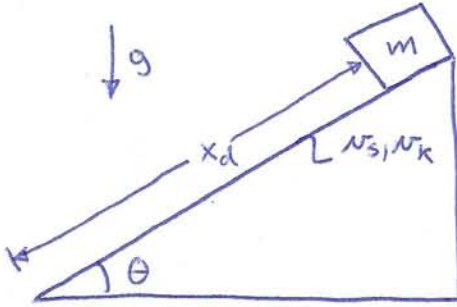


Given the following parameters determine:

- does the block slide
- what is the acceleration
- what is the final velocity at the bottom of the ramp



$$m = 1\text{ kg}$$

$$x_d = 10\text{ m}$$

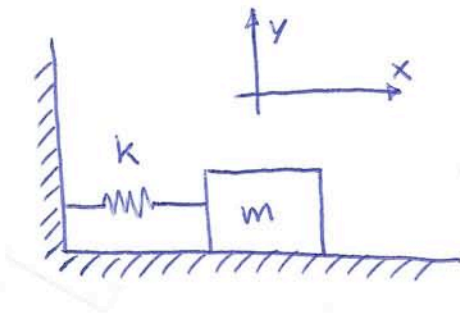
$$\mu_s = 0.2$$

$$\mu_k = 0.1$$

$$\theta = 30^\circ$$

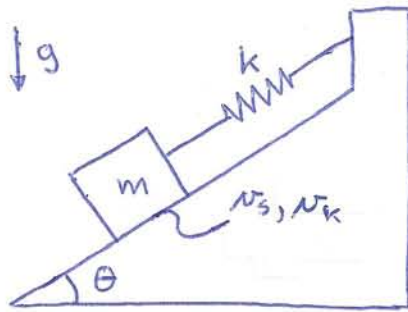
$$g = 9.81\text{ m/s}^2 \text{ (downward)}$$

Determine the maximum and minimum distances that the block could rest without slipping.



given
 m , k , μ_s , μ_k and g

Determine the maximum and minimum distances that the block could rest without slipping.



$$m = 1\text{ kg}$$

$$k = 10\text{ N/m}$$

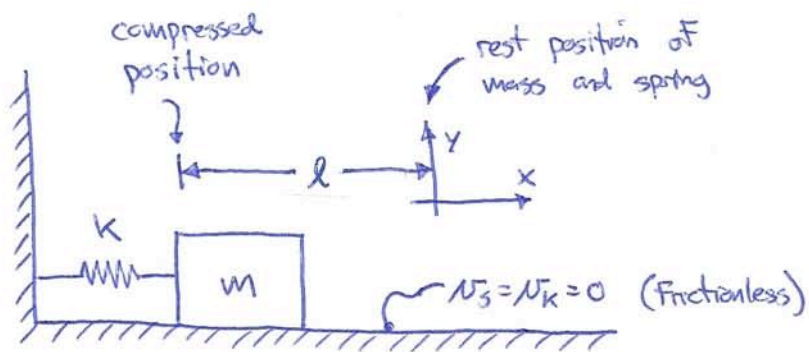
$$\mu_s = 0.2$$

$$\mu_k = 0.1$$

$$\theta = 30^\circ$$

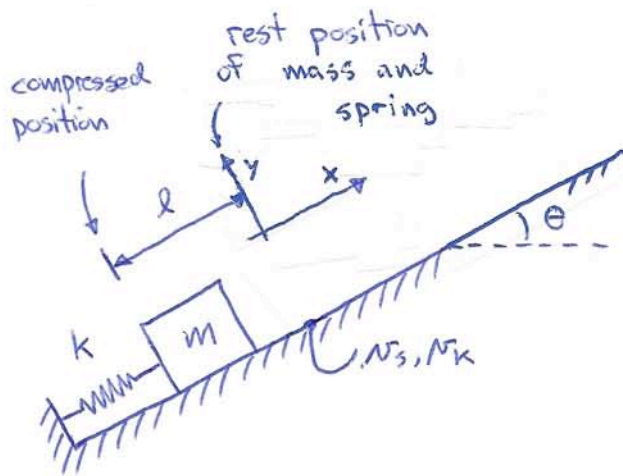
$$g = 9.81\text{ m/s}^2\text{ (downward)}$$

