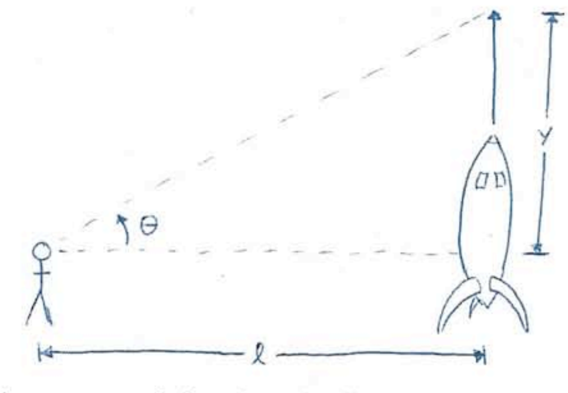
You are standing 100m away from a rocket ship that is taking off. If the height of the rocket ship follows the equation:

$$y = y_o + v_o t + \frac{1}{2} a t^2$$



where: y_o is the initial height ($y_o = 0$ m),
v_o is the initial velocity ($v_o = 0$ m/s),
t is the time and
a is the acceleration of the rocket ($a = 50$ m/s^2).

What is the velocity (dy/dt) of the rocket when 2 seconds have passed since it took off the ground? What is the rate of change that you have to rotate your head to track the rocket ship (i.e. $d\theta/dt$)?

given:

$y_0 = 0$

$V_0 = 0$

$a = 50$ m/s

so:

$y = 0 + (0)(t) + \frac{1}{2}(50)t^2$

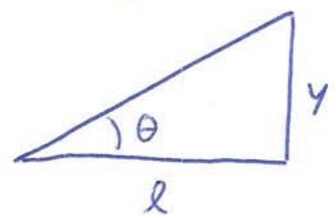
$\boxed{y = 25t^2}$

$\boxed{\frac{dy}{dt} = 50t}$

at t = 2 second.

$$\boxed{\begin{aligned} y &= 25(2)^2 = 100\text{ m} \\ \frac{dy}{dt} &= 50(2) = 100\text{ m/s} \end{aligned}}$$

rate of change:



$\tan\theta = \frac{y}{\ell}$

$\frac{d}{dt}$ $(\sec^2\theta)\left(\frac{d\theta}{dt}\right) = \frac{1}{\ell}\frac{dy}{dt}$

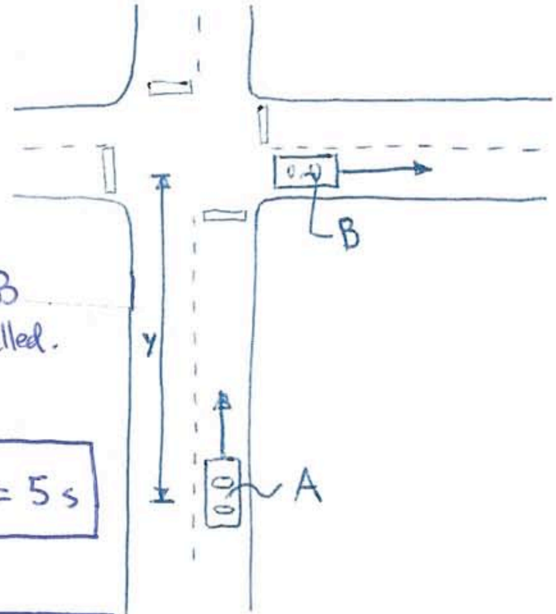
$$\boxed{\frac{d\theta}{dt} = \frac{\cos^2\theta}{\ell}\frac{dy}{dt}}$$

at t = 2 seconds

$\theta = \tan^{-1}\left(\frac{y}{\ell}\right) = \tan^{-1}\left(\frac{100}{100}\right) = 45°$

$$\boxed{\frac{d\theta}{dt} = \frac{\cos^2(45)}{100}(100) = 0.5 \text{ degrees/sec}}$$

Car A drives North at 10 m/s and car B drives East at 20 m/s. Cars A and B start in the orientation as shown in the figure when y = 100m. How fast will car A be travelling away from car B (how fast will they be moving apart) when y = 50 m?



