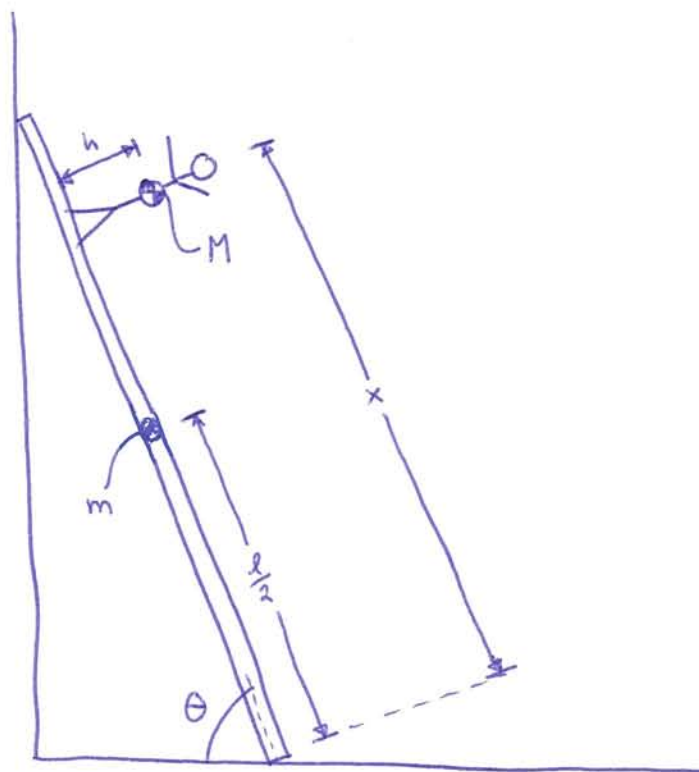
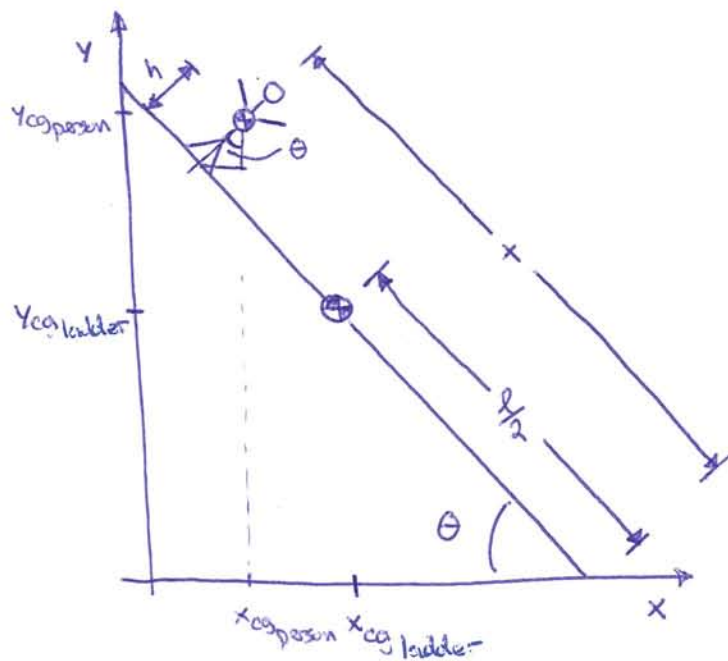


Person on a ladder (extended).



Determine centre of mass of person and ladder



$$x_{cg \text{ ladder}} = l \cos \theta - \frac{l}{2} \cos \theta = l \left[\cos \theta - \frac{1}{2} \cos \theta \right] = l \left[\frac{\cos \theta}{2} \right] = \frac{l}{2} \cos \theta$$

$$y_{cg \text{ ladder}} = l \sin \theta - \frac{l}{2} \sin \theta = l \left[\sin \theta - \frac{1}{2} \sin \theta \right] = l \left[\frac{\sin \theta}{2} \right] = \frac{l}{2} \sin \theta$$

$$x_{cg \text{ person}} = l \cos \theta - x \cos \theta + h \sin \theta = [l - x] \cos \theta + h \sin \theta$$

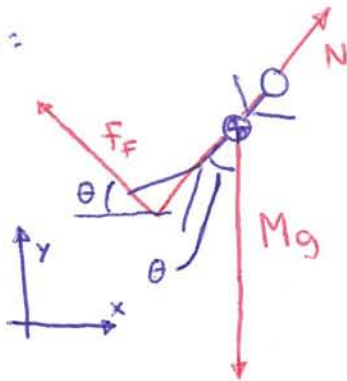
$$y_{cg \text{ person}} = x \sin \theta + h \cos \theta$$

$$x_{cg \text{ sys.}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(m) \left(\frac{l}{2} \cos \theta \right) + (M) ([l - x] \cos \theta + h \sin \theta)}{M + m}$$

$$y_{cg \text{ sys.}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(m) \left(\frac{l}{2} \sin \theta \right) + (M) (x \sin \theta + h \cos \theta)}{M + m}$$

