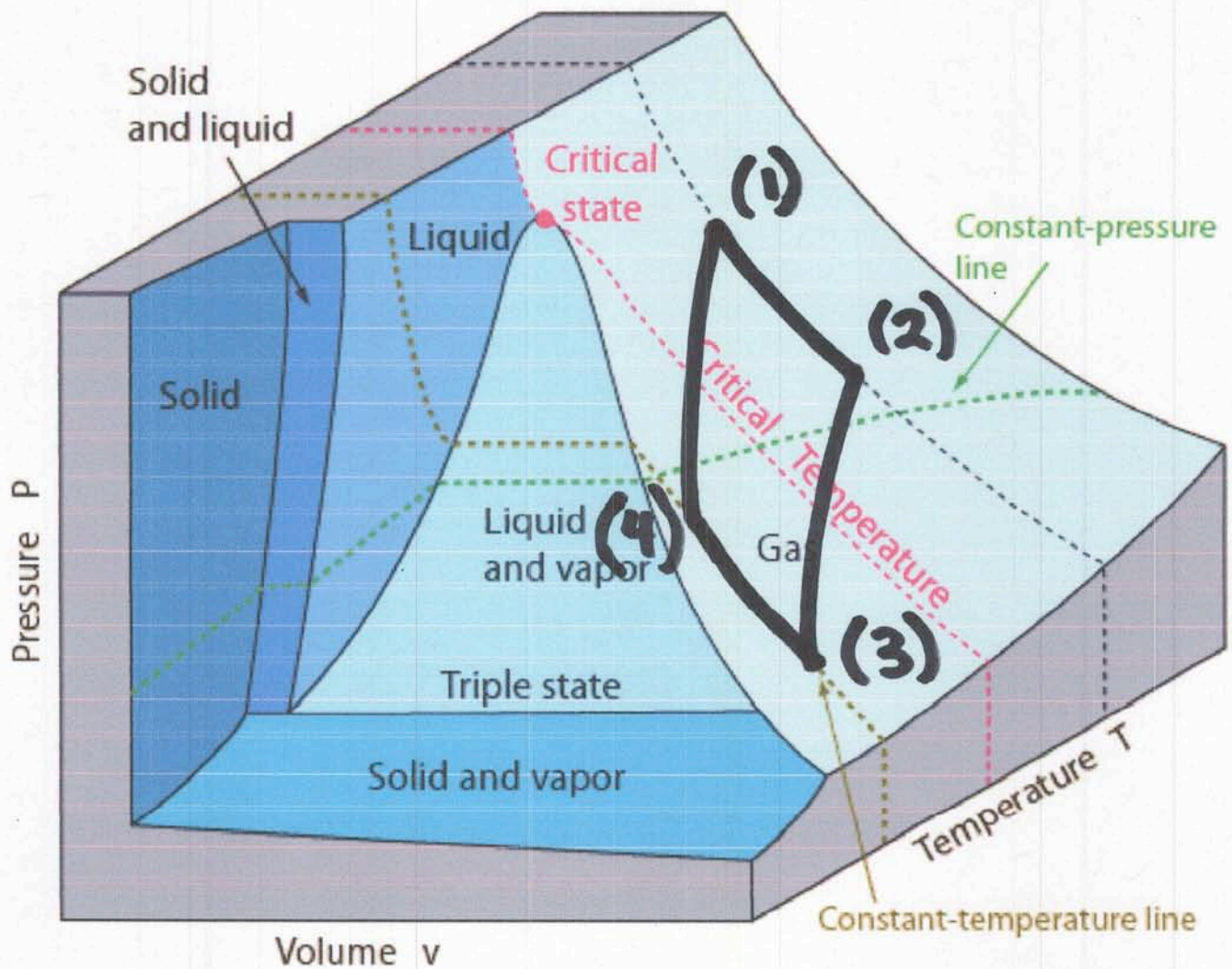


Carnot Cycle



Carnot Cycle

given:

$$T_{\text{Hot}}, T_{\text{Cold}}$$

$$\text{Compression ratio} = \frac{V_3}{V_1}$$

$$\text{and } P_1, V_1$$

determine:

all states

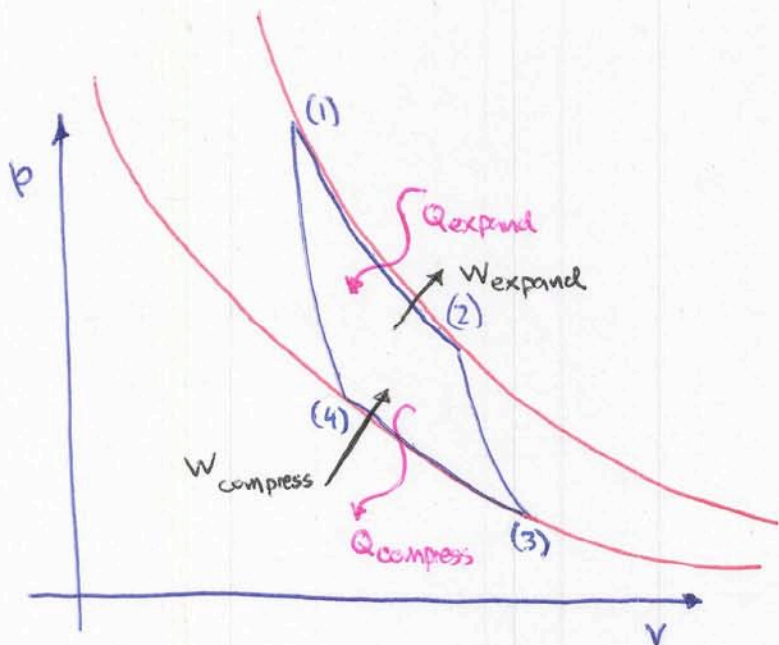


table:

	1	2	3	4
V	given	c.	given	c.
P	given	b.	a.	b.
T	given	given	given	given
m	ideal	/	/	/

path:

$$a. \frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \Rightarrow P_3 = P_1 \left(\frac{V_1}{V_3} \right) \left(\frac{T_3}{T_1} \right)$$

$$b. P_2^{1-\gamma} T_2^{\gamma} = P_3^{1-\gamma} T_3^{\gamma} \Rightarrow P_2 = P_3 \left(\frac{T_3}{T_2} \right)^{\frac{\gamma}{1-\gamma}}$$

$$P_1^{1-\gamma} T_1^{\gamma} = P_4^{1-\gamma} T_4^{\gamma} \Rightarrow P_4 = P_1 \left(\frac{T_1}{T_4} \right)^{\frac{\gamma}{1-\gamma}}$$

$$c. \frac{P_2 V_2}{T_2} = \frac{P_4 V_4}{T_4} \Rightarrow \frac{V_4}{V_2} = \left(\frac{P_2}{P_4} \right) \left(\frac{T_4}{T_2} \right)$$

general:

$$\frac{P_1 V_1}{T_H} = \frac{P_2 V_2}{T_H} = \frac{P_3 V_3}{T_C} = \frac{P_4 V_4}{T_C}$$

Similar ratios for adiabatic process

state $2 \rightarrow 3$ (adiabatic) : $P_2^{1-\gamma} T_2^\gamma = P_3^{1-\gamma} T_3^\gamma$

state $4 \rightarrow 1$ (adiabatic) : $P_4^{1-\gamma} T_4^\gamma = P_1^{1-\gamma} T_1^\gamma$

so:
$$\left(\frac{T_2}{T_3} \right)^{\frac{\gamma}{1-\gamma}} = \frac{P_3}{P_2} = \left(\frac{T_h}{T_c} \right)^{\frac{\gamma}{1-\gamma}} = \frac{P_4}{P_1} = \left(\frac{T_1}{T_4} \right)^{\frac{\gamma}{1-\gamma}}$$

so:

$$\boxed{\frac{P_3}{P_2} = \frac{P_4}{P_1}}$$

Further:

state $2 \rightarrow 3$ (adiabatic) :

$$P_2 V_2^\gamma = P_3 V_3^\gamma$$

state $4 \rightarrow 1$ (adiabatic) :

$$P_4 V_4^\gamma = P_1 V_1^\gamma$$

so

$$\frac{P_3}{P_2} = \frac{V_2^\gamma}{V_3^\gamma} = \frac{P_4}{P_1} = \frac{V_1^\gamma}{V_4^\gamma}$$

so

$$\boxed{\frac{V_2}{V_1} = \frac{V_3}{V_4}}$$

Energy Balance

Work
(W)

Heat transfer
(Q)

