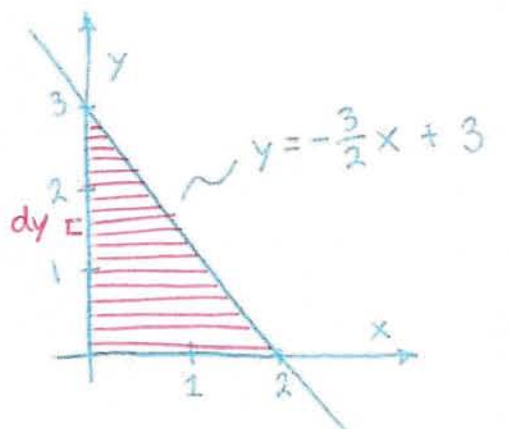
find the area described in the figure



area of enclosed triangle ?

$\Rightarrow x = -\frac{2}{3}y + 2$

use rows of height dy and width x(y)

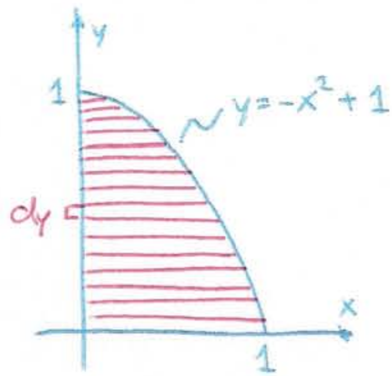
$$\text{Area} = \int_0^3 x(y)\,dy = \int_0^3 -\frac{2}{3}y + 2\,dy = \left[-\frac{2}{3}\left(\frac{1}{2}y^2\right) + 2y\right]_0^3$$

$$= \left[-\frac{1}{3}y^2 + 2y\right]_0^3 = \left[-\frac{1}{3}(3)^2 + 2(3)\right] - \left[-\frac{1}{3}(0)^2 + 2(0)\right]$$

$$= -3 + 6 = 3 \text{ units}^2$$

$$\boxed{\text{Area} = 3 \text{ units}^2}$$

find the area described in the figure



area of enclosed parabola

$x(y) = \sqrt{1-y}$

use rows of height dy and width $x(y)$

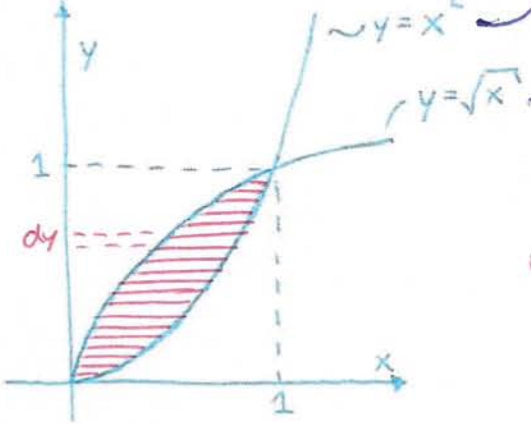
$$\text{Area} = \int_0^1 x(y)\,dy = \int_0^1 \left(\sqrt{1-y}\right) dy = \left[-\frac{2}{3}(1-y)^{\frac{3}{2}}\right]_0^1$$

$$= \left[-\frac{2}{3}(1-(1))^{\frac{3}{2}}\right] - \left[-\frac{2}{3}(1-(0))^{\frac{3}{2}}\right] = [0] - \left[-\frac{2}{3}\right] = \frac{2}{3}$$

$$\boxed{\text{Area} = \frac{2}{3} \cong 0.67 \text{ units}^2}$$

find the area described in the figure

$x_1(y) = \sqrt{y}$



area enclosed by
$y = \sqrt{x}$ and $y = x^2$

$x_2(y) = y^2$

use rows of height dy
and width $(x_1 - x_2)$

$$\text{Area} = \int_0^1 \left(x_1(y) - x_2(y)\right) dy = \int_0^1 \left(\sqrt{y} - y^2\right) dy = \left[\frac{2}{3} y^{3/2} - \frac{1}{3} y^3\right]_0^1$$

$$= \left[\frac{2}{3}(1)^{3/2} - \frac{1}{3}(1)^3\right] - \left[\frac{2}{3}(0)^{3/2} - \frac{1}{3}(0)^3\right]$$

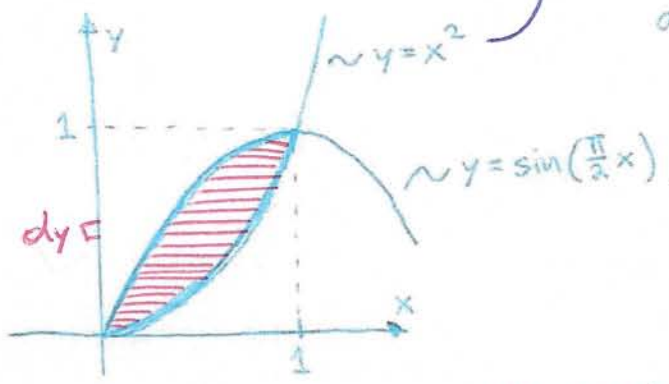
$$= \left[\frac{2}{3} - \frac{1}{3}\right] = \frac{1}{3} \cong 0.33 \text{ units}^2$$

$$\boxed{\text{Area} = \frac{1}{3} = 0.33 \text{ units}^2}$$

find the area described in the figure

$x_1(y) = \sqrt{y}$

area enclosed by $y = \sin\left(\frac{\pi x}{2}\right)$ and $y = x^2$?



$$x_2(y) = \frac{2}{\pi}\arcsin(y) = \frac{2}{\pi}\sin^{-1}(y)$$

use rows of height dy and width $(x_1(y) - x_2(y))$

$$\text{Area} = \int_0^1 \left(x_1(y) - x_2(y)\right) dy = \int_0^1 \sqrt{y} - \frac{2}{\pi}\sin^{-1}(y)\, dy$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{\pi}\left(y\sin^{-1}(y) + \sqrt{1-y^2}\right)\right]_0^1$$

$$\boxed{\begin{array}{l} \int \sin^{-1}(x)\,dx = \\ \quad x\sin^{-1}(x) - \sqrt{1-x^2} \end{array}}$$

would have been given

$$= \left[\frac{2}{3}(1)^{\frac{3}{2}} - \frac{2}{\pi}\left((1)\sin^{-1}(1) + \sqrt{1-(1)^2}\right)\right] - \left[\frac{2}{3}(0)^{\frac{3}{2}} - \frac{2}{\pi}\left((0)\sin^{-1}(0) + \sqrt{1-(0)^2}\right]\right.$$

$$= \left[\frac{2}{3} - \frac{2}{\pi}\left(\frac{\pi}{2} + \sqrt{0}\right)\right] - \left[0 - \frac{2}{\pi}(0 + 1)\right]$$

$$= \frac{2}{3} - \frac{2\pi}{2\pi} + \frac{2}{\pi} = \frac{2}{3} - 1 + \frac{2}{\pi} = \frac{2}{\pi} - \frac{1}{3} \cong 0.30 \text{ units}^2$$

$$\boxed{\text{Area} = \frac{2}{\pi} - \frac{1}{3} \cong 0.30 \text{ units}^2}$$