Determine the final velocity that the block would have after leaving the spring.



given
 m , k , L and g

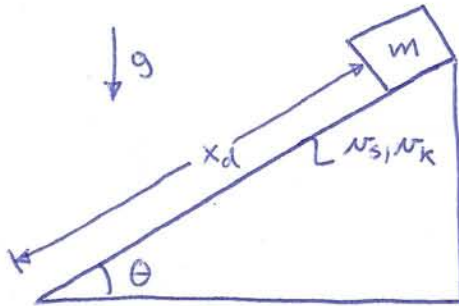
Determine the final velocity that the block would have after leaving the spring



given
 $m, k, \mu_s, \mu_k, \theta, L$ and g

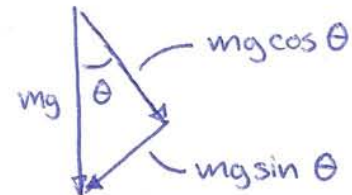
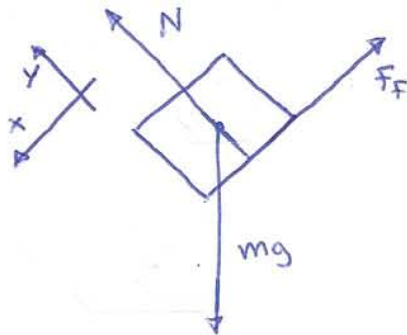
Given the following parameters determine:

- does the block slide
- what is the acceleration
- what is the final velocity at the bottom of the ramp



$$\begin{aligned} m &= 1 \text{ kg} \\ x_d &= 10 \text{ m} \\ \mu_s &= 0.2 \\ \mu_k &= 0.1 \\ \theta &= 30^\circ \\ g &= 9.81 \text{ m/s}^2 \text{ (downward)} \end{aligned}$$

Free Body Diagram (FBD) on block:



Summation of forces in x & y direction (Newton's Laws)

$$\sum F_x = m a_x = mg \sin \theta - F_f \quad (1)$$

$$\sum F_y = m a_y = -mg \cos \theta + N = 0 \quad (2)$$

Solve for N in equation (2)

$$-mg \cos \theta + N = 0 \quad \Rightarrow \quad \boxed{N = mg \cos \theta} \quad (2a)$$

Substitute $F_f = \mu_s N$ into equation (1)

$$mg \sin \theta - F_f = ma_x = mg \sin \theta - \mu_s N = ma_x$$

Substitute equation (2a) into previous and solve for a_x :

$$\cancel{mg} \sin \theta - \mu_s (\cancel{mg} \cos \theta) = \cancel{ma_x}$$

$$\begin{aligned} a_x &= g (\sin \theta - \mu_s \cos \theta) \\ &= (9.81) (\sin 30 - (0.2)(\cos 30)) \end{aligned}$$

$$\boxed{a_x = 3.2 \text{ m/s}^2 \text{ (it accelerates!)}}$$

solve for the acceleration using μ_k :

$$\begin{aligned} a_x &= g (\sin \theta - \mu_k \cos \theta) \\ &= (9.81) (\sin 30 - (0.1) \cos \theta) \end{aligned}$$

$$\boxed{a_x = 4.06 \text{ m/s}^2}$$

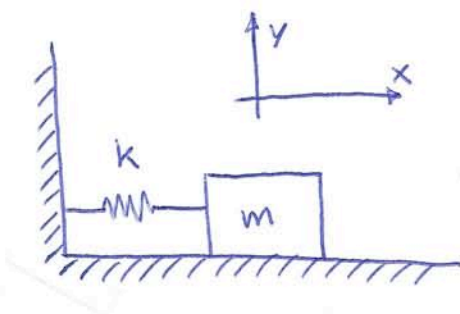
Use kinematics equation to solve for velocity:

