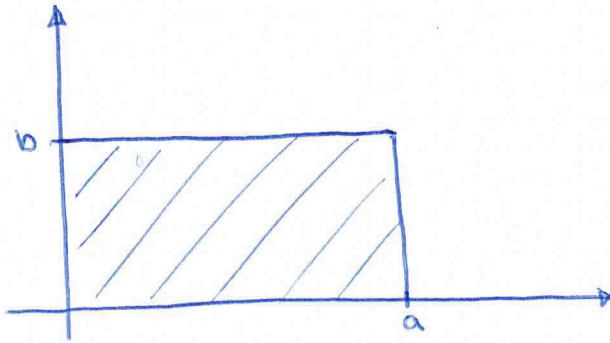


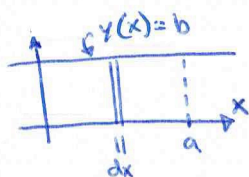
Find the centroid of



$$\vec{x}_c = \frac{\int \vec{x} dA}{A}$$

$$A = ab$$

x-axis:



$$\text{so } dA = (b)(dx)$$

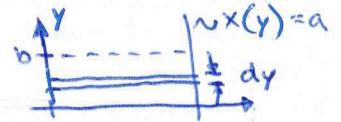
$$x_c = \frac{\int x dA}{A} = \frac{\int x [(b)(dx)]}{[ab]}$$

$$= \frac{\int x \cancel{b} dx}{a\cancel{b}} = \frac{\int x dx}{a} = \frac{[\frac{1}{2}x^2]_0^a}{a}$$

$$= \frac{[\frac{1}{2}a^2 - 0]}{a} = \frac{1}{2}a$$

y-axis:

$$y_c = \frac{\int y dA}{A}$$



$$\text{so } dA = (a)(dy)$$

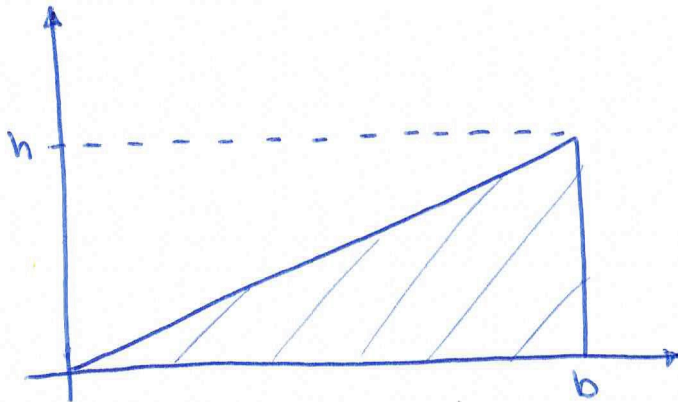
$$= \frac{\int y [a dy]}{[ab]}$$

$$= \frac{\int y \cancel{a} dy}{a\cancel{b}} = \frac{\int y dy}{b} = \frac{[\frac{1}{2}y^2]_0^b}{b}$$

$$= \frac{\frac{1}{2}b^2 - 0}{b} = \frac{1}{2}b$$

$$\vec{x}_c = \left( \frac{a}{2}, \frac{b}{2} \right)$$

Find the centroid of

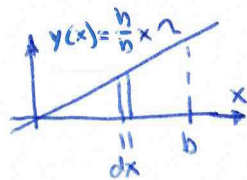


$$\vec{X}_c = \frac{\int \vec{x} dA}{A}$$

$$A = \frac{1}{2} h b$$

x-axis:

$$x_c = \frac{\int x dA}{A}$$



$$dA = \left(\frac{h}{b}x\right)(dx)$$

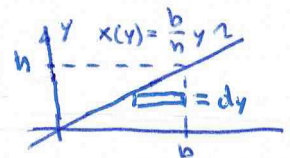
$$= \frac{\int x \left[\frac{h}{b}x dx\right]}{\left[\frac{1}{2}hb\right]}$$

$$= \frac{\int \frac{x}{b} x^2 dx}{\frac{1}{2}hb} = \frac{\int x^2 dx}{\frac{1}{2}b^2} = \frac{\left[\frac{1}{3}x^3\right]_0^b}{\frac{1}{2}b^2}$$

$$= \frac{\left[\frac{1}{3}b^3 - 0\right]}{\frac{1}{2}b^2} = \frac{2}{3}b$$

y-axis:

