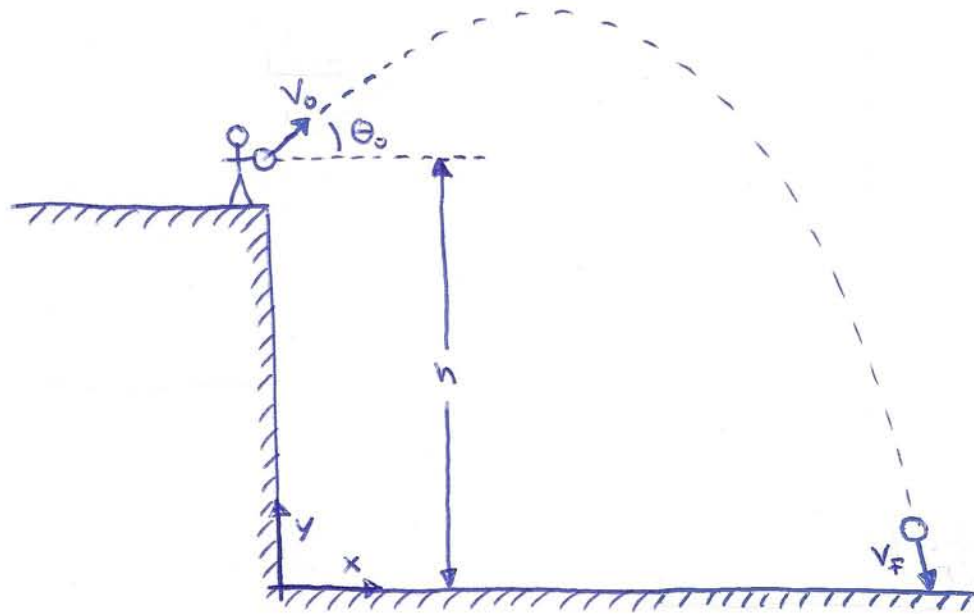
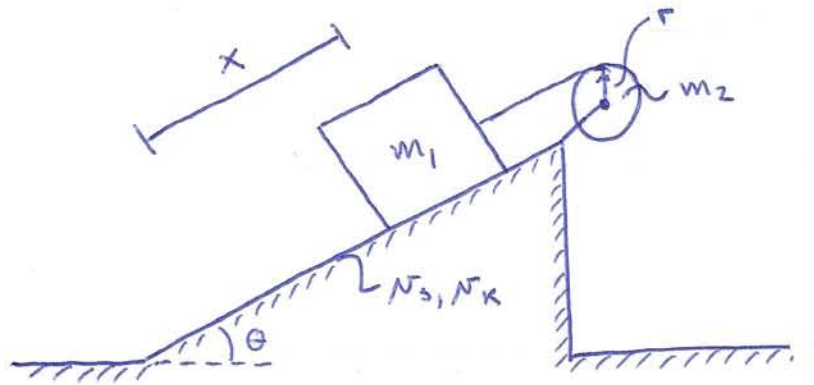


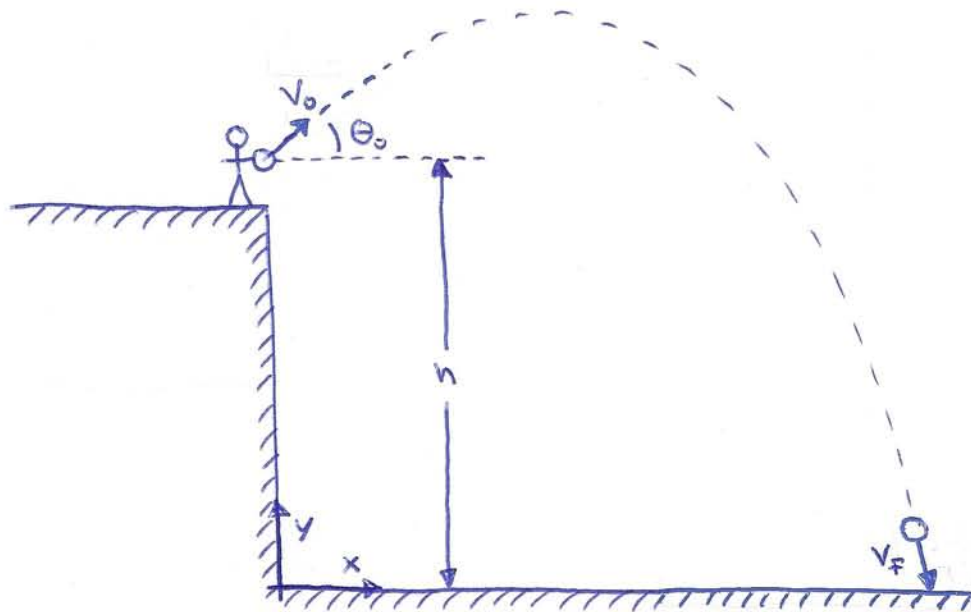
Determine the final velocity of the ball using:
(a) kinematics
(b) Thermodynamics