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\Delta\vec{x}) \quad \Rightarrow \quad v_x^2 = v_{x0}^2 + 2a_x(x_f - x_0)$$

$$\begin{aligned} v_x &= \sqrt{v_{x0}^2 + 2a_x(x_f - x_0)} \\ &= \sqrt{0^2 + (2)(4.06)(10 - 0)} \end{aligned}$$

$$\boxed{v_x = 9.0 \text{ m/s}}$$

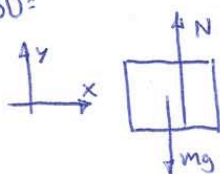
Determine the maximum and minimum distances that the block could rest without slipping.



given
 m, k, μ_s, μ_k and g

Rest position ($a_x = 0$)

FBD:



Newton's 2nd Law:

$$\sum F_x = m a_x = 0$$

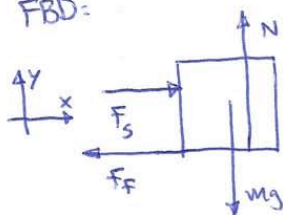
$$\sum F_y = m a_y = 0 = N - mg$$

where

$$F_s = -k(x - x_0) \text{ and } x = 0 \text{ so } \boxed{x_0 = 0}$$

Compression position

FBD:



Newton's 2nd Law

$$\sum F_x = m a_x = F_s - F_f \quad (1)$$

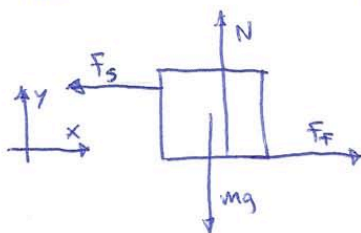
$$\sum F_y = m a_y = 0 = N - mg \quad (2)$$

use maximum friction so $F_f = \mu N = \mu mg$. From (2)
 solve eqn. (1)

$$m a_x = -k(x) - \mu mg \Rightarrow \boxed{x_{\min} = -\frac{\mu mg}{k}}$$

Tension position

FBD



Newton's 2nd Law:

$$\sum F_x = m a_x = F_f - F_s \quad (1)$$

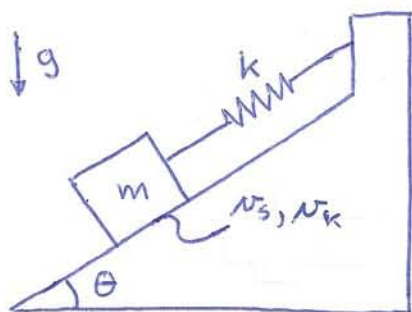
$$\sum F_y = m a_y = 0 = N - mg \quad (2)$$

use max. friction so $F_f = \mu N = \mu mg$ from eqn. (2)
 solve eqn. (1)

$$m a_x = \mu mg - k(x) \Rightarrow \boxed{x_{\max} = \frac{\mu mg}{k}}$$

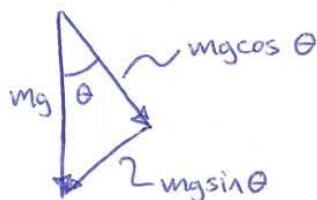
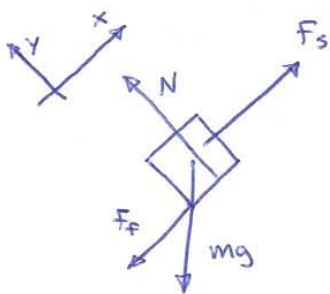
Note: don't include the negative sign as you already took the direction into account.

