Find two nonnegative numbers whose sum is 10 and the product of one number and the second number is a maximum.

$$x + y = 10 \qquad (1)$$

$$xy = F(x,y) \qquad (2).$$

to be a maximum.

sub (1) into (2)

$$F(x) = x(10-x) = 10x - x^2$$

$\frac{d}{dx}$

$$\frac{dF}{dx} = 10 - 2x$$

Find maximum/minimum. $\frac{dF}{dx} = 0$

$$\frac{dF}{dx} = 0 = 10 - 2x. \Rightarrow x = \frac{10}{2} = 5 \Rightarrow y = 10 - x = 10 - 5 = 5.$$

$$\boxed{\begin{matrix} x = 5 \\ y = 5 \end{matrix}}$$

Find two nonnegative numbers whose sum is 10 and the product of one number and the square of the second number is a maximum.

$$x + y = 10 \qquad (1)$$

$$xy^2 = F(x,y) \qquad (2)$$

to be a maximum

sub (1) into (2)

$$F(x) = x(10-x)^2$$

$\frac{d}{dx}$

$$\frac{dF}{dx} = 1(10-x)^2 + x(2)(10-x)(-1)$$

$$= (100 - 20x + x^2) - 20x + 2x^2$$

$$= 3x^2 - 40x + 100$$

Find max/min $\frac{dF}{dx} = 0$

$$\frac{dF}{dx} = 0 = 3x^2 - 40x + 100$$

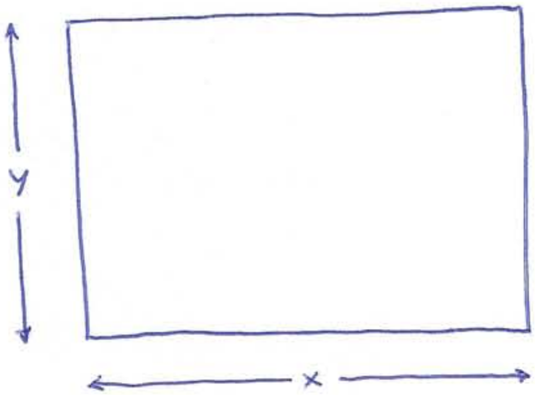
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{40 \pm \sqrt{(-40)^2 - (4)(3)(100)}}{(2)(3)} = \frac{40 \pm 20}{6}$$

$$x = 10 \text{ or } 3.\bar{3}$$

$$y = 10 - x = 0 \text{ or } 6.\bar{6}$$

$$\boxed{\begin{array}{l} x = 3.\bar{3} \\ y = 6.\bar{6} \end{array}}$$

You have 100 feet of fencing materials and would like to build a rectangular pen. What dimensions should the rectangular pen have to maximize the area.



to be a maximum.

$$\text{Area} = A(x,y) = xy \qquad (1)$$

$$\text{Primeter} = 100 = 2x + 2y \qquad (2)$$

$$\hookrightarrow y = 50 - x$$

sub (2) into (1)

$$A(x) = x(50 - x) = 50x - x^2$$

$\frac{d}{dx}$

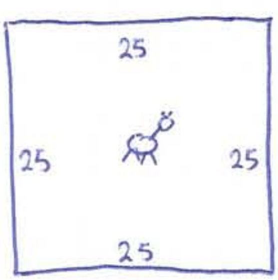
$$\frac{dA}{dx} = 50 - 2x$$

Find max/min $\frac{dA}{dx} = 0$

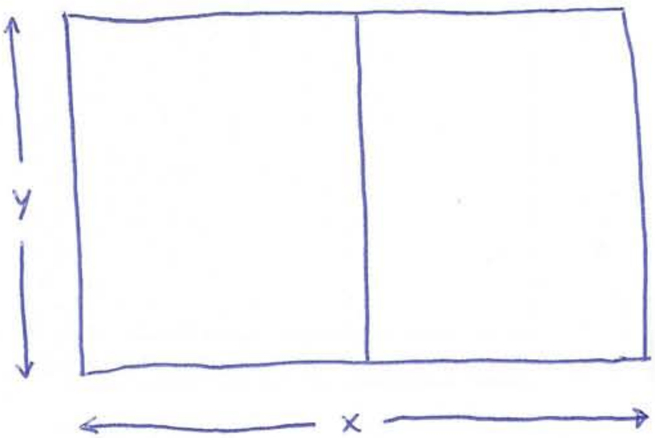
$$\frac{dA}{dx} = 0 = 50 - 2x \quad \Rightarrow \quad x = \frac{50}{2} = 25$$

$$\Rightarrow \quad y = 50 - x = 50 - 25 = 25.$$

$$\boxed{\begin{array}{l} x = 25 \text{ Feet} \\ y = 25 \text{ Feet} \end{array}}$$



