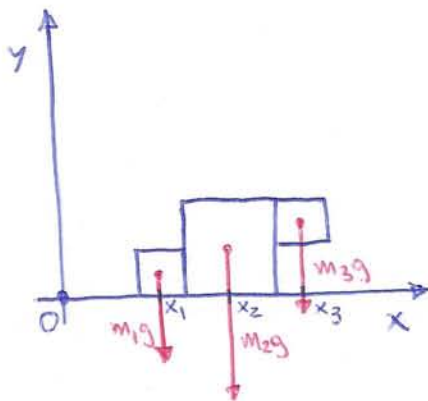
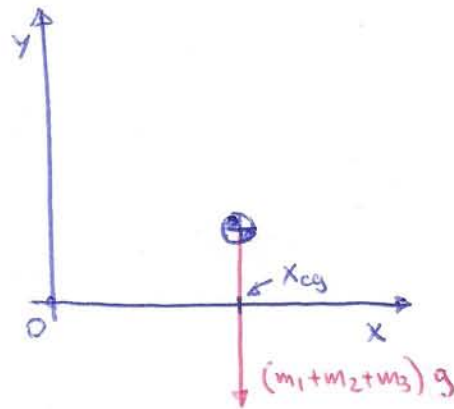


Centre of Gravity Proof:



$\equiv$



$$\sum M_o = \cancel{I_o \alpha_o} = m_1 g x_1 + m_2 g x_2 + m_3 g x_3$$

$$\sum M_o = \cancel{I_o \alpha_o} = (m_1 + m_2 + m_3) g x_{cg}$$

$$m_1 x_1 + m_2 x_2 + m_3 x_3 = (m_1 + m_2 + m_3) x_{cg}$$

so:

$$x_{cg} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

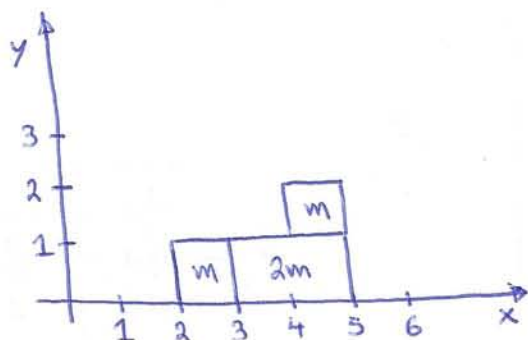
and similarly

$$y_{cg} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

so in general:

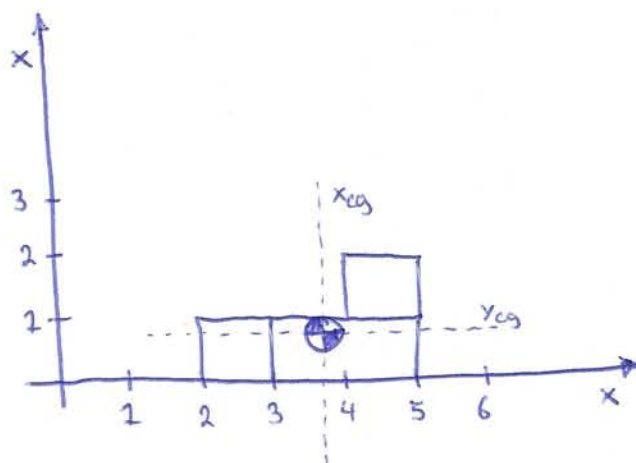
$$\boxed{\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}}$$

Centre of Gravity Example:

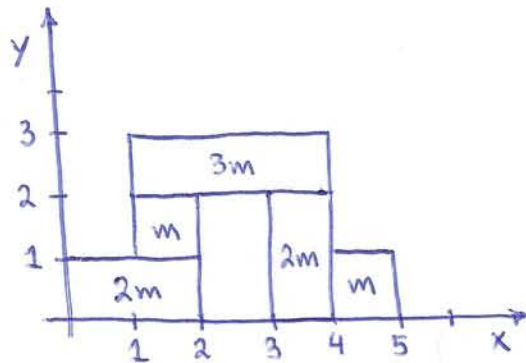


$$\begin{aligned}x_{cg} &= \frac{\sum m_i x_i}{\sum m_i} = \frac{(m)(2.5) + (2m)(4) + (m)(4.5)}{m + 2m + m} \\&= \frac{m(2.5 + (2)(4) + 4.5)}{m(1 + 2 + 1)} = 3.75\end{aligned}$$

$$\begin{aligned}y_{cg} &= \frac{\sum m_i y_i}{\sum m_i} = \frac{(m)(0.5) + (2m)(0.5) + (m)(1.5)}{m + 2m + m} \\&= \frac{m(0.5 + (2)(0.5) + 1.5)}{m(1 + 2 + 1)} = 0.75\end{aligned}$$



Centre of Gravity Example:



$$X_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(2m)(1) + (m)(1.5) + (3m)(2.5) + (2m)(3.5) + (m)(4.5)}{2m + m + 3m + 2m + m}$$

$$= \frac{m[(2)(1) + (1)(1.5) + (3)(2.5) + (2)(3.5) + (1)(4.5)]}{m[2 + 1 + 3 + 2 + 1]} = 2.5$$

$$Y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2m)(0.5) + (m)(1.5) + (3m)(2.5) + (2m)(1) + (m)(0.5)}{2m + m + 3m + 2m + m}$$

$$= \frac{m[(2)(0.5) + (1)(1.5) + (3)(2.5) + (2)(1) + (1)(0.5)]}{m[2 + 1 + 3 + 2 + 1]} = 1.3\bar{8}$$