Determine the maximum and minimum distances that the block could rest without slipping.



$$\begin{aligned} m &= 1 \text{ kg} \\ k &= 10 \text{ N/m} \\ \mu_s &= 0.2 \\ \mu_k &= 0.1 \\ \theta &= 30^\circ \\ g &= 9.81 \text{ m/s}^2 \text{ (downward)} \end{aligned}$$

FBD For maximum tension in spring



Newton's laws

$$\sum F_x = ma_x = 0 = F_s - F_f - mg \sin \theta = -k\Delta x - \mu_s N - mg \sin \theta \quad (1)$$

$$\sum F_y = ma_y = 0 = N - mg \cos \theta \quad (2)$$

Solve for N in equation (2):

$$0 = N - mg \cos \theta \Rightarrow \boxed{N = mg \cos \theta} \quad (2a)$$

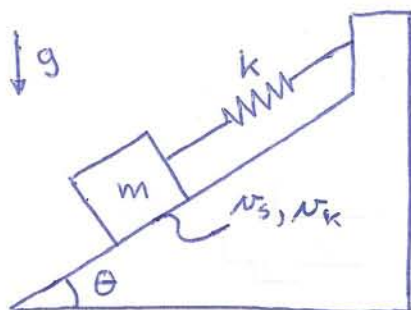
Substitute equation (2a) into equation (1) and solve for Δx :

$$0 = -k\Delta x - \mu_s (mg \cos \theta) - mg \sin \theta$$

$$\Delta x = \frac{-\mu_s mg \cos \theta - mg \sin \theta}{k} = \frac{-(0.2)(1)(9.81) \cos 30 - (1)(9.81) \sin 30}{10}$$

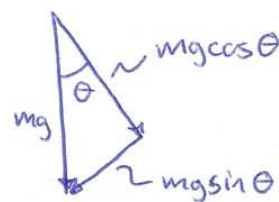
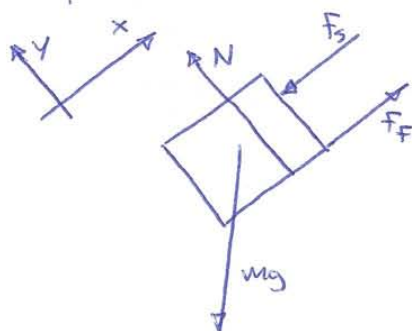
$$\boxed{\Delta x = -0.66 \text{ m}} \quad \text{max. tension.}$$

Determine the maximum and minimum distances that the block could rest without slipping.



$$\begin{aligned} m &= 1 \text{ kg} \\ k &= 10 \text{ N/m} \\ \mu_s &= 0.2 \\ \mu_k &= 0.1 \\ \theta &= 30^\circ \\ g &= 9.81 \text{ m/s}^2 \text{ (downward)} \end{aligned}$$

FBD For maximum compression in spring:



Newton's Laws

$$\sum F_x = m a_x = 0 = +F_f - F_s - mg \sin \theta = \mu_s N - (-k \Delta x) - mg \sin \theta \quad (1)$$

$$\sum F_y = m a_y = 0 = N - mg \cos \theta \quad (2)$$

Solve for N in equ. (2)

$$N = mg \cos \theta \quad (2a)$$

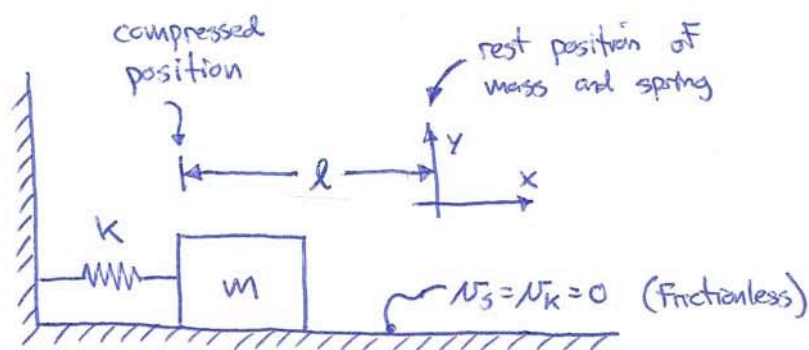
Substitute equ. (2a) into equ. (1) and solve for Δx :

$$\mu_s (mg \cos \theta) + k \Delta x - mg \sin \theta = 0$$

$$\Delta x = \frac{-\mu_s mg \cos \theta + mg \sin \theta}{k} = \frac{-(0.2)(1)(9.81) \cos 30 + (1)(9.81) \sin 30}{10}$$