You have 100 feet of fencing materials and would like to build a rectangular pen with 2 identical partitions. What dimensions should the rectangular pen have to maximize the area.



to be a maximum.

$$\text{Area} = A(x,y) = xy \qquad (1)$$

$$\begin{matrix}\text{Primeter +}\\ \text{partition}\end{matrix} = 100 = 2x + 3y \qquad (2)$$

$$\hookrightarrow y = \frac{100-2x}{3}$$

sub (2) into (1)

$$A(x) = x\left(\frac{100-2x}{3}\right) = \frac{100x}{3} - \frac{2x^2}{3} = 33.\bar{3}x - 0.\bar{6}x^2$$

$\frac{d}{dx}$

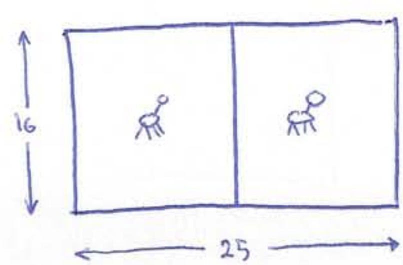
$$\frac{dA}{dx} = 33.\bar{3} - 1.\bar{3}x$$

Find max/min $\frac{dA}{dx} = 0$

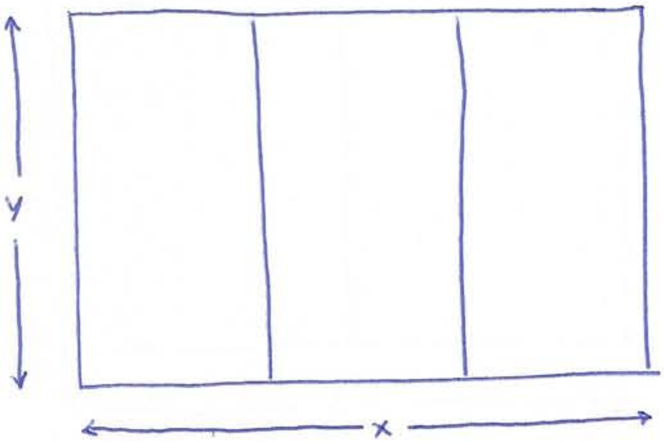
$$\frac{dA}{dx} = 0 = 33.\bar{3} - 1.\bar{3}x \quad \Rightarrow \quad x = \frac{33.\bar{3}}{1.\bar{3}} = 25$$

$$\Rightarrow \quad y = \frac{100-2x}{3} = 16.\bar{6}$$

$$\boxed{\begin{matrix} x = 25 \text{ feet} \\ y = 16.\bar{6} \text{ feet}\end{matrix}}$$



You have 100 feet of fencing materials and would like to build a rectangular pen with 3 identical partitions. What dimensions should the rectangular pen have to maximize the area.



to be a maximum.

$$\text{Area} = A(x,y) = xy \qquad (1)$$

$$\begin{matrix}\text{Primeter +}\\ \text{partition}\end{matrix} = 100 = 2x + 4y \qquad (2)$$

$$\hookrightarrow y = \frac{100-2x}{4}$$

sub (2) into (1)