$$y_c = \frac{\int y dA}{A}$$



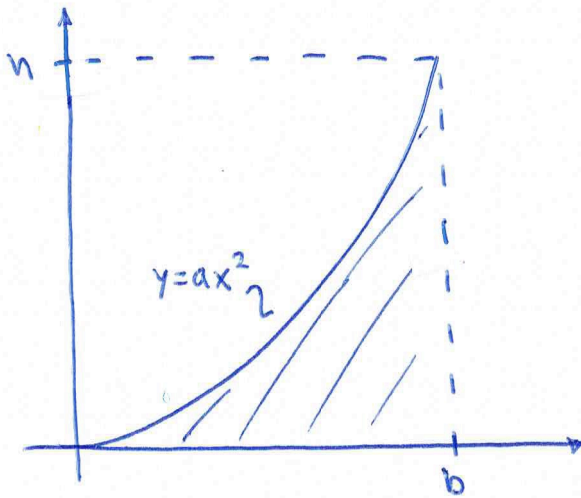
$$= \frac{\int y \left[b\left(1 - \frac{1}{h}y\right) dy\right]}{\frac{1}{2}hb} \quad \text{so } dA = \left(b - \frac{b}{h}y\right) dy = b\left(1 - \frac{1}{h}y\right) dy$$

$$= \frac{\int y \left(y - \frac{1}{h}y^2\right) dy}{\frac{1}{2}hb} = \frac{\left[\frac{1}{2}y^2 - \frac{1}{3h}y^3\right]_0^h}{\frac{1}{2}h}$$

$$= \frac{1}{3}h$$

$$\vec{X}_c = \left(\frac{2}{3}b, \frac{1}{3}h\right)$$

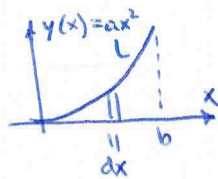
Find the centroid of



$$\vec{X}_c = \frac{\int \vec{x} dA}{\int dA}$$

$$x_c = \frac{\int x dA}{\int dA}$$

x-axis:



$$\text{so } dA = (ax^2)(dx)$$

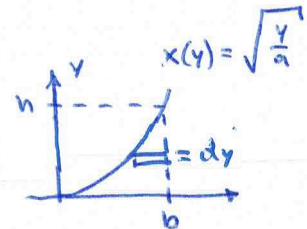
$$= \frac{\int x [ax^2 dx]}{\int [ax^2 dx]}$$

$$= \frac{\int ax^3 dx}{\int ax^2 dx} = \frac{\left[ \frac{1}{4} x^4 \right]_0^b}{\left[ \frac{1}{3} x^3 \right]_0^b} = \frac{3}{4} b$$

$$= \frac{3}{4} b$$

y-axis:

$$y_c = \frac{\int y dA}{\int dA}$$



$$\text{so } dA = \left( b - \sqrt{\frac{y}{a}} \right) dy$$

and  $h = ab^2$

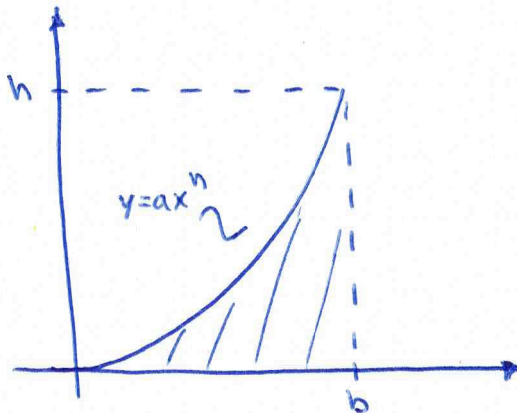
$$= \frac{\int y \left[ b - \sqrt{\frac{y}{a}} \right] dy}{\int \left[ b - \sqrt{\frac{y}{a}} \right] dy}$$

$$= \frac{\int by - \frac{y^{3/2}}{\sqrt{a}} dy}{\int b - \frac{y^{1/2}}{\sqrt{a}} dy} = \frac{\left[ \frac{1}{2} by^2 - \frac{2}{5} \frac{y^{5/2}}{\sqrt{a}} \right]_0^{ab^2}}{\left[ by - \frac{2}{3} \frac{y^{3/2}}{\sqrt{a}} \right]_0^{ab^2}}$$