Determine time and distance car B travelled.

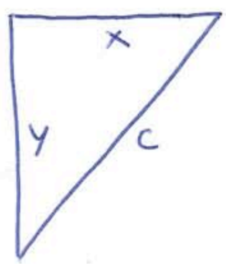
Car A:

$$V = \frac{d}{t} \Rightarrow \boxed{t = \frac{d}{V} = \frac{(100-50)}{10} = 5\,s}$$

Car B:

$$V = \frac{d}{t} \Rightarrow \boxed{d = vt = (20)(5) = 100\,m}$$

Rate of Change:



$$\boxed{x^2 + y^2 = c^2}$$

$$\frac{d}{dt}$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$$

$$\boxed{\frac{x}{c}\frac{dx}{dt} + \frac{y}{c}\frac{dy}{dt} = \frac{dc}{dt}}$$

Solve.

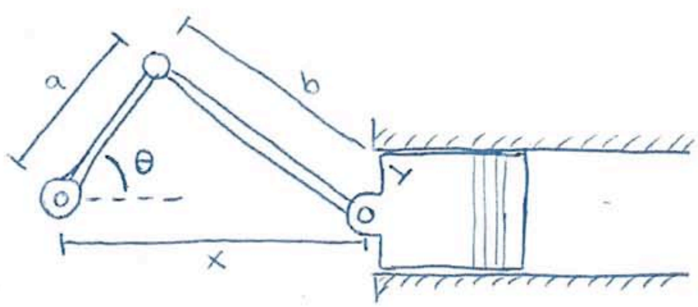
$$c = \sqrt{x^2+y^2} = \sqrt{100^2 + 50^2} = 112\,m$$

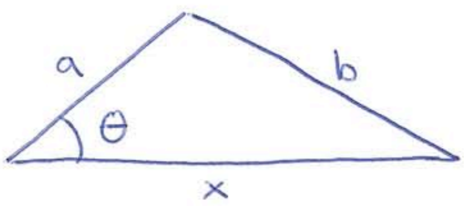
$$\frac{dc}{dt} = \frac{x}{c}\frac{dx}{dt} + \frac{y}{c}\frac{dy}{dt} = \frac{100}{112}(20) + \frac{50}{112}(-10) = 17.9 - 4.5$$

$$= 13.4$$

$$\boxed{\frac{dc}{dt} = 13\,m/s}$$

A simplified car piston is shown in the figure. Determine the velocity of the piston (i.e. dx/dt) as a function of a, b, x, θ and dθ/dt.





cosine law:

$$b^2 = a^2 + x^2 - 2ax\cos\theta$$

$$2b\cancel{\frac{db}{dt}}^{0} = 2a\cancel{\frac{da}{dt}}^{0} + 2x\frac{dx}{dt} - 2\cancel{\frac{da}{dt}}^{0} x\cos\theta - 2a\frac{dx}{dt}\cos\theta + 2ax\sin\theta\frac{d\theta}{dt}$$

$$0 = x\frac{dx}{dt} - a\frac{dx}{dt}\cos\theta + ax\sin\theta\frac{d\theta}{dt}$$

$$\frac{dx}{dt}\left(a\cos\theta - x\right) = ax\frac{d\theta}{dt}\sin\theta$$

$$\boxed{\frac{dx}{dt} = \frac{ax\sin\theta}{a\cos\theta - x}\frac{d\theta}{dt}}$$