$$A(x) = x\left(\frac{100-2x}{4}\right) = 25x - 0.5x^2$$

$\frac{d}{dx}$

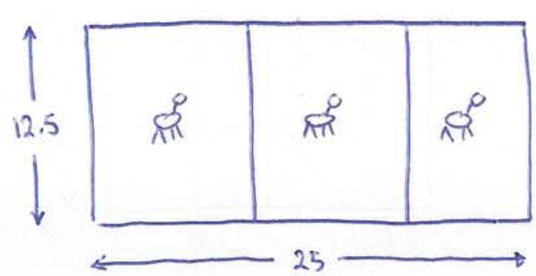
$$\frac{dA}{dx} = 25 - x$$

find max/min $\frac{dA}{dx} = 0$

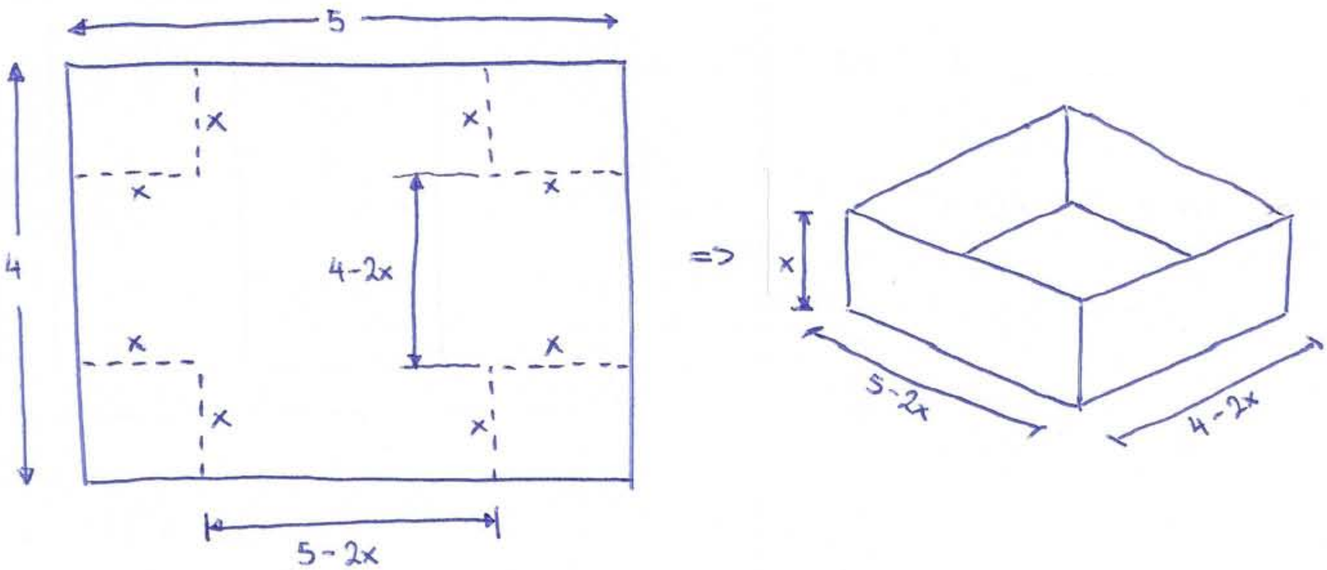
$$\frac{dA}{dx} = 0 = 25 - x \quad \Rightarrow \quad x = 25$$

$$\Rightarrow \quad y = \frac{100-2x}{4} = 12.5$$

$$\boxed{\begin{matrix} x = 25 \text{ feet} \\ y = 12.5 \text{ feet}\end{matrix}}$$



You have to make an open rectangular box (i.e. 5 sides to the box). You must create the box with one sheet measuring 4 meters by 5 meters. Equal sized squares pieces are cut from each corner and the four edges are folded up to create the box. What should the dimensions be to maximize the volume of the box.



to be a maximum.

$$\text{Volume} = V(x) = (x)(5-2x)(4-2x) = x\left(20 - 18x + 4x^2\right) = 4x^3 - 18x^2 + 20x$$

$\frac{d}{dx}$

$$\frac{dV}{dx} = 12x^2 - 36x + 20$$

find max/min $\frac{dV}{dx} = 0$

$$\frac{dV}{dx} = 0 = 12x^2 - 36x + 20 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+36 \pm \sqrt{(-36)^2 - (4)(12)(20)}}{(2)(12)}$$

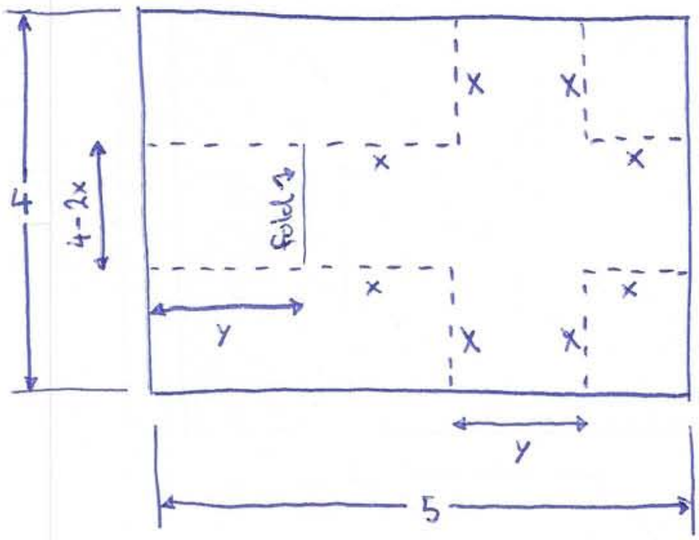
$$= \frac{36 \pm 18.3}{24} = 2.26 \text{ or } 0.74$$