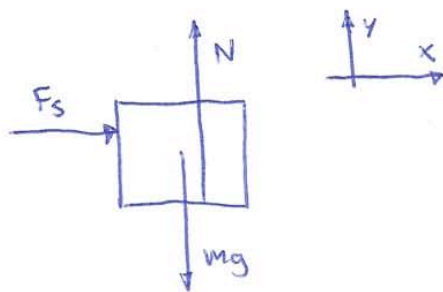
$$\Delta x = 0.32 \text{ m} \quad \text{max. compression,}$$

Determine the final velocity that the block would have after leaving the spring.



given
 m, k, L and g

FBD:



Newton's 2nd law:

$$\sum F_x = ma_x = F_s \quad (1)$$

$$\sum F_y = \cancel{ma_y} = N - mg \quad (2)$$

where

$$F_s = -k(x - x_0)$$

Rest position ($a_x = 0$)

Solve eq. (1) for x_0

$$ma_x = -k(x - x_0)$$

$$0 = kx_0$$

$$x_0 = 0$$

set $x = 0$ (rest position)

Compressed position ($a_x \neq 0$)

Solve eq. (1) for a_x :

$$ma_x = -k(x - x_0)$$

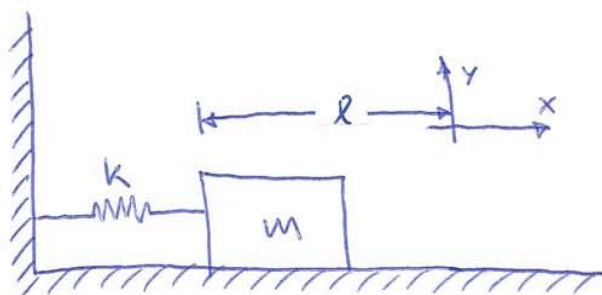
$$ma_x = -kx + k[0]$$

$$a_x = -\frac{kx}{m}$$

general

$$a_x = -\frac{k(-l)}{m} = \frac{kl}{m}$$

initial acceleration ($x = -l$)



a_x changes as " x " changes

$$a_x = -\frac{kx}{m}$$

Use the definition of kinematics

$$v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v} \quad \swarrow \quad a \left[\frac{dx}{v} \right] = dv \Rightarrow \boxed{a dx = v dv}$$

$$a = \frac{dv}{dt} \Rightarrow a dt = dv \quad \searrow$$

Sub in a_x and integrate from $x = -l$ to $x = 0$

$$\int_{-l}^0 -\frac{kx}{m} dx = \int_0^{v_f} v dv$$

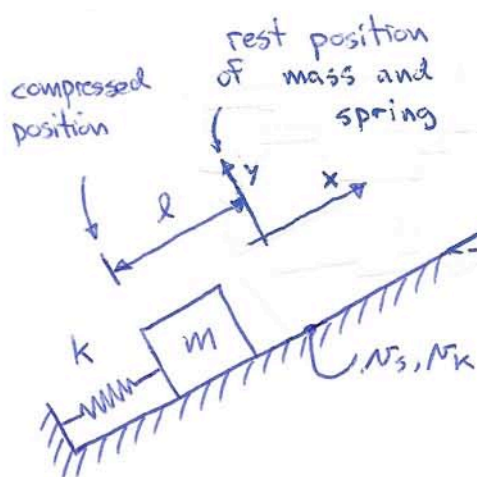
$$\left[-\frac{kx^2}{2m} \right]_{-l}^0 = \left[\frac{1}{2} v^2 \right]_0^{v_f}$$

