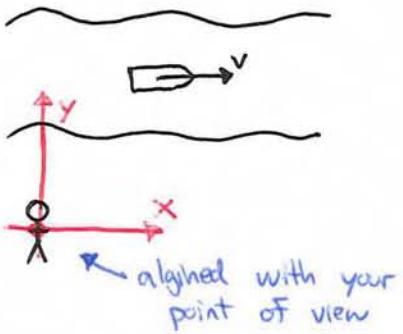
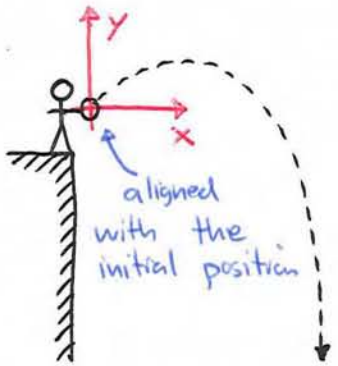
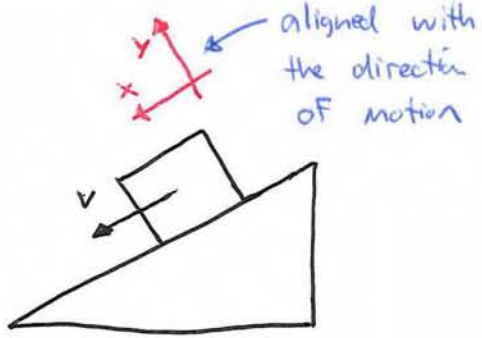
Vectors

The first thing you do when dealing with a physics problem is draw a picture, which means set a coordinate system.

examples:







We set a coordinate system to solve governing equations (i.e. Laws) in each of the coordinate system axis (dimension). Each equation is INDEPENDENT of each other equation for independent axis. For this course we assume 3 independent space dimensions (i.e. x, y, z) which "flows" through a constantly increasing time dimension.

Vectors

General Equation

x direction, y direction ...

$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$$

V_x: x component; V_y: y component ...

$\vec{V}$: The vector

Definitions

vector

$$\text{displacement} = \Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

$\vec{x}_f$: final; $\vec{x}_i$: initial

$$\text{distance} = d = |\vec{x}_1| + |\vec{x}_2| + \ldots$$

distance: scalar

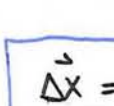
Rules

When adding/subtracting vectors only add/subtract individual components, remember they are independent of each other.

Magnitudes

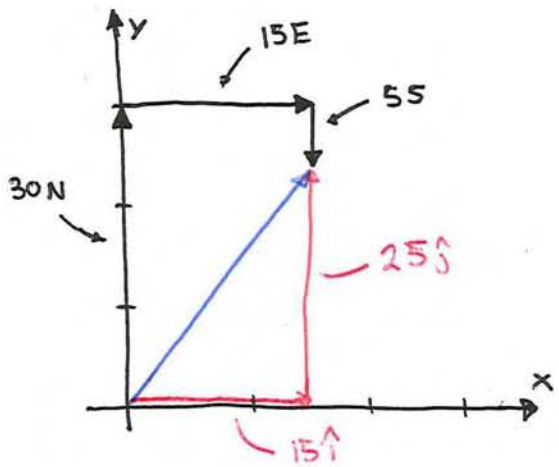
$$|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$\angle\vec{V} = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$



Example:

You walk 30 m north, 15 m east & 5 m south. Calculate displacement & distance travelled.



$$\vec{x}_i = 0\hat{i} + 0\hat{j}$$

$$\vec{x}_f = [(+15)]\hat{i} + [(+30) + (-5)]\hat{j}$$

$$= 15\hat{i} + 25\hat{j}$$

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i$$

$$= [15 - 0]\hat{i} + [25 - 0]\hat{j}$$

$$\Delta\vec{x} = 15\hat{i} + 25\hat{j}$$

$$|\Delta\vec{x}| = \sqrt{15^2 + 25^2} = 29 \text{ m}$$

$$d = |\vec{x}_1| + |\vec{x}_2| + |\vec{x}_3|$$

$$d = 30 + 15 + 5 = 50 \text{ m}$$

Relative Velocity

General Equation

$$\vec{V}_{A/C} = \vec{V}_{A/B} + \vec{V}_{B/C}$$

"B" with respect to "C"

Velocity of "A" with respect to "C"

"A" with respect to "B"

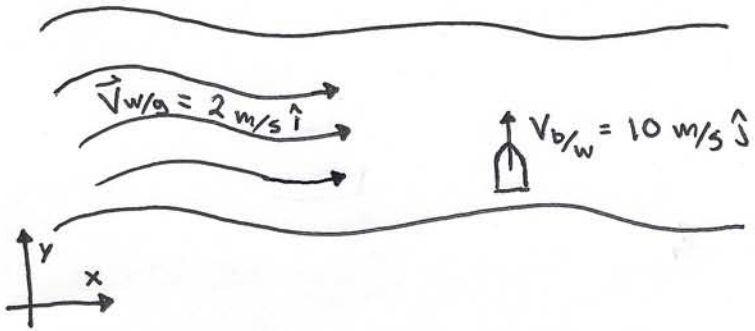
Notes:

Can be expanded for more and more complicated systems:

i.e.

$$\vec{V}_{A/E} = \vec{V}_{A/B} + \vec{V}_{B/C} + \ldots + \vec{V}_{D/E}$$

Example:

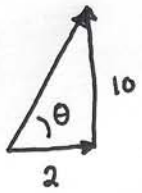


Question: what is the velocity of the boat with respect to the ground?

Answer:

$$V_{b/g} = V_{b/w} + V_{w/g}$$

$$= 10\,\hat{j} + 2\,\hat{i}$$



$$|V_{b/g}| = \sqrt{10^2 + 2^2} = 10.2\ m/s$$

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = 79°$$