$V(2.26) = -0.56 \text{ m}^3$

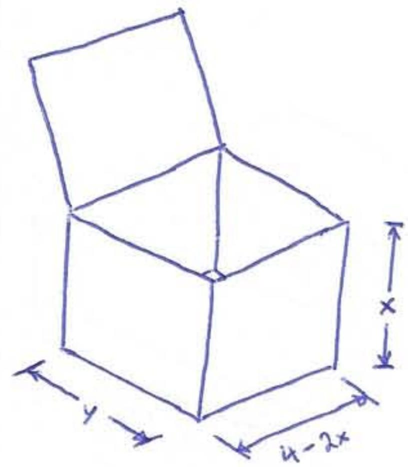
$\boxed{V(0.74) = 6.56 \text{ m}^3}$

$\boxed{x = 0.74 \text{ m}}$

You have to make a closed rectangular box (i.e. 6 sides to the box). You must create the box with one sheet measuring 4 meters by 5 meters. Pieces are cut from each corner and the four edges are folded up to create the box while one of the edges is folded twice to create the top. What should the dimensions be to maximize the volume of the box.



$\Rightarrow$



$$2y + 2x = 5$$

$$\Rightarrow y = \frac{5}{2} - x$$

$$\text{Volume} = (x)(4-2x)\left(\frac{5}{2} - x\right) = x\left(10 - 9x + 2x^2\right) = 2x^3 - 9x^2 + 10x$$

$\frac{d}{dx}$

$$\frac{dV}{dx} = 6x^2 - 18x + 10$$

find max/min $\frac{dV}{dx} = 0$

$$\frac{dV}{dx} = 0 = 6x^2 - 18x + 10 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{18 \pm \sqrt{(-18)^2 - (4)(6)(10)}}{(2)(6)} = \frac{18 \pm 9.17}{12}$$

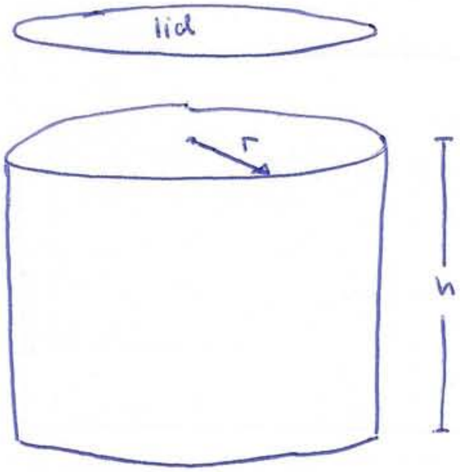
$$= 2.26 \text{ or } 0.74$$

$V(2.26) = -0.28 \text{ m}^3$

$\boxed{V(0.74) = 3.28 \text{ m}^3}$

$\boxed{x = 0.74 \text{ m}}$

You have to make an circular cylinder container with a lid and it must have a volume of 10 m³. The material for the top and bottom costs 10 \$/m² whereas the material for the sides costs 7 \$/m². What should the dimensions be to minimize the cost of the container.



Volume $= \pi r^2 h = 10$ (1) $\rightarrow h = \dfrac{10}{\pi r^2}$

Top + Bottom Area $= 2\pi r^2$

Side Area $= 2\pi r h$

$$\text{Cost} = (10)(2\pi r^2) + (7)(2\pi r h) \quad (2)$$

sub (1) into (2)

$$\text{Cost} = C(x) = 20\pi r^2 + 14\pi r\left(\frac{10}{\pi r^2}\right) = 20\pi r^2 + \frac{140}{r}$$

$\frac{d}{dr}$

$$\frac{dC}{dr} = 40\pi r - 140 r^{-2}$$

Find max/min $\frac{dC}{dr} = 0$

$$\frac{dC}{dr} = 0 = 40\pi r - \frac{140}{r^2} \quad \Rightarrow \quad 0 = 40\pi r^3 - 140$$

$$r = \sqrt[3]{\frac{140}{40\pi}} = 1.04 \text{ m}$$

$$h = \frac{10}{\pi r^2} = \frac{10}{\pi (1.04)^2} = 2.96 \text{ m}$$

$$\boxed{\begin{array}{l} r = 1.04\text{ m} \\ h = 2.96\text{ m} \end{array}}$$