

Governing equations

$$A = xy \quad (1)$$

$$R^2 = \left(\frac{x}{2}\right)^2 + y^2 \quad (2)$$

solve for y in (2)

$$y = \sqrt{R^2 - \left(\frac{x}{2}\right)^2} \quad (2a)$$

sub (2a) into (1)

$$A = x \left[R^2 - \frac{x^2}{4} \right]^{\frac{1}{2}}$$

take derivative with respect to x:

$$\begin{aligned} \frac{dA}{dx} &= (1) \left[R^2 - \frac{x^2}{4} \right]^{\frac{1}{2}} + (x) \left[\frac{1}{2} \left(R^2 - \frac{x^2}{4} \right)^{-\frac{1}{2}} \left(-\frac{2x}{4} \right) \right] \\ &= \sqrt{R^2 - \frac{x^2}{4}} - \frac{x^2}{4} \frac{1}{\sqrt{R^2 - \frac{x^2}{4}}} = 0 \quad (\text{max/min}) \end{aligned}$$

$$R^2 - \frac{x^2}{4} - \frac{x^2}{4} = 0$$

$$R^2 - \frac{x^2}{2} = 0$$

$$x = \sqrt{2} R \approx 1.41 R$$

sub into (2a)

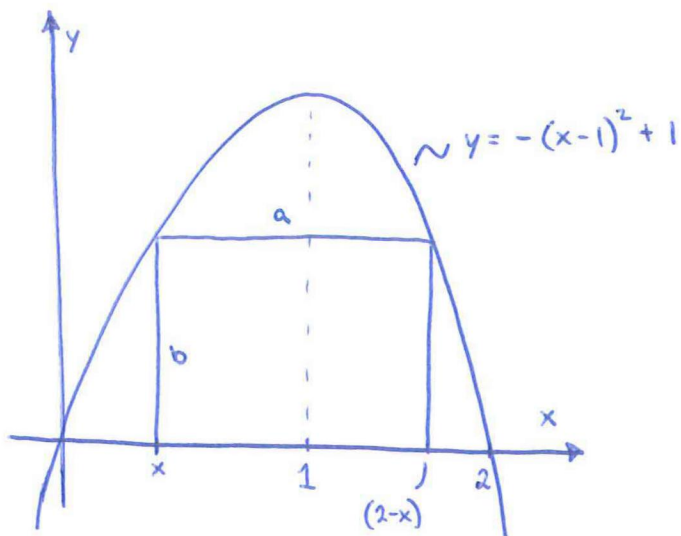
$$y = \sqrt{R^2 - \frac{1}{4} [\sqrt{2} R]^2} = \sqrt{R^2 - \frac{2}{4} R^2} = \sqrt{\frac{R^2}{2}}$$

$$y = \frac{R}{\sqrt{2}} \approx 0.707 R$$

For $R = 1 \text{ m}$

$$x = 1.41 \text{ m}$$

$$y = 0.71 \text{ m}$$



Governing equations

$$A = ab \quad (1)$$

$$\begin{aligned} 2 &= x + a + [2 - (2-x)] \\ &= 2x + a \end{aligned} \quad (2)$$

$$\begin{aligned} y &= -(x-1)^2 + 1 \\ &= -x^2 + 2x - 1 + 1 = b \end{aligned} \quad (3)$$

solve for a in (2)

$$a = 2 - 2x \quad (2a)$$

sub (2a) and (3) into (1)

$$\begin{aligned} A &= (2-2x)(-x^2+2x) \\ &= -2x^2 + 4x + 2x^3 - 4x^2 \\ &= 2x^3 - 6x^2 + 4x \end{aligned}$$

take derivative with respect to x :

$$\frac{dA}{dx} = 6x^2 - 12x + 4 = 0 \quad (\text{max/min})$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{+12 \pm \sqrt{144 - 96}}{12} = 1.58, 0.423 \end{aligned}$$

$x = 0.423$

$$\begin{aligned} a &= 2 - 2x \\ &= 2 - 2(0.423) \\ &= 1.15 \end{aligned}$$

$$\begin{aligned} b &= -x^2 + 2x \\ &= -(0.423)^2 + 2(0.423) \\ &= 0.6 \end{aligned}$$