$$= \frac{3}{10} ab^2$$

$$\vec{X}_c = \left( \frac{3}{4} b, \frac{3}{10} h \right)$$

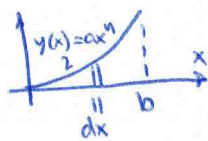
Find the centroid of



$$\vec{X}_c = \frac{\int \vec{x} dA}{\int dA}$$

x-axis:

$$X_c = \frac{\int x dA}{\int dA}$$



$$dA = (ax^n)(dx)$$

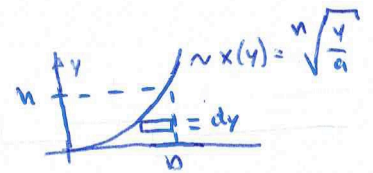
$$= \frac{\int x [ax^n dx]}{\int ax^n dx}$$

$$= \frac{\int x^{n+1} dx}{\int x^n dx} = \frac{\left[ \frac{1}{n+2} x^{n+2} \right]_0^b}{\left[ \frac{1}{n+1} x^{n+1} \right]_0^b}$$

$$= \frac{n+1}{n+2} b$$

y-axis

$$Y_c = \frac{\int y dA}{\int dA}$$



$$dA = (b - \sqrt[n]{\frac{y}{a}}) dy$$

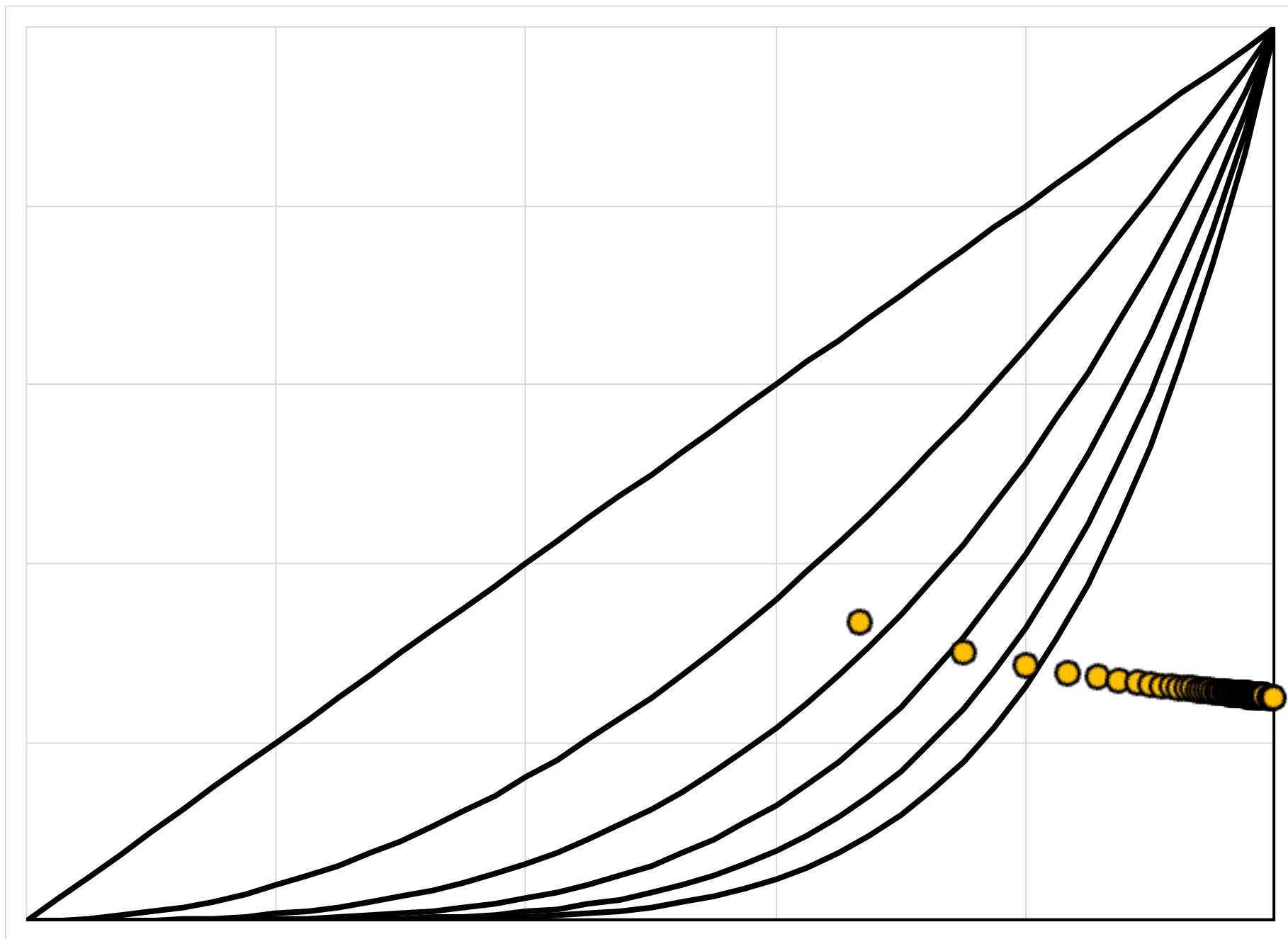
and  $n = ab^n$

$$= \frac{\int y [b - \sqrt[n]{\frac{y}{a}}] dy}{\int [b - \sqrt[n]{\frac{y}{a}}] dy}$$

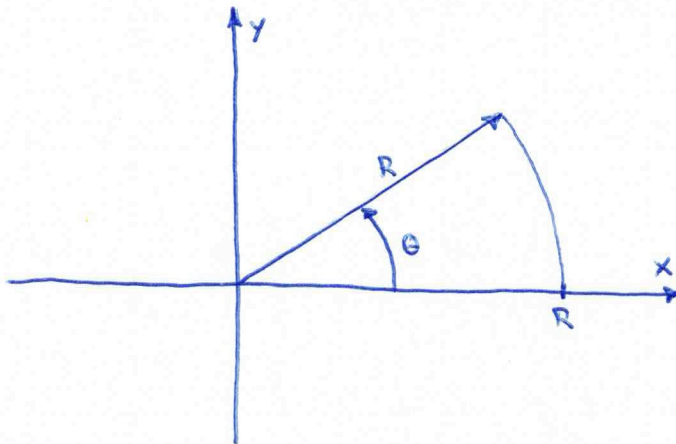
$$= \frac{\int y b - \frac{y^{\frac{n+1}{n}}}{\sqrt[n]{a}} dy}{\int b - \frac{y^{\frac{1}{n}}}{\sqrt[n]{a}} dy} = \frac{\left[ \frac{b}{2} y^2 - \frac{n}{2n+1} y^{\frac{2n+1}{n}} \frac{1}{\sqrt[n]{a}} \right]_0^{ab^n}}{\left[ by - \frac{n}{n+1} y^{\frac{n+1}{n}} \frac{1}{\sqrt[n]{a}} \right]_0^{ab^n}}$$

