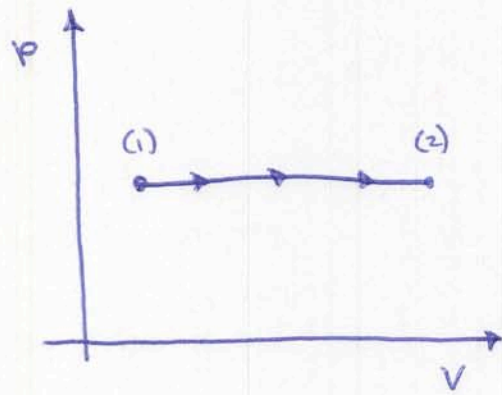


Isobaric Process ($P_1 = P_2$)

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$

(Charles' law)



Work

$$\boxed{W = PdV = p(V_2 - V_1)}$$

Heat transfer

1st Law of Thermodynamics

$$E_f - E_o = Q - W$$

$$m c_v (T_2 - T_1) = Q - p(V_2 - V_1)$$

$$Q = m c_v (T_2 - T_1) + p(V_2 - V_1)$$

sub Ideal Gas law

$$Q = m c_v (T_2 - T_1) + m R_s (T_2 - T_1)$$

$$Q = m \underbrace{(c_v + R_s)}_{c_p} (T_2 - T_1)$$

$$\boxed{Q = m c_p (T_2 - T_1)}$$

where

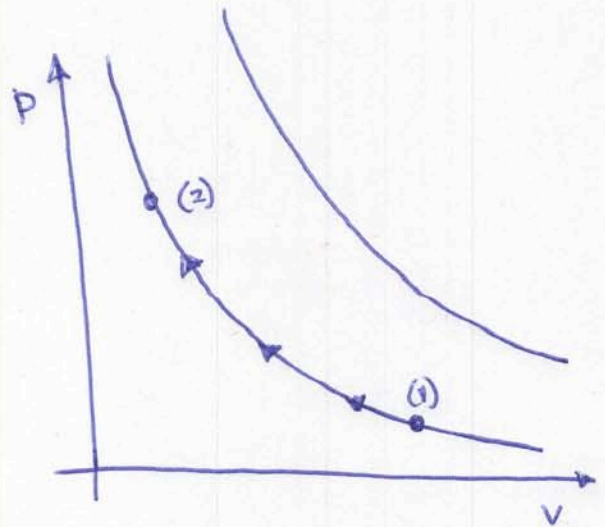
c_p = specific heat capacity at constant pressure

$$= c_v + R_s \quad (\text{Mayer's relation})$$

Isothermal Process ($T_1 = T_2$)

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\boxed{p_1 V_1 = p_2 V_2} \quad (\text{Boyle's Law})$$



Work

$$\int_1^2 dW = \int_1^2 p dV$$

$$W = \int_1^2 \frac{m R_s T}{V} dV = m R_s T \ln(V) \Big|_{V_1}^{V_2} = m R_s T \ln\left(\frac{V_2}{V_1}\right)$$

ideal gas law

$$\boxed{W = m R_s T \ln\left(\frac{V_2}{V_1}\right)}$$

Heat transfer

1st Law of Thermodynamics

$$E_f - E_o = Q - W$$

$$m c_v (T_2 - T_1) = Q - m R_s T \ln\left(\frac{V_2}{V_1}\right)$$

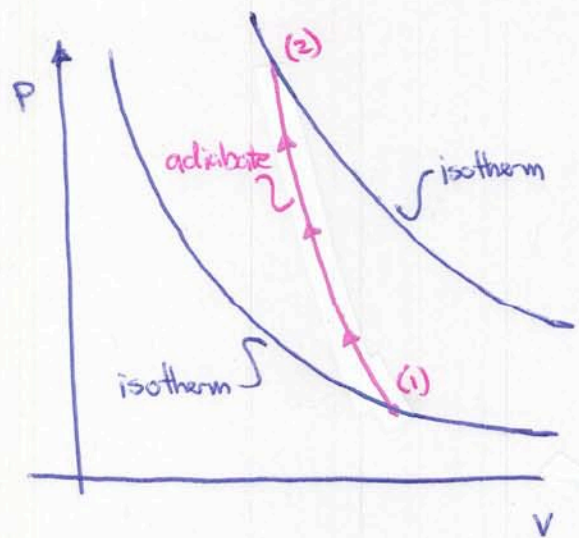
$$\boxed{Q = m R_s T \ln\left(\frac{V_2}{V_1}\right)}$$