Internal energy
(ΔE)

$$1 \rightarrow 2 \quad m R_s T_h \ln\left(\frac{V_2}{V_1}\right)$$

$$m R_s T_h \ln\left(\frac{V_2}{V_1}\right)$$

$$0$$

$$2 \rightarrow 3 \quad \frac{P_3 V_3 - P_2 V_2}{1 - \gamma}$$

$$m C_v (T_2 - T_3)$$

$$0$$

$$\frac{P_3 V_3 - P_2 V_2}{1 - \gamma}$$

$$3 \rightarrow 4 \quad m R_s T_c \ln\left(\frac{V_4}{V_3}\right)$$

$$m R_s T_c \ln\left(\frac{V_4}{V_3}\right)$$

$$0$$

$$4 \rightarrow 1 \quad \frac{P_1 V_1 - P_4 V_4}{1 - \gamma}$$

$$m C_v (T_4 - T_1)$$

$$0$$

$$\frac{P_1 V_1 - P_4 V_4}{1 - \gamma}$$

$$W_{\text{cycle}} = m R_s T_h \ln\left(\frac{V_2}{V_1}\right) + m C_v (T_h - T_c) + m R_s T_c \ln\left(\frac{V_4}{V_3}\right) + m C_v (T_c - T_h)$$

$$= \underbrace{m R_s T_h \ln\left(\frac{V_2}{V_1}\right)}_{W_{\text{expand}}} + \underbrace{m R_s T_c \ln\left(\frac{V_4}{V_3}\right)}_{W_{\text{compress}}} + \cancel{m C_v (T_h - T_c)} - \cancel{m C_v (T_h - T_c)}$$

$$Q_{\text{in}} = \underbrace{m R_s T_h \ln\left(\frac{V_2}{V_1}\right)}_{Q_{\text{expand}}}$$

$$Q_{\text{out}} = \underbrace{m R_s T_c \ln\left(\frac{V_4}{V_3}\right)}_{Q_{\text{compress}}}$$

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{m R_s T_h \ln\left(\frac{V_2}{V_1}\right) + m R_s T_c \ln\left(\frac{V_4}{V_3}\right)}{m R_s T_h \ln\left(\frac{V_2}{V_1}\right)}$$

Efficiency

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{mR_s T_h \ln\left(\frac{V_2}{V_1}\right) + mR_s T_c \ln\left(\frac{V_4}{V_3}\right)}{mR_s T_h \ln\left(\frac{V_2}{V_1}\right)}$$

similar ratios:

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

substitution:

$$\eta = \frac{\cancel{mR_s T_h} \ln\left(\frac{V_2}{V_1}\right) + \cancel{mR_s T_c} \ln\left(\frac{V_4}{V_3}\right)}{\cancel{mR_s T_h} \ln\left(\frac{V_2}{V_1}\right)} = \frac{T_h - T_c}{T_h}$$

$$\boxed{\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}}$$

Carnot Cycle

Constants

T_abs	273 K
R_gas	287 J/(kg K)
gamma	1.4

Given

T_hot	250 oC
T_cold	0 oC
V_3/V_1	20 (compression ratio)

Additional Parameters

P_1	50 kPa
V_1	2 m3
m =	0.666 kg

Volume Ratio

V2/V1 =	3.94	(> 1)
V3/V4 =	3.94	

(Volume Ratio)^gamma

(V2/V3)^gamma =	0.10
(V1/V4)^gamma =	0.10

Pressure Ratio

P3/P2 =	0.10
P4/P1 =	0.10

States Table

	state 1	state 2	state 3	state 4
V [m3]	2	8	40	10
P [kPa]	50	13	1	5
T [oC]	250	250	0	0
m [kg]	1	1	1	1

Graph Tables

1->2 (isothermal)		2->3 (adiabatic)		3->4 (isothermal)		4->1 (adiabatic)	
V	P	V	P	V	P	V	P
2	50	8	13	40	1	10	5
3	31	14	6	34	2	9	7
4	23	21	3	28	2	7	9
6	18	27	2	22	2	5	13
7	15	34	2	16	3	4	22
8	13	40	1	10	5	2	50

Energy Balance [kJ]

	W	Q	dE	
1->2	137	137	0	
2->3	119	0	119	Cv = 717.5 J/(kg K)
3->4	-72	-72	0	
4->1	-119	0	-119	Cv = 717.5 J/(kg K)
	66	66	0	
eta =	47.8	% (Wcycle/Qin)		
	47.8	% (1-Tc/Th)		