$$[0] - \left[-\frac{k(-l)^2}{2m} \right] = \left[\frac{1}{2} v_f^2 \right] - [0]$$

$$\frac{kl^2}{2m} = \frac{v_f^2}{2}$$

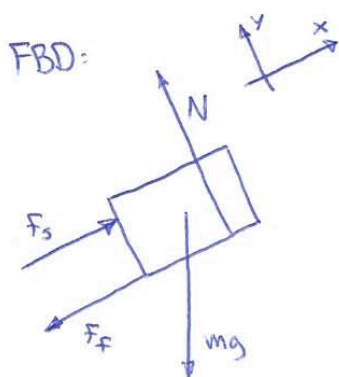
$$\boxed{v_f = \sqrt{\frac{kl^2}{m}}}$$

Determine the final velocity that the block would have after leaving the spring



given

$m, k, \mu_s, \mu_k, \theta, L$ and g



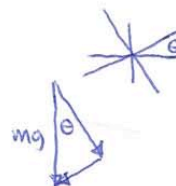
Newton's 2nd Law:

$$\sum F_x = ma_x = F_s - F_f - mg \sin \theta$$

$$\sum F_y = ma_y = N - mg \cos \theta$$

where

$$F_s = -k(x - x_0)$$



Rest Position ($a_x = 0$)

solve eq. (1) for x_0

$$0 = -k(x - x_0) - F_f - mg \sin \theta, \text{ where } x = 0 \text{ (rest position)}$$

$$0 = kx_0 - mg \sin \theta$$

$$x_0 = \frac{mg \sin \theta}{k}$$

Compressed position ($a_x \neq 0$)

solve eq. (1) & (2) for a_x :

$$ma_x = -k(x - x_0) - \mu N - mg \sin \theta$$

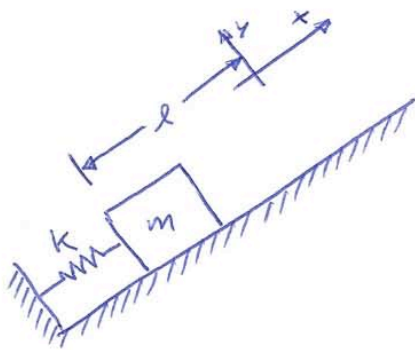
$$ma_x = -kx + k \left[\frac{mg \sin \theta}{k} \right] - \mu [mg \cos \theta] - mg \sin \theta$$

$$a_x = -\frac{kx}{m} - \mu g \cos \theta$$

general

$$a_x = \frac{kL}{m} - \mu g \cos \theta$$

initial acceleration
($x = -L$)



$$a_x = -\frac{kx}{m} - N g \cos \theta$$

↑ changes as the mass accelerates,
so the acceleration is a function
of x .

Use the definition of kinematics

$$v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v} \quad \searrow \quad a \left[\frac{dx}{v} \right] = dv \Rightarrow \boxed{a dx = v dv}$$

$$a = \frac{dv}{dt} \Rightarrow a dt = dv \quad \nearrow$$

sub in a_x and integrate from $x = -l$ to $x = 0$:

$$\int_{-l}^0 \left[-\frac{kx}{m} - N g \cos \theta \right] dx = \int_0^{v_F} v dv$$

$$\left[-\frac{1}{2} \frac{k}{m} x^2 - N g x \cos \theta \right]_{-l}^0 = \left[\frac{1}{2} v^2 \right]_0^{v_F}$$

$$[0] - \left[-\frac{1}{2} \frac{k}{m} (-l)^2 - N g (-l) \cos \theta \right] = \left[\frac{1}{2} v_F^2 \right] - [0]$$

$$\frac{1}{2} \frac{k}{m} l^2 - N g l \cos \theta = \frac{1}{2} v_F^2$$

$$\boxed{v_F = \sqrt{\frac{k l^2}{m} - 2 N g l \cos \theta}}$$