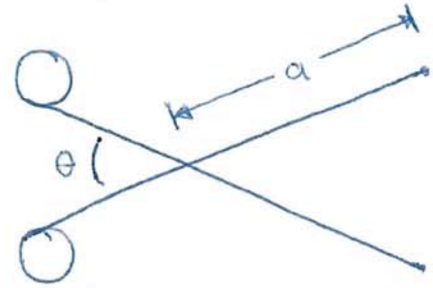
additional:

$$x^2 - 2a\cos\theta\, x + a^2 - b^2 = 0$$

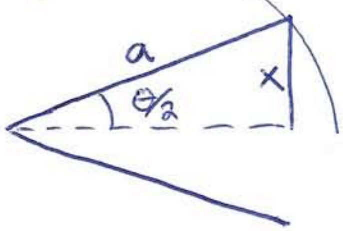
$$x = \frac{2a\cos\theta \pm \sqrt{4a^2\cos^2\theta - (4)(a^2-b^2)}}{2}$$

$$= a\cos\theta \pm a\sqrt{\cos^2\theta + b^2/a^2 - 1}$$

If a pair of scissors (a = 10 cm) close at a rate of 30 degrees/sec, how fast are the tips of the scissors coming together when θ = 30 degrees?



using triangles:



$$\tan \frac{\theta}{2} = \frac{x}{a}$$

$$\sec^2\left(\frac{\theta}{2}\right)\left(\frac{1}{2}\right)\left(\frac{d\theta}{dt}\right) = \frac{1}{a}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(\frac{a}{2}\right)\frac{1}{\cos^2(\theta/2)}\frac{d\theta}{dt}$$

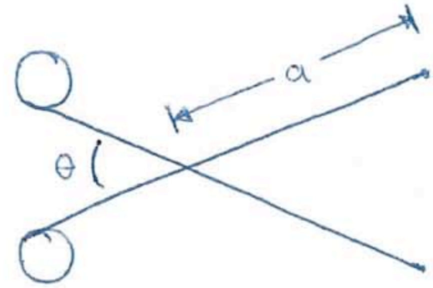
$$= \frac{10}{2}\,\frac{1}{\cos^2(30/2)}\,(30)$$

$$= 160.8 \text{ cm/s}$$

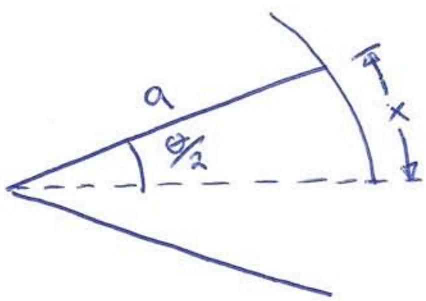
how Fast are the tips coming together?

$$\boxed{2\,\frac{dx}{dt} = 322 \text{ cm/s}}$$

If a pair of scissors (a = 10 cm) close at a rate of 30 degrees/sec, how fast are the tips of the scissors coming together when θ = 30 degrees?



Using arc lenght



$$x = \left(\frac{\theta}{2}\right) a$$

$$x = \frac{a\theta}{2}$$

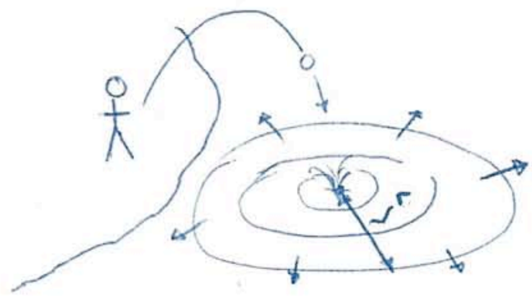
$$\frac{dx}{dt} = \cancel{\frac{da}{dt}\left(\frac{\theta}{2}\right)}^{\,0 \quad \frac{da}{dt}=0} + \frac{d\theta}{dt}\left(\frac{a}{2}\right)$$

$$= (30)\frac{(10)}{2} = 150 \text{ cm/s}$$

how fast are the tips coming together?

$$\boxed{2\,\frac{dx}{dt} = 300 \text{ cm/s}}$$

If i throw a rock into a pond ripple will emit from where the rock landed in the water. How much will the area be increasing as a function of the radius (r) and the velocity of the wave (dr/dt)?



area $= A = \pi r^2$

$$A = \pi r^2$$

$$\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$$