

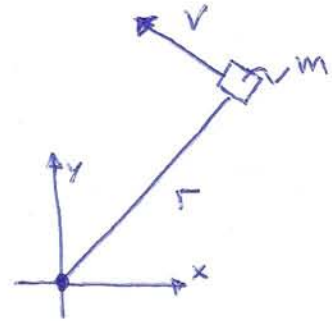
Determine the tension in the rope at the top and bottom of the swing, given:

$$m = 0.5 \text{ kg}$$

$$v = 1 \text{ m/s}$$

$$r = 2 \text{ m}$$

$g \downarrow$

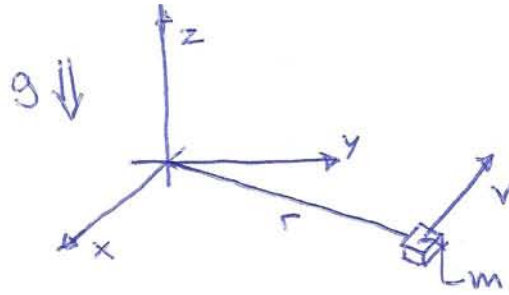


What is the deflection that the rope make with the horizontal plane, given:

$$m = 0.5 \text{ kg}$$

$$v = 1 \text{ m/s}$$

$$r = 2 \text{ m}$$

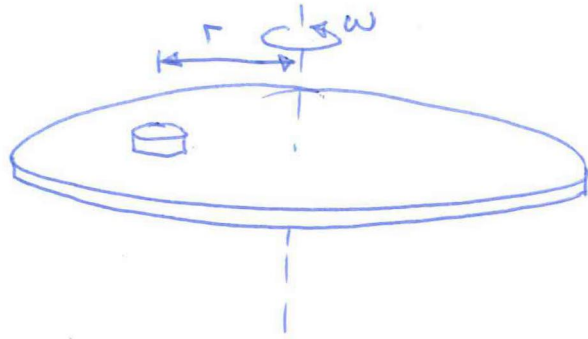


What is the maximum angular velocity that the disk can rotate at until the mass slips, given:

$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.1$$

$$r = 0.1 \text{ m}$$

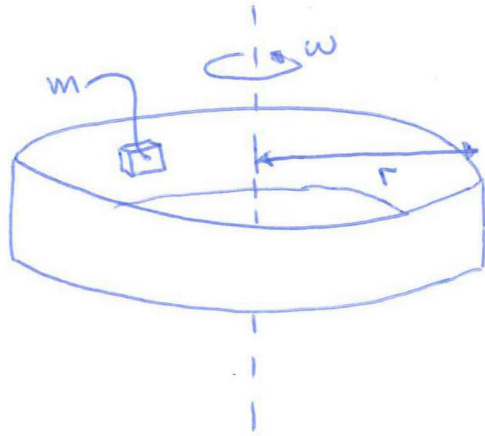


What is the minimum angular velocity that the hoop can rotate at until the mass slips, given:

$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.1$$

$$r = 0.1 \text{ m}$$



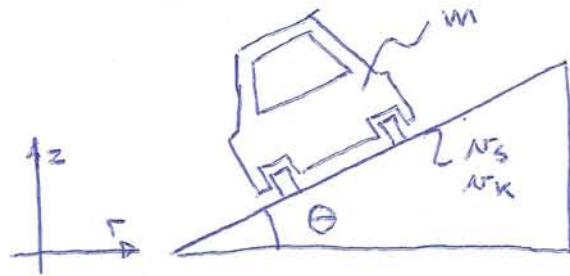
What is the maximum and minimum theta that car can drive at until it slips, given:

$$m = 1000 \text{ kg}$$

$$\mu_s = 0.5$$

$$v = 30 \text{ m/s}$$

$$r = 100 \text{ m}$$

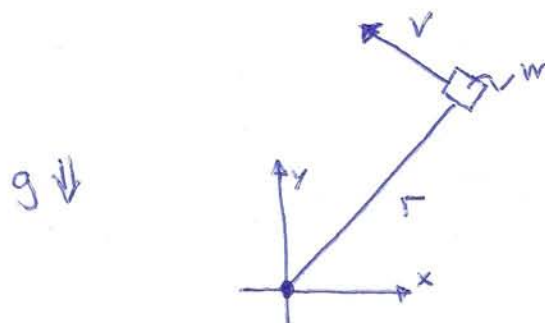


Determine the tension in the rope at the top and bottom of the swing, given:

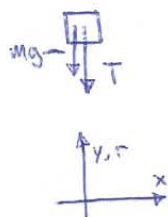
$$m = 0.5 \text{ kg}$$

$$v = 1 \text{ m/s}$$

$$r = 2 \text{ m}$$



Top of circle:



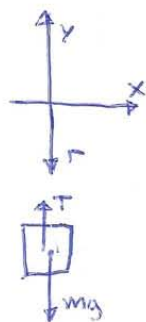
$$\sum F_r = ma_r = -T - mg$$

$$m\left(-\frac{v^2}{r}\right) = -T - mg$$

$$T = m\left(\frac{v^2}{r} - g\right) = (0.5)\left(\frac{(1)^2}{2} - 9.81\right) = -4.7 \text{ N}$$

This means that the rope is not tight, i.e. it doesn't make it for the top of the circle

Bottom of circle:



$$\sum F_r = ma_r = -T + mg$$

$$m\left(-\frac{v^2}{r}\right) = -T + mg$$

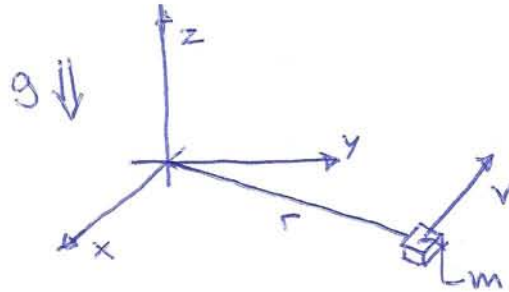
$$T = m\left(\frac{v^2}{r} + g\right) = (0.5)\left(\frac{(1)^2}{2} + 9.81\right) = 5.2 \text{ N}$$

What is the deflection that the rope make with the horizontal plane, given:

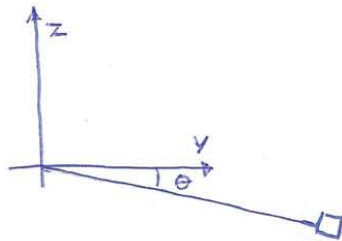
$$m = 0.5 \text{ kg}$$

$$v = 1 \text{ m/s}$$

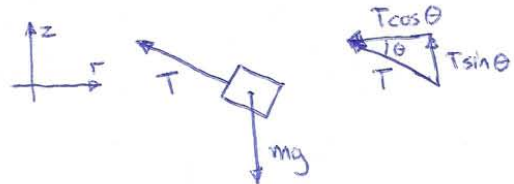
$$r = 2 \text{ m}$$



View in horizontal plane:



FBD:



$$\sum F_r = m a_r = -T \cos \theta \quad (1)$$

$$\sum F_z = m a_z = -mg + T \sin \theta \quad (2)$$

Solve for T in eqn (1) and sub $a_r = -\frac{v^2}{r}$

$$T = -\frac{m a_r}{\cos \theta} = -\frac{m}{\cos \theta} \left(-\frac{v^2}{r} \right) = \frac{m v^2}{r \cos \theta} \quad (1a)$$

sub. (1a) into (2)

$$0 = -mg + \left(\frac{m v^2}{r \cos \theta} \right) \sin \theta \Rightarrow mg = \frac{m v^2}{r} \left(\frac{\sin \theta}{\cos \theta} \right) = \frac{m v^2}{r} \tan \theta$$

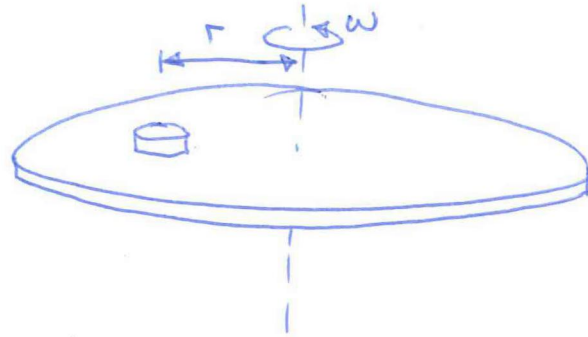
$$\text{so } \theta = \tan^{-1} \left(\frac{g r}{v^2} \right) = \tan^{-1} \left(\frac{(9.81)(2)}{(1)^2} \right) = 87^\circ$$

What is the maximum angular velocity that the disk can rotate at until the mass slips, given:

$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.1$$

$$r = 0.1 \text{ m}$$



Newton's 2nd Law

$$\sum F_r = m(-\omega^2 r) = -\mu N \quad (1)$$

$$\sum F_z = \cancel{m a_z^0} = N - mg \quad (2)$$

solve (2) for N :

$$N = mg \quad (2a)$$

sub (2a) into (1) and solve for ω :

$$-\mu \omega^2 r = -\mu [mg]$$

$$\omega^2 r = \mu g$$

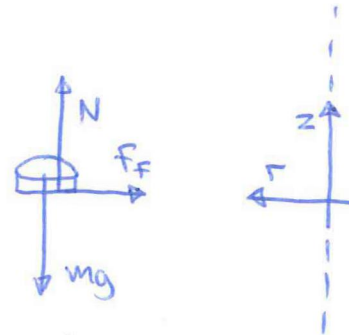
$$\boxed{\omega_{\max} = \sqrt{\frac{\mu g}{r}}}$$

maximum velocity:

$$v = \omega r = \sqrt{\frac{\mu g}{r}} r$$

$$\boxed{v_{\max} = \sqrt{\mu g r}}$$

FBD:



$$\omega_{\max} = \sqrt{\frac{(0.1)(9.81)}{(0.1)}}$$

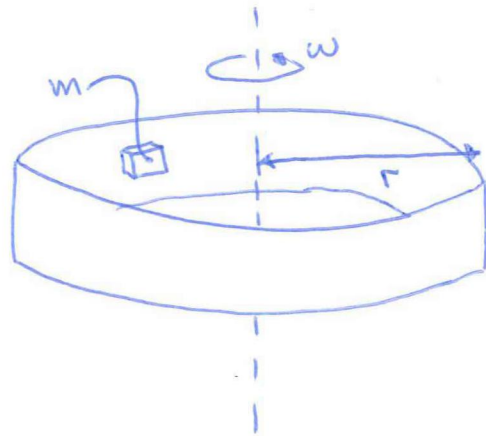
$$= 3.13 \text{ rad/s}$$

What is the minimum angular velocity that the hoop can rotate at until the mass slips, given:

$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.1$$

$$r = 0.1 \text{ m}$$



Newton's 2nd law:

$$\sum F_r = m(-\omega^2 r) = -N \quad (1)$$

$$\sum F_z = m a_z = \mu N - mg \quad (2)$$

solve eq. (2) for N:

$$N = \frac{mg}{\mu} \quad (2a)$$

sub (2a) into (1) and solve for ω:

$$-m\omega^2 r = -\left[\frac{\mu mg}{\mu}\right]$$

$$\omega^2 r = \frac{g}{\mu}$$

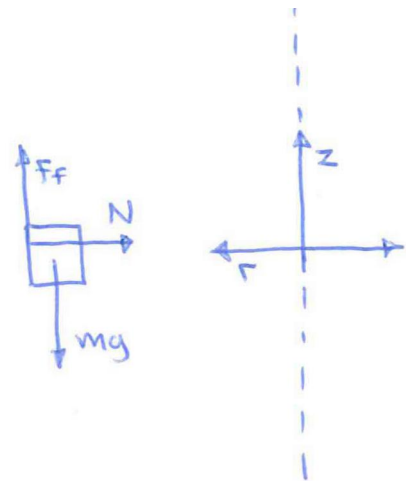
$$\boxed{\omega_{\min} = \sqrt{\frac{g}{r\mu}}}$$

minimum velocity

$$V = \omega r = \sqrt{\frac{g}{r\mu}} r$$

$$\boxed{V_{\min} = \sqrt{\frac{gr}{\mu}}}$$

FBD:



$$\omega_{\min} = \sqrt{\frac{9.81}{(0.1)(0.1)}}$$

$$= 31.3 \text{ rads/sec}$$

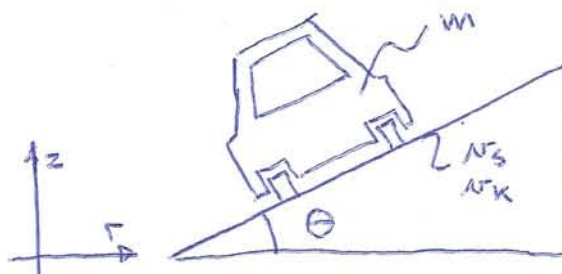
What is the maximum and minimum theta that car can drive at until it slips, given:

$$m = 1000 \text{ kg}$$

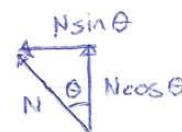
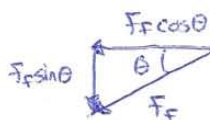
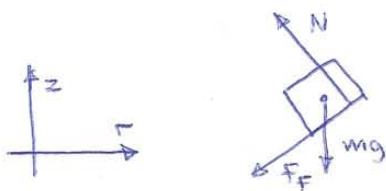
$$\mu_s = 0.5$$

$$v = 30 \text{ m/s}$$

$$r = 100 \text{ m}$$



Slip outwards



$$\sum F_r = m a_r = -F_f \cos \theta - N \sin \theta \quad (1)$$

$$\sum F_z = m a_z = N \cos \theta - F_f \sin \theta - mg \quad (2)$$

Sub $a_r = -\frac{v^2}{r}$ into eqn (1) and solve for N:

$$m \left(-\frac{v^2}{r} \right) = -\mu N \cos \theta - N \sin \theta \Rightarrow N = \frac{m v^2}{r \sin \theta + \mu r \cos \theta} \quad (1a)$$

sub (1a) into eqn (2):

$$0 = \left(\frac{m v^2}{r \sin \theta + \mu r \cos \theta} \right) (-\mu \sin \theta + \cos \theta) - mg$$

solve for θ :

$$\theta = \tan^{-1} \left[\frac{\left(1 - \frac{\mu g r}{v^2} \right)}{\left(\frac{g r}{v^2} + \mu \right)} \right] = \tan^{-1} \left[\frac{\left(1 - \frac{(0.5)(9.81)(100)}{(30)^2} \right)}{\left(\frac{(9.81)(100)}{(30)^2} + 0.5 \right)} \right] = 16^\circ$$

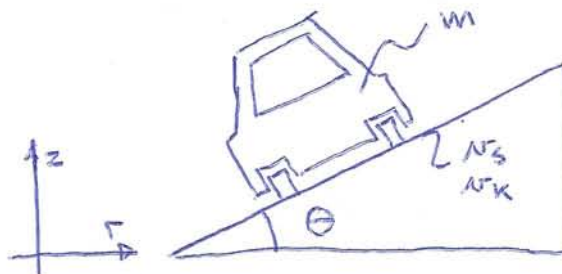
What is the maximum and minimum theta that car can drive at until it slips, given:

$$m = 1000 \text{ kg}$$

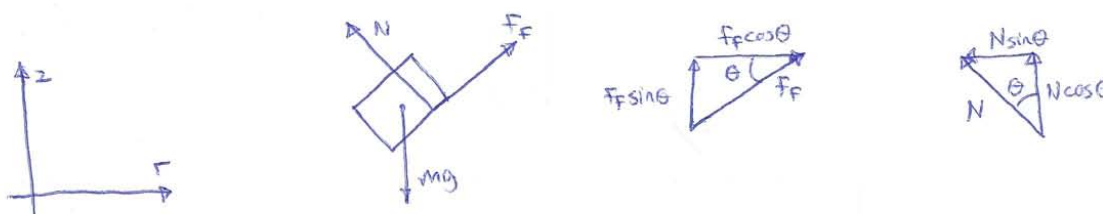
$$\mu_s = 0.5$$

$$v = 30 \text{ m/s}$$

$$r = 100 \text{ m}$$



Slip downward:



$$\sum F_r = m a_r = -N \sin \theta + f \cos \theta \quad (1)$$

$$\sum F_z = m a_z = N \cos \theta + f \sin \theta - mg \quad (2)$$

sub $a_r = -\frac{v^2}{r}$ into equ. (1) and solve for N:

$$m \left(-\frac{v^2}{r} \right) = -N \sin \theta + \mu N \cos \theta \Rightarrow N = \frac{m v^2}{r \sin \theta - \mu r \cos \theta} \quad (1a)$$

sub (1a) into equ. (2)

$$0 = \left(\frac{m v^2}{r \sin \theta - \mu r \cos \theta} \right) (\mu \sin \theta + \cos \theta) - mg$$

Solve for θ

$$\theta = \tan^{-1} \left[\frac{\left(1 + \frac{\mu g r}{v^2} \right)}{\left(\frac{g r}{v^2} - \mu \right)} \right] = \tan^{-1} \left[\frac{\left(1 + \frac{(0.5)(9.81)(100)}{(30)^2} \right)}{\left(\frac{(9.81)(100)}{(30)^2} - 0.5 \right)} \right] = 69^\circ$$