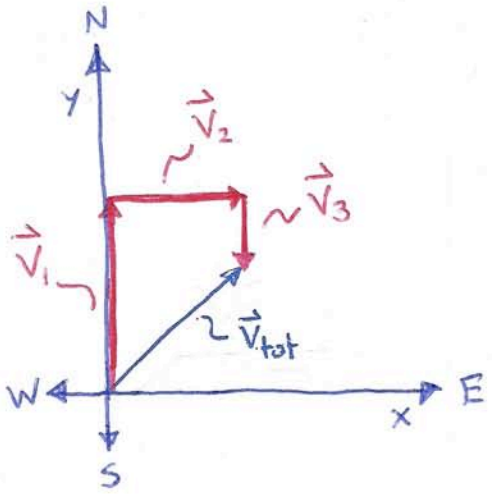
You walk 20m north, 10m east and 5m south. How far are you from where you started?

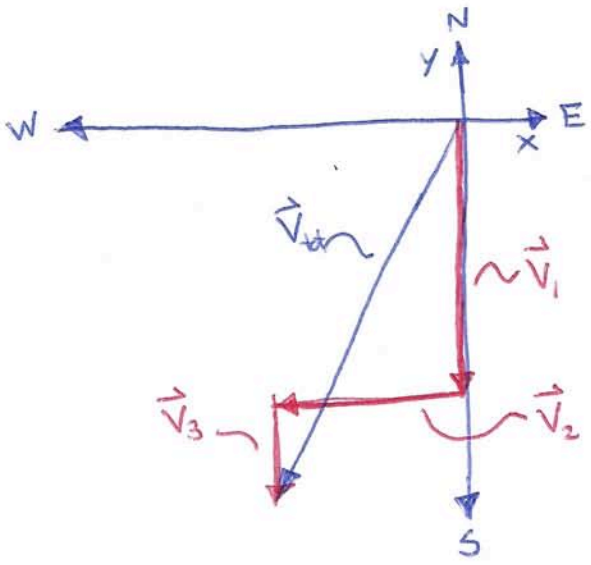


$$\vec{V}_1 = 0\hat{x} + 20\hat{y}$$
$$\vec{V}_2 = 10\hat{x} + 0\hat{y}$$
$$\vec{V}_3 = 0\hat{x} - 5\hat{y}$$
$$\vec{V}_{tot} = 10\hat{x} + 15\hat{y}$$

$$|\vec{V}_{tot}| = \sqrt{(10)^2 + (15)^2}$$
$$= 18.03$$

$$\boxed{|\vec{V}_{tot}| = 18\ m}$$

You walk 20m south, 10m west and 5m south. How far are you from where you started?



$$\vec{V}_1 = 0\hat{x} - 20\hat{y}$$

$$\vec{V}_2 = -10\hat{x} + 0\hat{y}$$

$$\vec{V}_3 = 0\hat{x} - 5\hat{y}$$

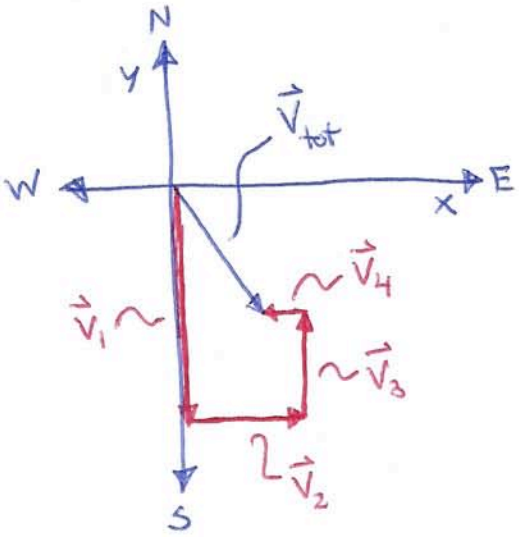
$$\vec{V}_{tot} = -10\hat{x} - 25\hat{y}$$

$$|\vec{V}_{tot}| = \sqrt{(-10)^2 + (-25)^2}$$

$$= 26.93$$

$$\boxed{|\vec{V}_{tot}| = 27\ m}$$

You run 100m south, 30m east, 40m north and 10m west. How far are you from where you started and at what angle?