$$= \frac{n+1}{4n+2} h$$

$$\vec{X}_c = \left( \frac{n+1}{n+2} b, \frac{n+1}{4n+2} h \right)$$



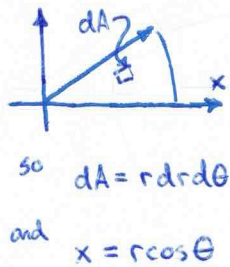
Find the centroid of



$$\vec{X}_c = \frac{\int \vec{x} dA}{\int dA}$$

x-axis:

$$x_c = \frac{\int x dA}{\int dA}$$

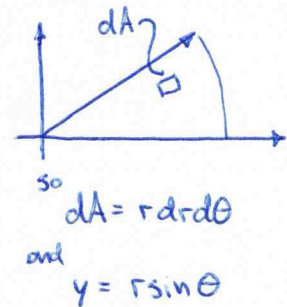


$$x_c = \frac{\iint (r \cos \theta) (r dr d\theta)}{\iint r dr d\theta} = \frac{\int \frac{1}{3} R^3 \cos \theta d\theta}{\int \frac{1}{2} R^2 d\theta}$$

$$= \frac{\frac{1}{3} R^3 \sin \theta}{\frac{1}{2} R^2 \theta} = \frac{2R \sin \theta}{3\theta}$$

y-axis:

$$y_c = \frac{\int y dA}{\int dA}$$



$$= \frac{\iint (r \sin \theta) (r dr d\theta)}{\iint r dr d\theta} = \frac{\int \frac{1}{3} R^3 \sin \theta d\theta}{\int \frac{1}{2} R^2 d\theta}$$

$$= \frac{-\frac{1}{3} R^3 \cos \theta \Big|_0^\theta}{\frac{1}{2} R^2 \theta} = \frac{2R(1 - \cos \theta)}{3\theta}$$

$$\vec{X}_c = \left( \frac{2R \sin \theta}{3\theta}, \frac{2R(1 - \cos \theta)}{3\theta} \right)$$