Free body diagram on person and ladder

Person:



$$\sum F_x = M a_x = N \sin \theta - F_f \cos \theta \quad (1)$$

$$\sum F_y = M a_y = N \cos \theta + F_f \sin \theta - Mg \quad (2)$$

solve (1) for F_f

$$F_f = N \frac{\sin \theta}{\cos \theta} \quad (1a)$$

sub (1a) into (2)

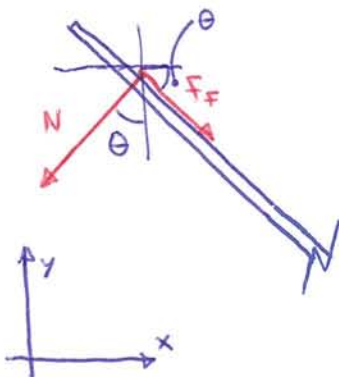
$$0 = N \cos \theta + \left(N \frac{\sin \theta}{\cos \theta} \right) \sin \theta - Mg$$

$$0 = N \cos^2 \theta + N \sin^2 \theta - Mg \cos \theta$$

$$0 = N (\cos^2 \theta + \sin^2 \theta) - Mg \cos \theta$$

$$\boxed{N = Mg \cos \theta} \Rightarrow \boxed{F_f = Mg \sin \theta}$$

ladder:



$$\sum F_x = F_f \cos \theta - N \sin \theta$$

$$= (Mg \sin \theta)(\cos \theta) - (Mg \cos \theta)(\sin \theta)$$

$$= 0$$

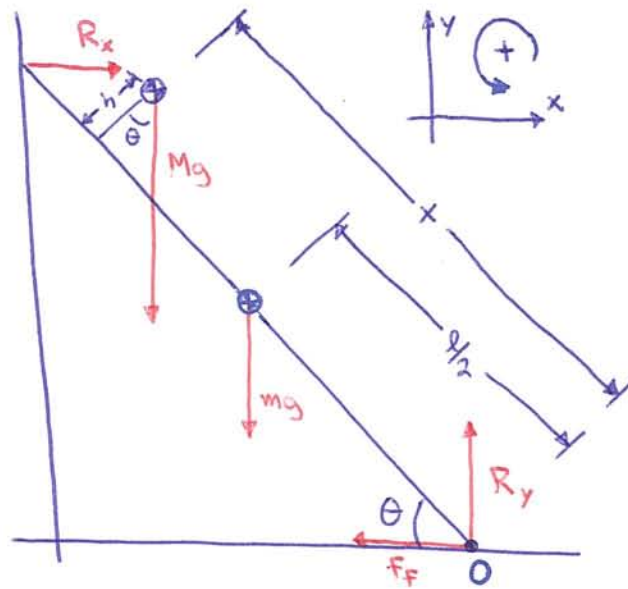
$$\sum F_y = -N \cos \theta - F_f \sin \theta$$

$$= -(Mg \cos \theta)(\cos \theta) - (Mg \sin \theta)(\sin \theta)$$

$$= -Mg (\cos^2 \theta + \sin^2 \theta)$$

$$= -Mg$$

Free body diagram



$$\sum F_x = (M+m)a_x = R_x - F_f \quad (1)$$

$$\sum F_y = (M+m)a_y = R_y - Mg - mg \quad (2)$$

$$\sum M_o = I_o \alpha_o = (mg)\left(\frac{l}{2} \cos \theta\right) + (Mg)(x \cos \theta - h \sin \theta) - (R_x)(l \sin \theta) \quad (3)$$

solve (3) for R_x

$$R_x = \frac{\frac{mg l \cos \theta}{2} + (Mg)(x \cos \theta - h \sin \theta)}{l \sin \theta} \quad (3a)$$

sub (3a) into (1)

$$F_f = R_x = \frac{\frac{mg l \cos \theta}{2} + (Mg)(x \cos \theta - h \sin \theta)}{l \sin \theta}$$

solve (2) for R_y

$$R_y = Mg + mg$$

Ladder Problem
Feb. 28, 2017

Constants

g 9.81 [m/s²] acceleration due to gravity

Variables

l 10 [m] length of ladder
 h 0.75 [m] height of cg of person
 x 9 [m] distance person is up the ladder
 M 70 [kg] mass of person
 m 10 [kg] mass of ladder
 θ 60 [degrees] angle of ladder from ground

