 t \cos \theta$$

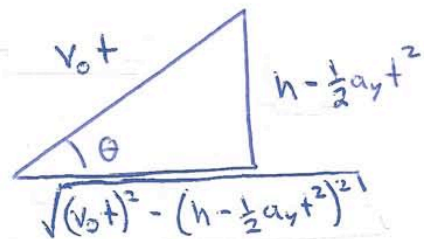
$$l = \cancel{V_0 t} \left[\frac{\sqrt{V_0^2 t^2 - [h - \frac{1}{2} a_y t^2]^2}}{\cancel{V_0 t}} \right]$$

$$l^2 = V_0^2 t^2 - h^2 + h a_y t^2 - \frac{a_y^2}{4} t^4$$

$$\frac{a_y^2}{4} t^4 + (-h a_y - V_0^2) t^2 + (l^2 + h^2) = 0 \quad \text{let } t^2 = b$$

$$\frac{(-9.81)^2}{4} b^2 + [-(5)(-9.81) - (15)^2] b + [(2)^2 + (5)^2] = 0$$

$$\boxed{24 b^2 - 176 b + 29 = 0}$$



$$\cos \theta = \frac{\sqrt{V_0^2 t^2 - (h - \frac{1}{2} a_y t^2)^2}}{V_0 t}$$

$$24b^2 - 176b + 29 = 0$$

$$b = \frac{-(-176) \pm \sqrt{(-176)^2 - 4(24)(29)}}{2(24)} = 3.7 \pm 3.5$$

$$= 7.2, 0.2$$

$$t = \sqrt{b} = 2.7, 0.4 \text{ sec}$$

eqn. (4)

$$l = v_0 t \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{l}{v_0 t} \right) = \cos^{-1} \left(\frac{2}{(15)(t)} \right)$$

$$\theta = 87^\circ, 71^\circ$$

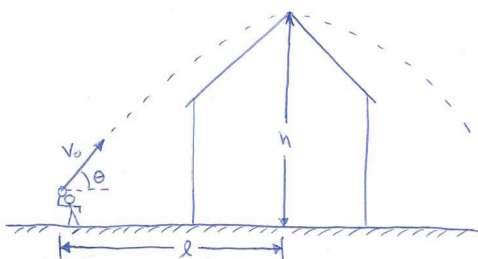
eqn. (2)

$$V_{fy} = v_0 \sin \theta + a_y t$$

$$= 15 \sin \theta - 9.81 t$$

$$V_{fy} = -11.5 \text{ m/s}, 10.2 \text{ m/s}$$

Determine the initial throw velocity (v_0) and angle (θ) to have the ball follow the drawn path, i.e. the maximum height will be at the top of the building. The height of the building (h) is 50m and you are 40m horizontally away from the centre (l).



Kinematic equations

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$x_F = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$l = v_0 \cos \theta t \quad (1)$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{Fy}^2 = v_{0y}^2 + 2a_y(y_F - y_0)$$

$$0 = v_0^2 \sin^2 \theta + 2a_y h \quad (2)$$

$$\vec{v} = \vec{v}_0 + \vec{a} t$$

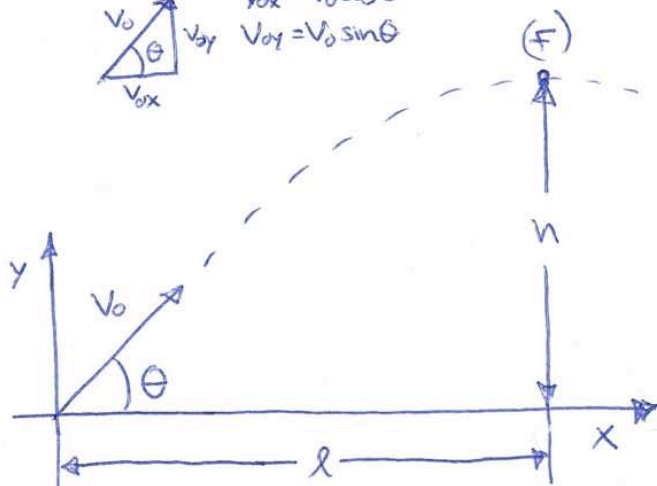
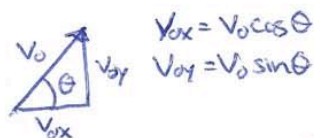
$$v_{Fy} = v_{0y} + a_y t$$

$$0 = v_0 \sin \theta + a_y t \quad (3)$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$y_F = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$h = v_0 \sin \theta t + \frac{1}{2} a_y t^2 \quad (4)$$



X-dir	-y-dir	time
$x_0 = 0$	$y_0 = 0$	$t =$
$x_F = l$	$y_F = h$	
$v_{0x} = v_0 \cos \theta$	$v_{0y} = v_0 \sin \theta$	
$v_{Fx} = v_0 \cos \theta$	$v_{Fy} = 0$	
$a_x = 0$	$a_y = -9.81$	

Solve for $v_0 \sin \theta$ in eq. (2) & (3)

$$0 = v_0^2 \sin^2 \theta + 2a_y h$$

$$v_0^2 \sin^2 \theta = -2a_y h$$

$$v_0 \sin \theta = \sqrt{-2a_y h} \quad (2a)$$

$$0 = v_0 \sin \theta + a_y t$$

$$v_0 \sin \theta = -a_y t \quad (3a)$$

Set (2a) and (3a) equal:

$$v_0 \sin \theta = \sqrt{-2a_y h} = -a_y t$$

$$(\sqrt{-2a_y h})^2 = (-a_y t)^2$$

$$-2a_y h = a_y^2 t^2$$

$$t = \sqrt{\frac{-2h}{a_y}}$$

$$= \sqrt{\frac{-2(50)}{-9.81}}$$

$$= 3.2 \text{ sec.}$$

Solve for $v_0 \cos \theta$ in eqn. (1)

$$l = v_0 \cos \theta t$$

$$v_0 \cos \theta = \frac{l}{t} \quad (1a)$$

divide (3a) by (1a)

$$\frac{v_0 \sin \theta}{v_0 \cos \theta} = \frac{-a_y t}{l/t}$$

$$\tan \theta = -\frac{a_y t^2}{l}$$

$$\theta = \tan^{-1} \left(-\frac{a_y t^2}{l} \right)$$

$$\theta = \tan^{-1} \left(-\frac{(-9.81)(3.2)^2}{40} \right)$$

$$\boxed{\theta = 68^\circ}$$

Solve for v_0 in eqn. (1)

$$l = v_0 \cos \theta t$$

$$\boxed{v_0 = \frac{l}{t \cos \theta} = \frac{40}{(3.2) \cos 68} = 34 \text{ m/s.}}$$