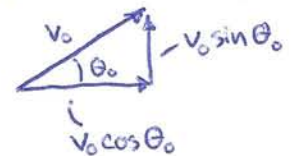
Determine the final velocity of the sliding block attached by a massless stretchless rope which is rapped around a pulley using:

- (a) Dynamics and kinematics
- (b) Thermodynamics





initial:



Kinematics:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}\Delta\vec{x} \Rightarrow v_y^2 = v_{y0}^2 + 2a_y\Delta y$$

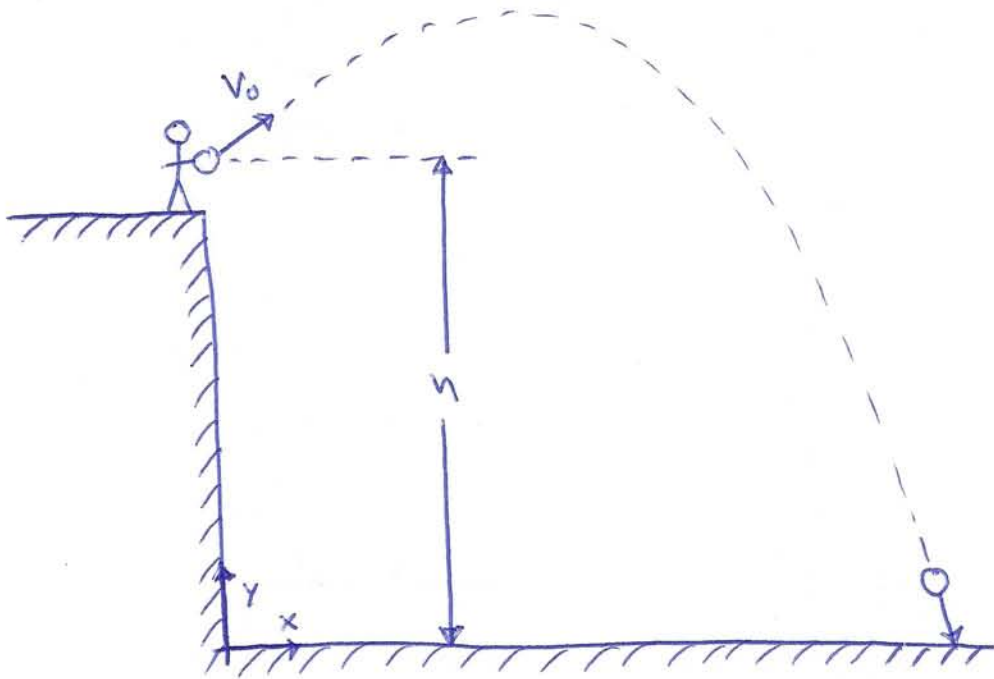
$$\begin{aligned} v_y &= \sqrt{v_{y0}^2 + 2(-g)(y_f - y_0)} \\ &= \sqrt{(v_0 \sin \theta_0)^2 + 2(-g)(0 - h)} \\ &= \sqrt{v_0^2 \sin^2 \theta_0 + 2gh} \end{aligned}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \Rightarrow v_x = v_{x0} + \cancel{a_x t}^0 \quad (a_x = 0)$$

$$v_x = v_{x0} = v_0 \cos \theta_0$$

$$\begin{aligned} v_f &= \sqrt{v_{xf}^2 + v_{yf}^2} \\ &= \sqrt{(v_0 \cos \theta_0)^2 + (\sqrt{v_0^2 \sin^2 \theta_0 + 2gh})^2} \\ &= \sqrt{v_0^2 \cos^2 \theta_0 + v_0^2 \sin^2 \theta_0 + 2gh} \\ &= \sqrt{v_0^2 (\cos^2 \theta_0 + \sin^2 \theta_0) + 2gh} \\ &= \sqrt{v_0^2 + 2gh} \end{aligned}$$