Numbers for graph

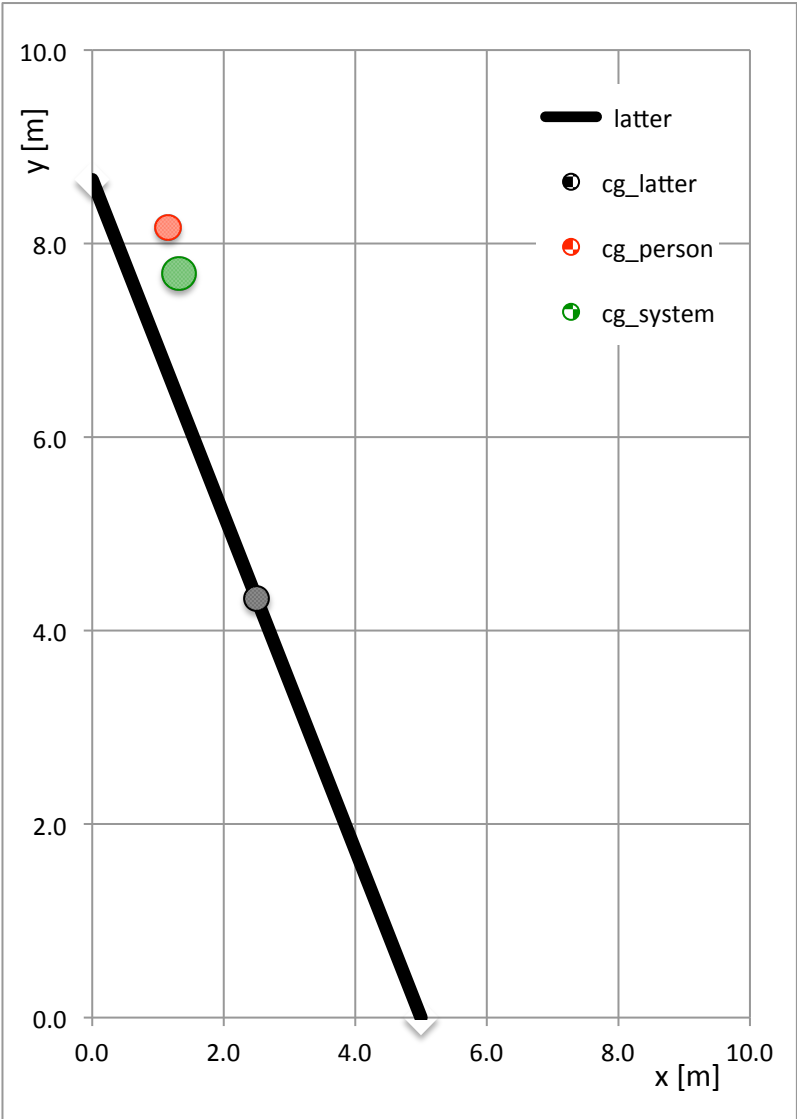
$x_{0ladder}$ = 0.0 [m] top of ladder x location
 $y_{0ladder}$ = 8.7 [m] top of ladder y location
 $x_{1ladder}$ = 5.0 [m] bottom of ladder x location
 $y_{1ladder}$ = 0.0 [m] bottom of ladder y location

Centre of Gravity

$x_{cg_lad.}$ = 2.5 [m] x cg of ladder
 $x_{cg_per.}$ = 1.1 [m] x cg of person
 $x_{cg_sys.}$ = 1.3 [m] x cg of system
 $y_{cg_lad.}$ = 4.3 [m] y cg of ladder
 $y_{cg_per.}$ = 8.2 [m] y cg of person
 $y_{cg_sys.}$ = 7.7 [m] y cg of system

Forces

R_x = 334
 R_y = 785
 ff = 334



Ladder Problem
Feb. 28, 2017

Constants

g	9.81 [m/s2] acceleration due to gravity
Variables	
l	10 [m] length of ladder
h	0.75 [m] height of cg of person
x	9 [m] distance person is up the ladder
M	70 [kg] mass of person
m	10 [kg] mass of ladder
mu	0.5 [] coefficient of static friction

Forces

theta [degrees]	theta ratio	Rx [N]	Ry [N]	ff [N]	ffs [N]
0.1	0.0	382157	784.8	382157	392.4
10	0.2	3732	784.8	3732	392.4
20	0.4	1781	784.8	1781	392.4
30	0.6	1104	784.8	1104	392.4
40	0.8	743	784.8	743	392.4
50	1.2	508	784.8	508	392.4
60	1.7	334	784.8	334	392.4
70	2.7	191	784.8	191	392.4
80	5.7	66	784.8	66	392.4
89.99	5729.6	-51	784.8	-51	392.4