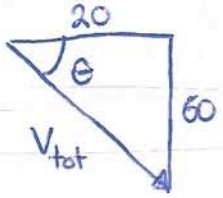


$$\vec{V}_1 = 0\hat{x} - 100\hat{y}$$
$$\vec{V}_2 = 30\hat{x} + 0\hat{y}$$
$$\vec{V}_3 = 0\hat{x} + 40\hat{y}$$
$$\vec{V}_4 = -10\hat{x} + 0\hat{y}$$

$$\vec{V}_{tot} = 20\hat{x} - 60\hat{y}$$

$$|\vec{V}_{tot}| = \sqrt{(20)^2 + (-60)^2}$$
$$= 63.25$$

$$\theta = \tan^{-1}\left(\frac{60}{20}\right)$$
$$= 71.57^\circ$$

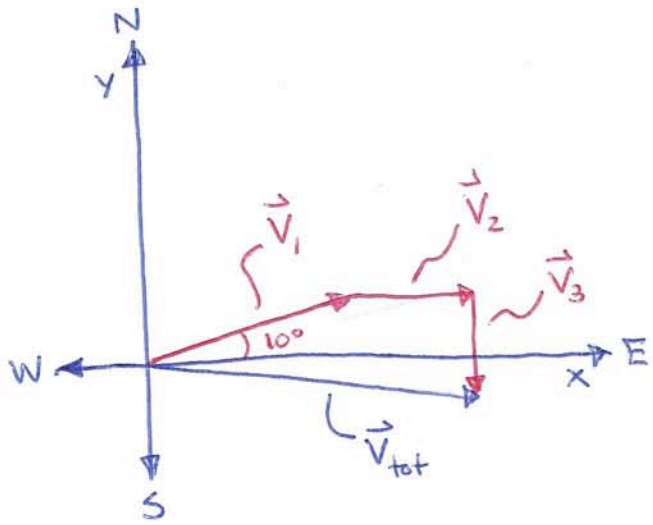


$|\vec{V}_{tot}| = 63$ m

$\angle \vec{V}_{tot} = 72^\circ$ S of E

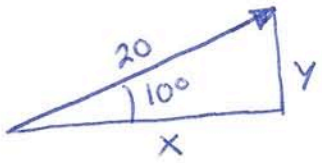
"South of East"

You run 20m at 10 degrees north of east, 10m east and 5m south. How far are you from where you started and at what angle?



$$\vec{V}_1 = 19.7\hat{x} + 3.5\hat{y}$$
$$\vec{V}_2 = 10\hat{x} + 0\hat{y}$$
$$\vec{V}_3 = 0\hat{x} - 5\hat{y}$$

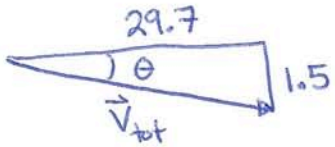
$$\vec{V}_{tot} = 29.7\hat{x} - 1.5\hat{y}$$



$x = 20\cos 10 = 19.70$
$y = 20\sin 10 = 3.47$

$$|\vec{V}_{tot}| = \sqrt{(29.7)^2 + (-1.5)^2}$$
$$= 29.7$$

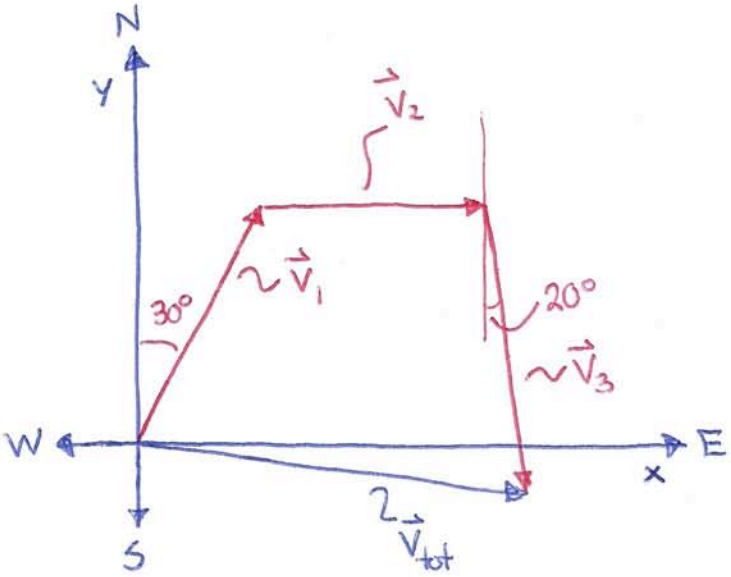
$$\theta = \tan^{-1}\left(\frac{1.5}{29.7}\right)$$
$$= 2.89°$$

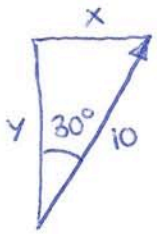


$|\vec{V}_{tot}| = 30$ m

$\angle\vec{V}_{tot} = 3°$ S of E

You sprint 10m at 30 degrees east of north, 10m east and 10m at 20 degrees east of south. How far are you from where you started and at what angle?