$$\boxed{v_f = \sqrt{v_0^2 + 2gh}}$$



1st Law of Thermodynamics:

$$\Delta E = \cancel{Q} - \cancel{W} = 0 \quad (1)$$

$$E_o = \frac{1}{2} m v_o^2 + mgh \quad (2)$$

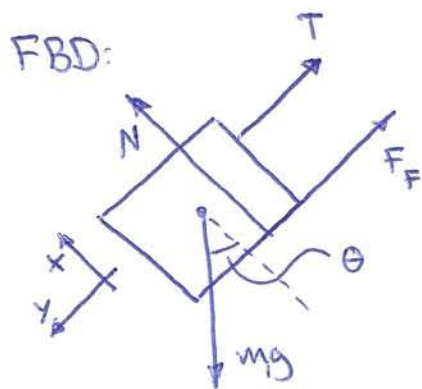
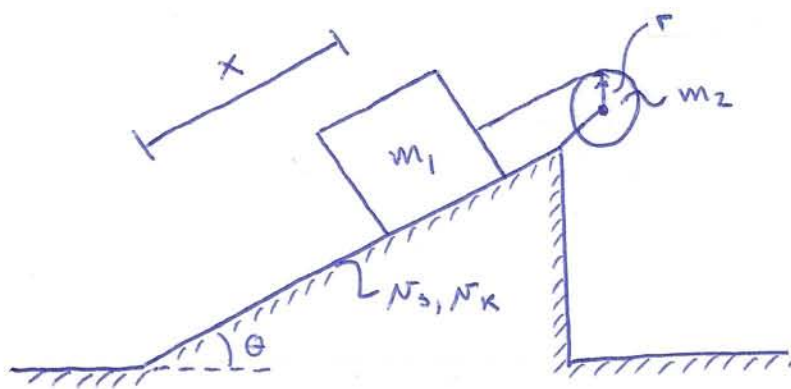
$$E_f = \frac{1}{2} m v_f^2 \quad (3)$$

sub (2) & (3) into (1):

$$\Delta E = E_f - E_o = \left[\frac{1}{2} m v_f^2 \right] - \left[\frac{1}{2} m v_o^2 + mgh \right] = 0$$

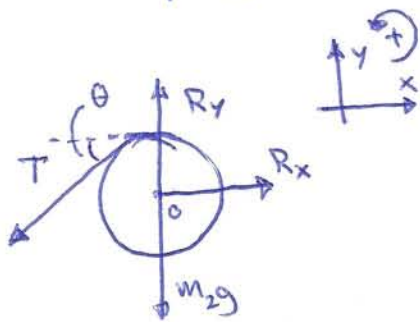
$$v_f^2 = v_o^2 + 2gh$$

$$\boxed{v_f = \sqrt{v_o^2 + 2gh}}$$



$$\sum F_x = m_1 a_x = m_1 g \sin \theta - T - F_f \quad (1)$$

$$\sum F_y = m_1 a_y = N - m_1 g \cos \theta \quad (2)$$



$$\sum F_x = m_2 a_{x2} = R_x - T \cos \theta \quad (3)$$

$$\sum F_y = m_2 a_{y2} = R_y - m_2 g - T \sin \theta \quad (4)$$

$$\sum M_o = I_o \alpha_o = T r \quad (5)$$

sub (2) into (1) and solve for T:

$$m_1 a_x = m_1 g \sin \theta - T - [m_1 g \cos \theta] \mu$$

$$T = m_1 g \sin \theta - \mu m_1 g \cos \theta - m_1 a_x$$

(1a)

sub (2a) into (3):

$$I_0 \alpha = [m_1 g \sin \theta - \mu m_1 g \cos \theta - m_1 a_x] r$$

sub $a_x = \alpha r$ and solve for a_x :

$$I_0 \left[\frac{a_x}{r} \right] = m_1 g r \sin \theta - \mu m_1 g r \cos \theta - m_1 a_x r$$

$$I_0 \frac{a_x}{r} + m_1 a_x r = m_1 g r (\sin \theta - \mu \cos \theta)$$

$$a_x \left[\frac{I_0}{r^2} + m_1 \right] = m_1 g (\sin \theta - \mu \cos \theta)$$