Adiabatic Process

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_1 \left(\frac{m R_s T}{P_1} \right)^\gamma = P_2 \left(\frac{m R_s T}{P_2} \right)^\gamma$$

$$P_1^{1-\gamma} T_1^\gamma = P_2^{1-\gamma} T_2^\gamma$$



Work

$$\int_1^2 dw = \int_1^2 p dv$$

$$\begin{aligned} W &= \int_1^2 P_1 \left(\frac{V_1}{V} \right)^\gamma dV = P_1 V_1^\gamma \int_1^2 V^{-\gamma} dV = P_1 V_1^\gamma \left[\frac{1}{1-\gamma} V^{1-\gamma} \right]_{V_1}^{V_2} \\ &= P_1 V_1^\gamma \left[\frac{V_2^{1-\gamma} - V_1^{1-\gamma}}{1-\gamma} \right] = \frac{P_1 V_1^\gamma V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma}}{1-\gamma} \\ &= \frac{(P_2 V_2^\gamma) V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma}}{1-\gamma} = \frac{P_2 V_2 - P_1 V_1}{1-\gamma} \end{aligned}$$

$$W = \frac{P_2 V_2 - P_1 V_1}{1-\gamma}$$

Further, sub Ideal gas Law

$$W = \frac{m R_s T_2 - m R_s T_1}{1-\gamma} = m \left(\frac{R_s}{\gamma-1} \right) (T_1 - T_2)$$

$$W = m C_v (T_1 - T_2)$$

Heat Transfer

$$Q = 0$$

Adiabatic Derivation (molar form)

1st Law of Thermodynamics

$$E_f - E_o = Q - W$$

differential form

$$dE = \cancel{dQ}^0 - dW \quad (dQ=0 \text{ for adiabatic process}) \quad (1)$$

Definition of Work

$$dW = Fdx = PA dx = P dV \quad (2)$$

Definition of internal energy

$$dE = \frac{F}{2} nR dT \quad (3)$$

where:

F = degrees of freedom of the gas molecule
aside:

$$dE = \frac{F}{2} nR dT = n C_V dT = \frac{m}{M} C_V dT = m c_v dT$$

where:

C_V = heat capacity at constant volume

$$\left[\frac{J}{mol \cdot K} \right]$$

c_v = specific heat capacity at constant volume

$$\left[\frac{J}{kg \cdot K} \right]$$

Sub (2) and (3) into (1):

$$\frac{F}{2} nR dT = -P dV \quad (1a)$$

Ideal Gas Law

$$PV = nRT \Rightarrow T = \frac{PV}{nR} \quad (4)$$

sub (4) into (1a)

$$\frac{F}{2} nR d\left(\frac{PV}{nR}\right) = -P dV$$

cancel nR as it's a constant and use chain rule to expand d(PV)

$$\frac{F}{2} (P dV + V dP) = -P dV$$

continued

$$\frac{F}{2}(pdV + Vdp) = -pdV$$

$$\left(\frac{F}{2} + 1\right)pdV = -\frac{F}{2}Vdp$$

$$\left(\frac{1 + \frac{F}{2}}{\frac{F}{2}}\right) \frac{dV}{V} = -\frac{dp}{p}$$

let:

$$\gamma = \frac{1 + \frac{F}{2}}{\frac{F}{2}} = 1 + \frac{2}{F} = \text{heat capacity ratio}$$