$x = 10 \sin 30 = 5$

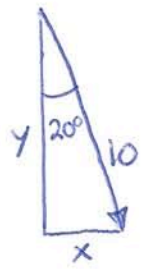
$y = 10 \cos 30 = 8.66$

$$\vec{V}_1 = 5\hat{x} + 8.7\hat{y}$$

$$\vec{V}_2 = 10\hat{x} + 0\hat{y}$$

$$\vec{V}_3 = 3.4\hat{x} - 9.4\hat{y}$$

$$\vec{V}_{tot} = 18.4\hat{x} - 0.7\hat{y}$$



$x = 10 \sin 20 = 3.42$

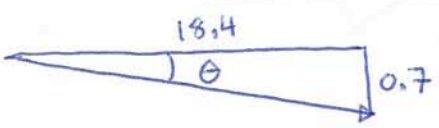
$y = 10 \cos 20 = 9.40$

$$|\vec{V}_{tot}| = \sqrt{(18.4)^2 + (0.7)^2}$$

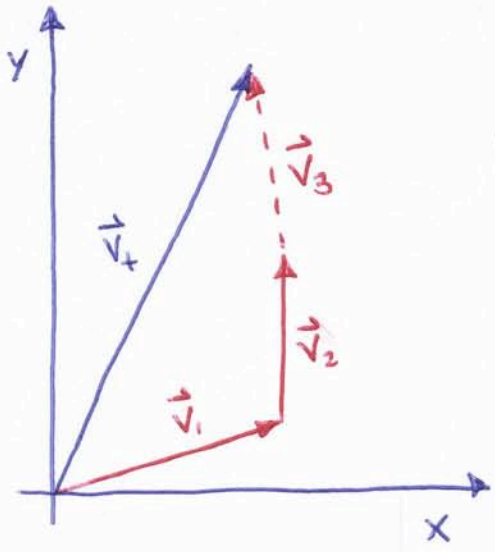
$$= 18.41$$

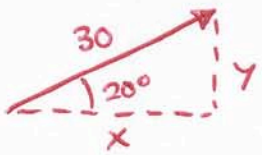
$$\theta = \tan^{-1}\left(\frac{0.7}{18.4}\right)$$

$$= 2.18°$$



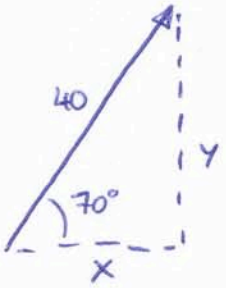
You run 30m at 20 degrees north of east, 10m north and an unknown distance and direction. You end up 40m at 70 degrees north of east, what was the unknown distance and direction that you travelled?





$x = 30\cos(20) = 28.2$

$y = 30\sin(20) = 10.3$



$x = 40\cos(70) = 13.7$

$y = 40\sin(70) = 37.6$

$$\vec{V}_+ = \vec{V}_1 + \vec{V}_2 + \vec{V}_3 \quad \Rightarrow \quad \vec{V}_3 = \vec{V}_+ - \vec{V}_1 - \vec{V}_2$$

$$= \vec{V}_+ + (-\vec{V}_1) + (-\vec{V}_2)$$

$$\vec{V}_+ = 13.7\,\hat{x} + 37.6\,\hat{y}$$

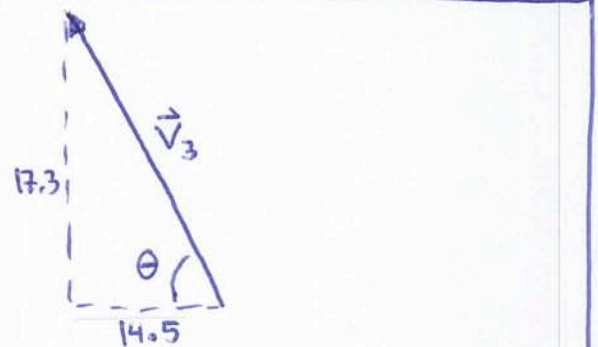
$$-\vec{V}_1 = -28.2\,\hat{x} - 10.3\,\hat{y}$$