$$a_x = \frac{m_1 g (\sin \theta - \mu \cos \theta)}{m_1 + I_0 / r^2}$$

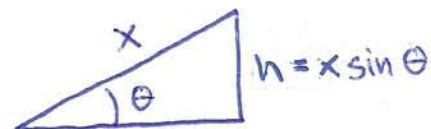
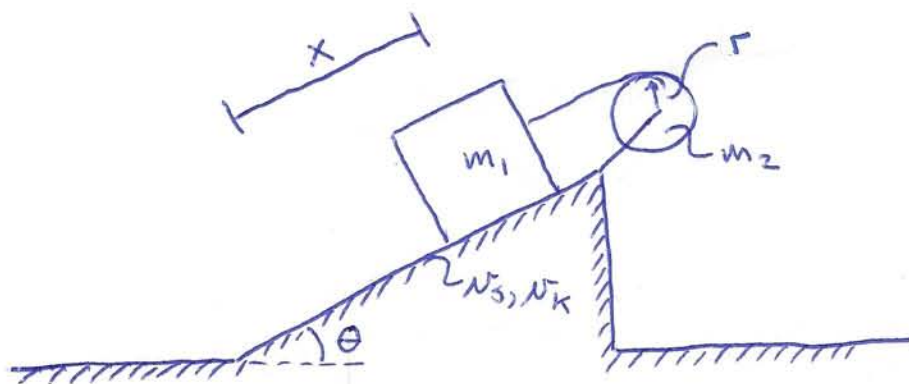
Kinematics on sliding block

$$\vec{V}^2 = \vec{V}_0^2 + 2 \vec{a} \cdot \vec{\Delta x} \Rightarrow V_{x_f}^2 = \cancel{V_{x_0}^2} + 2 a_x (x_f - x_0)$$

sub a_x into kinematics:

$$V_{x_f}^2 = 2 \left[\frac{m_1 g (\sin \theta - \mu \cos \theta)}{m_1 + I_0 / r^2} \right] (x - 0)$$

$$V_{x_f} = \sqrt{\frac{2 m_1 g x (\sin \theta - \mu \cos \theta)}{m_1 + I_0 / r^2}}$$

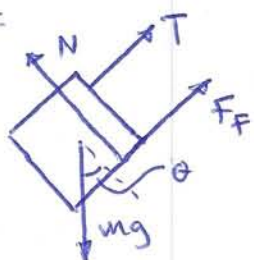


1st law of Thermodynamics

$$\Delta E = Q - W$$

Need to determine W , therefore need F_f .

FBD:



$$\sum F_x = m_1 a_x = m_1 g \sin \theta - T - F_f \quad (1)$$

$$\sum F_y = m_1 a_y = N - m_1 g \cos \theta \quad (2)$$

solve equation (2) for N :

$$N = m_1 g \cos \theta$$

solve for Friction:

$$F_f = N \mu = m_1 g \mu \cos \theta$$

Energy Balance:

$$E_o = mgh = m_1 g (x \sin \theta) \quad (3)$$

$$E_f = \frac{1}{2} m_1 v_f^2 + \frac{1}{2} I_o \omega_f^2 \quad (4)$$

$$W = + F_f x = m_1 g \mu x \cos \theta \quad (5)$$

↑ work done by the system

sub (3), (4) & (5) into 1st law of thermodynamics:

$$\Delta E = E_F - E_0 = -W$$

$$\left[\frac{1}{2} m_1 v_F^2 + \frac{1}{2} I_0 \omega_F^2 \right] - \left[m_1 g x \sin \theta \right] = - \left[m_1 g r x \cos \theta \right]$$

sub $v = \omega r$ and solve for v_F :

$$\frac{1}{2} m_1 v_F^2 + \frac{1}{2} I_0 \left(\frac{v_F}{r} \right)^2 - m_1 g x \sin \theta = - m_1 g r x \cos \theta$$

$$v_F^2 \left(m_1 + \frac{I_0}{r^2} \right) = 2 m_1 g x \sin \theta - 2 m_1 g r x \cos \theta$$

$$v_F = \sqrt{\frac{2 m_1 g x (\sin \theta - r \cos \theta)}{m_1 + I_0 / r^2}}$$