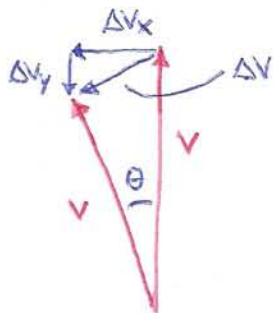
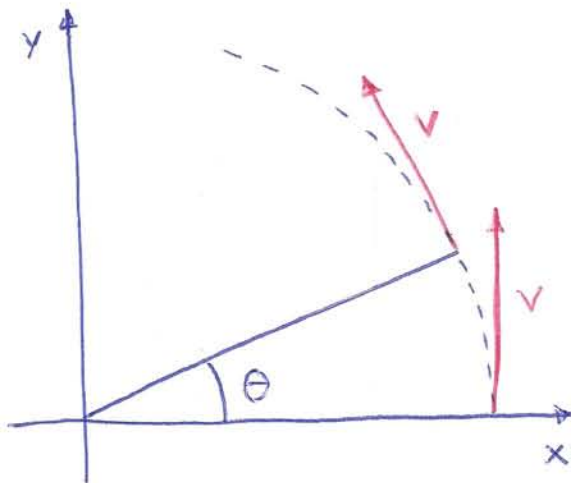


Uniform Circular Motion (cartesian coordinate system)



$$\Delta v_x = 0 - v \sin \theta \Rightarrow \frac{d}{dt}(\Delta v_x) = -(v \cos \theta)(\dot{\theta}) = a_x$$

$$\Delta v_y = v - v \cos \theta \Rightarrow \frac{d}{dt}(\Delta v_y) = -(v)(\sin \theta)(\dot{\theta}) = a_y$$

$$|a| = \sqrt{a_x^2 + a_y^2} = \sqrt{(-v\dot{\theta} \cos \theta)^2 + (-v\dot{\theta} \sin \theta)^2}$$

$$= \sqrt{v^2 \dot{\theta}^2 (\sin^2 \theta + \cos^2 \theta)} = v\dot{\theta} = \omega^2 r = \frac{v^2}{r}$$

$$\angle a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-v\dot{\theta} \sin \theta}{-v\dot{\theta} \cos \theta}\right) = \tan^{-1}(\tan(\theta)) = \theta$$

$$a_{\text{centripetal}} = \frac{v^2}{r} = \omega^2 r \quad \left(\begin{array}{l} \text{direction towards centre} \\ \text{of rotation} \end{array} \right)$$