$$-\vec{V}_2 = 0\,\hat{x} - 10\,\hat{y}$$

$$\vec{V}_3 = -14.5\,\hat{x} + 17.3\,\hat{y}$$

$$|\vec{V}_3| = \sqrt{(-14.5)^2 + (17.3)^2}$$

$$\boxed{|\vec{V}_3| = 22.6}$$



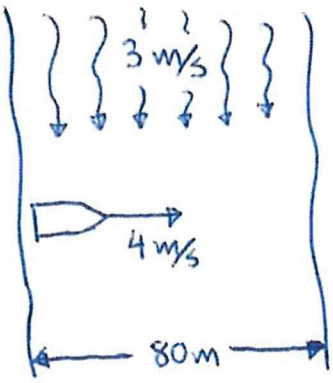
$$\theta = \tan^{-1}\left(\frac{17.3}{14.3}\right) = 50.4^\circ$$

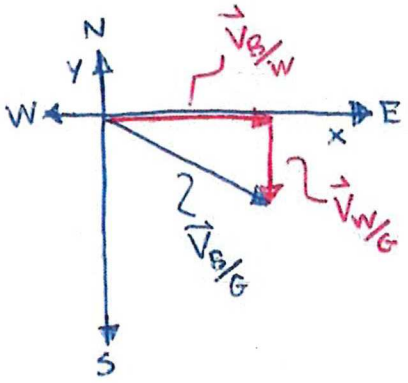
If motorboat is traveling at 4 m/s east encounters a water current traveling 3 m/s south.

a. What is the resultant velocity of the motorboat?

b. If the width of the river is 80 m wide, how much time does it take the boat to cross?

c. What distance downstream does the boat reach the opposite shore?





B = boat

W = water

G = ground.

$$\vec{V}_{B/W} = 4\hat{x} + 0\hat{y}$$

$$\vec{V}_{W/G} = 0\hat{x} - 3\hat{y}$$

$$\vec{V}_{B/G} = 4\hat{x} - 3\hat{y}$$

$$|\vec{V}_{B/G}| = \sqrt{(4)^2 + (-3)^2}$$

$$= 5$$

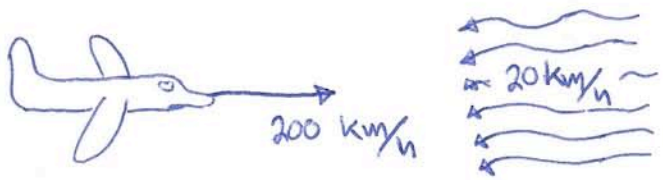
a.	$\lvert\vec{V}_{B/G}\rvert$	$= 5$ m/s
b.	Δt	$= 20$ s
c.	Δy	$= -60$ m

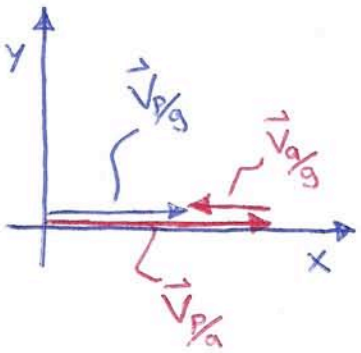
$$\vec{V} = \frac{\vec{\Delta x}}{\Delta t}$$

$$V_x = \frac{\Delta x}{\Delta t} \Rightarrow \Delta t = \frac{\Delta x}{V_x} = \frac{80}{4} = 20 \text{ s}$$

$$V_y = \frac{\Delta y}{\Delta t} \Rightarrow \Delta y = V_y \Delta t = (-3)(20) = -60 \text{ m}$$

A plane can travel with a speed of 200 km/h with respect to the air. Determine the velocity of the plane relative to the ground if there is a 20 km/h headwind (i.e. the wind is in the opposite direction of the plane).