note:

$$F = \frac{2}{\gamma - 1} = \text{degrees of freedom of the gas molecule}$$

so

$$\gamma \frac{dV}{V} = -\frac{dp}{p}$$

Integrate both sides from initial to final states

$$\gamma \int_{V_0}^V \frac{dV}{V} = - \int_{P_0}^P \frac{dp}{p}$$

$$\gamma \ln\left(\frac{V}{V_0}\right) = -\ln\left(\frac{P}{P_0}\right)$$

$$\left(\frac{V}{V_0}\right)^\gamma = \left(\frac{P}{P_0}\right)^{-1}$$

$$\left(\frac{V}{V_0}\right)^\gamma \left(\frac{P}{P_0}\right) = 1$$

so

$$pV^\gamma = p_0V_0^\gamma = \text{constant}$$

Adiabatic Derivation (mass form)

1st Law of thermodynamics

$$E_f - E_o = Q - W$$

differential form

$$dE = \cancel{dQ} - dW \quad (dQ=0 \text{ for adiabatic process}) \quad (1)$$

Definition of Work

$$dW = Fdx = PA dx = PdV \quad (2)$$

Definition of internal energy

$$dE = mc_v dT \quad (3)$$

sub (2) and (3) into (1)

$$mc_v dT = -PdV \quad (4)$$

Ideal Gas Law

$$PV = mR_s T \Rightarrow T = \frac{PV}{mR_s}, \text{ where } R_s = \text{specific gas constant} \quad (4)$$

sub (4) into (4)

$$mc_v d\left(\frac{PV}{mR_s}\right) = -PdV$$

$$\frac{c_v}{R_s} d(PV) = -PdV$$

use chain rule to expand $d(PV)$

$$\frac{c_v}{R_s} (Vdp + PdV) = -PdV$$

$$\left(1 + \frac{c_v}{R_s}\right) PdV = \frac{c_v}{R_s} Vdp$$

$$\left(\frac{1 + \frac{c_v}{R_s}}{\frac{c_v}{R_s}}\right) \frac{dV}{V} = -\frac{dp}{p}, \text{ let } \gamma = \frac{1 + \frac{c_v}{R_s}}{\frac{c_v}{R_s}} = \frac{c_v + R}{c_v} = \text{heat capacity ratio}$$

sub heat capacity ratio and integrate from initial to final states.

$$\gamma \ln\left(\frac{V}{V_o}\right) = -\ln\left(\frac{p}{p_o}\right)$$

so

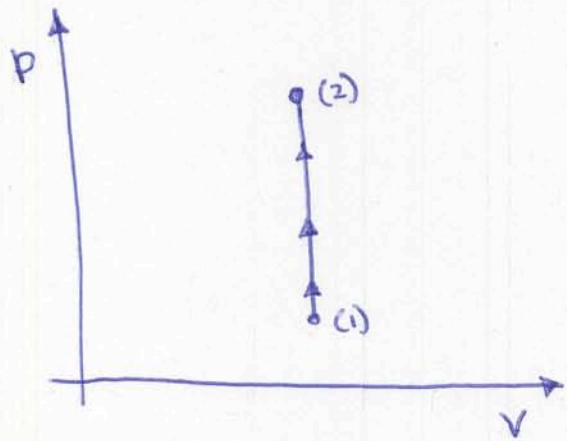
$$\boxed{PV^\gamma = pV_o^\gamma = \text{constant}}$$

Iso metric Process ($V_1 = V_2$)

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\boxed{\frac{p_1}{T_1} = \frac{p_2}{T_2}}$$

(Gay-Lussac's Law)



Work

$$\boxed{W = PdV = p(V_2 - V_1) = 0}$$

Heat transfer

1st Law of thermodynamics

$$E_f - E_o = Q - \cancel{W}^0$$

$$mc_v(T_2 - T_1) = Q$$

$$\boxed{Q = mc_v(T_2 - T_1)}$$