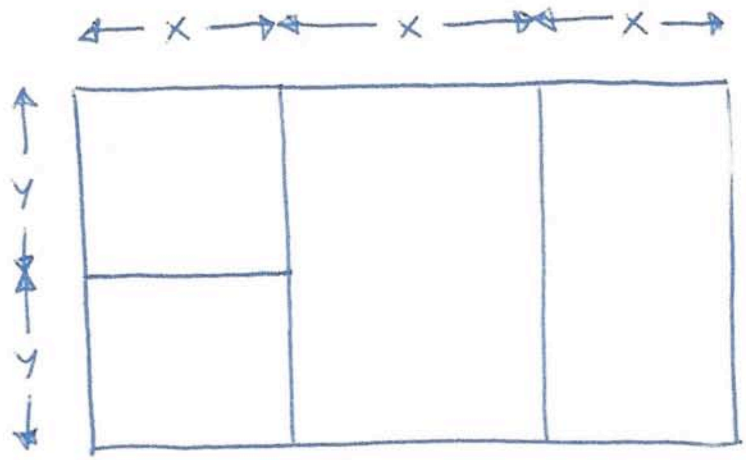
Given 100 meters in fencing determine the maximum area of the proposed enclosure.



fencing $= F$

$= 7x + 8y \qquad (1)$

area $= a$

$= (3x)(2y)$

$= 6xy \qquad (2)$

solve for x in (1)

$$F = 7x + 8y \quad \Rightarrow \quad x = \frac{F - 8y}{7} \qquad (1a)$$

sub into (2)

$$a = 6\left[\frac{F - 8y}{7}\right] y$$

$$= \frac{6F}{7} y - \frac{48}{7} y^2$$

take derivative with respect to y

$$\frac{da}{dy} = \frac{6F}{7} - \frac{96}{7} y = 0 \quad (\text{max}/\text{min})$$

$$y = \left(\frac{7}{96}\right)\left(\frac{6F}{7}\right) = \frac{F}{16} = \frac{100}{16} = 6.25$$

sub into (1a)

$$x = \frac{F - 8y}{7} = \frac{100 - 8(6.25)}{7} = 7.14$$

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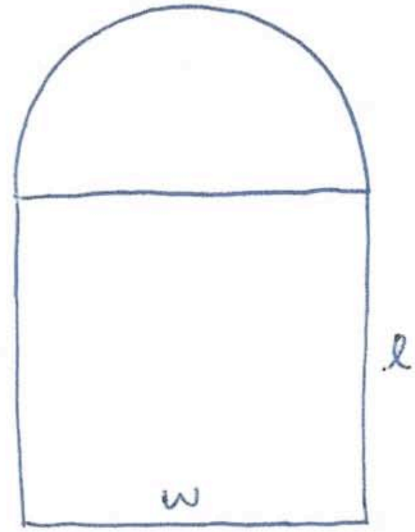

$$\text{Fencing} = F = 2l + 2w + \pi\left(\frac{w}{2}\right) \qquad (1)$$

$$\text{area} = a = wl + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2 \qquad (2)$$

solve for l in equ. (1)

$$F = 2l + 2w + \frac{\pi}{2}w$$

$$\boxed{\begin{aligned} l &= \frac{F}{2} - w - \frac{\pi}{4}w \\ &= \frac{F}{2} - w\left(1 + \frac{\pi}{4}\right) \end{aligned}} \qquad (1a)$$

sub (1a) into (2)

$$a = w\left[\frac{F}{2} - w\left(1 + \frac{\pi}{4}\right)\right] + \frac{1}{2}\pi\left(\frac{w}{2}\right)^2$$

$$a = \frac{F}{2}w - w^2\left(1 + \frac{\pi}{4}\right) + \frac{\pi}{8}w^2$$

take derivative of area with respect to w.

$$\frac{da}{dw} = \frac{F}{2} - 2w\left(1 + \frac{\pi}{4}\right) + \frac{\pi}{4}w = 0 \quad (\text{max/min})$$

$$\frac{F}{2} = w\left[2\left(1 + \frac{\pi}{4}\right) - \frac{\pi}{4}\right] = w\left[2 + \frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$w = \frac{F}{4 + \frac{\pi}{2}} = \frac{100}{5.57} = 18 \text{ m}$$

sub backinto (1a)

$$l = \frac{F}{2} - w\left(1 + \frac{\pi}{4}\right) = 50 - 18\left(1 + \frac{\pi}{4}\right) = 18 \text{ m}$$

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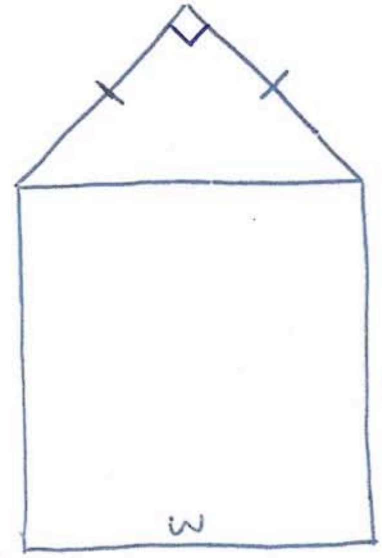

$$\sqrt{a^2+a^2} = w$$
$$a\sqrt{2} = w$$

$$\text{fencing} = F = 2w + 2l + 2a$$
$$= 2w + 2l + 2\left(\frac{w}{\sqrt{2}}\right) \qquad (1)$$
$$\text{area} = a = wl + \frac{a^2}{2} = wl + \frac{1}{2}\left(\frac{w}{\sqrt{2}}\right)^2 = wl + \frac{w^2}{4} \qquad (2)$$

solve for l in equ. (1)

$$F = w\left(2 + \frac{2}{\sqrt{2}}\right) + 2l \Rightarrow l = \frac{F}{2} - w\left(1 + \frac{1}{\sqrt{2}}\right) \qquad (1a)$$

sub (1a) into (2)

$$a = w\left[\frac{F}{2} - w\left(1 + \frac{1}{\sqrt{2}}\right)\right] + \frac{w^2}{4}$$
$$= \frac{F}{2}w - w^2\left(1 + \frac{1}{\sqrt{2}}\right) + \frac{w^2}{4}$$

take derivative of area with respect to w.

$$\frac{da}{dw} = \frac{F}{2} - 2w\left(1 + \frac{1}{\sqrt{2}}\right) + \frac{w}{2} = 0 \quad (\text{max}/\text{min})$$

$$\frac{F}{2} = w\left[2\left(1 + \frac{1}{\sqrt{2}}\right) - \frac{1}{2}\right] = w\left[\frac{3}{2} + \frac{2}{\sqrt{2}}\right] = w\left[\frac{3\sqrt{2} + 4}{2\sqrt{2}}\right]$$

$$w = \frac{F\sqrt{2}}{3\sqrt{2} + 4} = \frac{(100)(1.41)}{8.24} = \frac{100}{5.83} = 17 \text{ m}$$

sub back into (1a)

$$l = \frac{F}{2} - w\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{100}{2} - 17\left(1 + \frac{1}{\sqrt{2}}\right) = 21 \text{ m}$$