p = plane
a = air
g = ground

$$\vec{V}_{p/a} = 200\,\hat{x} + 0\,\hat{y}$$

$$\vec{V}_{a/g} = -20\,\hat{x} + 0\,\hat{y}$$

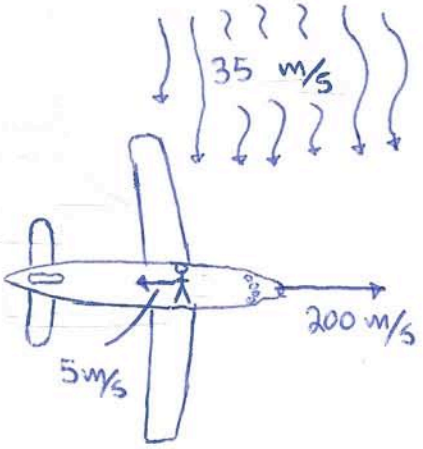
$$\vec{V}_{p/g} = 180\,\hat{x} + 0\,\hat{y}$$

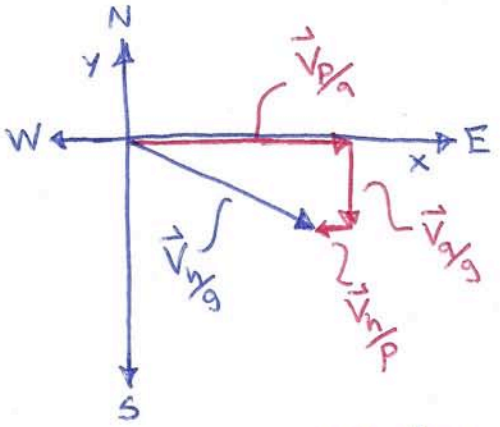
$$\left|\vec{V}_{p/g}\right| = \sqrt{(180)^2 + (0)^2}$$

$$= 180$$

$$\boxed{\left|\vec{V}_{p/g}\right| = 180 \text{ km/h}}$$

An airplane is flying at 200 m/s to the east with respect to the air. The air is moving at 35 m/s to the ~~south~~ with respect to the ground. And a passenger is traveling at 5 m/s towards the back of the plane (west). What is the velocity of the passenger with respect to the ground.





p = plane
a = air
g = ground
h = human

$$\vec{V}_{a/g} = 0\hat{x} - 35\hat{y}$$

$$\vec{V}_{p/a} = 200\hat{x} + 0\hat{y}$$

$$\vec{V}_{h/p} = -5\hat{x} + 0\hat{y}$$

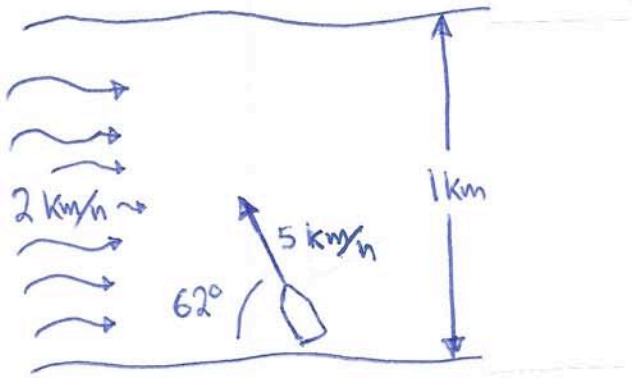
$$\vec{V}_{h/g} = 195\hat{x} - 35\hat{y}$$

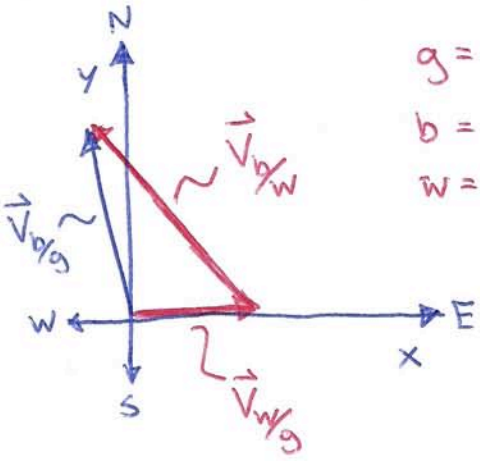
$$|\vec{V}_{h/g}| = \sqrt{(195)^2 + (-35)^2}$$

$$= 198$$

$$\boxed{|\vec{V}_{h/g}| = 198 \text{ m/s}}$$

A boat crosses a river with a velocity of 5 km/h at an angle 62 degrees north of west relative to the water. The river is 1 km wide and has an eastward current of 2 km/h. How far upstream is the boat when it reaches the opposite shore?



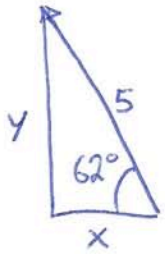


g = ground
b = boat
w = water

$$\vec{V}_{b/w} = -2.4\,\hat{x} + 4.4\,\hat{y}$$

$$\vec{V}_{w/g} = 2\,\hat{x} + 0\,\hat{y}$$

$$\vec{V}_{b/g} = -0.4\,\hat{x} + 4.4\,\hat{y}$$



$x = 5\cos 62 = 2.35$

$y = 5\sin 62 = 4.41$

Find time to travel across:

$$\vec{V} = \frac{\vec{\Delta x}}{\Delta t}$$

$$V_y = \frac{\Delta y}{\Delta t} \Rightarrow \Delta t = \frac{\Delta y}{V_y} = \frac{1}{4.4} = 0.23\ h$$

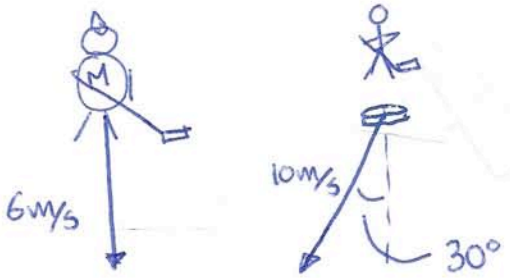
Find x-distance travelled.

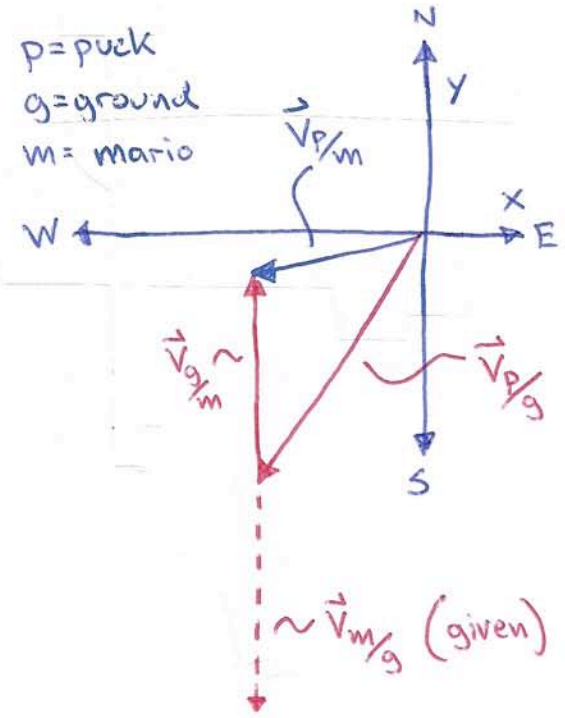
$$\vec{V} = \frac{\vec{\Delta x}}{\Delta t}$$

$$V_x = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = V_x\,\Delta t = (-0.4)(0.23) = -0.092\ km$$

$\Delta x = -92\ m$

Mario skates south at 6 m/s (relative to the ice). A fellow hockey player passes the puck to him at 10 m/s 30° west of south (relative to the ice). What is the magnitude and direction of the puck's velocity, as observed by Mario?

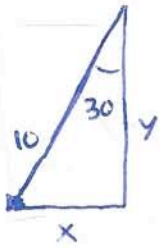




Relative velocity:

$$\vec{V}_{p/m} = \vec{V}_{p/g} + \vec{V}_{g/m}$$

$$\vec{V}_{g/m} = -\vec{V}_{m/g}$$



$x = 10 \sin 30 = 5$

$y = 10 \cos 30 = 8.66$

so

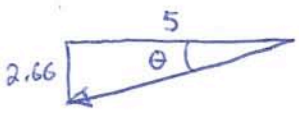
$$\vec{V}_{p/g} = -5\hat{x} - 8.66\hat{y}$$

$$\vec{V}_{g/m} = 0\hat{x} + 6\hat{y}$$

$$\vec{V}_{p/m} = -5\hat{x} - 2.66\hat{y}$$

$$|\vec{V}_{p/m}| = \sqrt{(-5)^2 + (-2.66)^2} = 5.7$$

$$\theta = \tan^{-1}\left(\frac{2.66}{5}\right) = 28°$$



$$|\vec{V}_{p/m}| = 5.7 \text{ m/s}$$

$$\angle \vec{V}_{p/m} = 28° \text{ S of W}$$