

Classical Mechanics Project

This project is designed to explore and discuss Newton's Laws of motion. A physical model of an appropriate physics event will be constructed and analyzed using the techniques learnt in class. For more complicated events, numerical models will have to be implemented to determine the motion of the object(s). Experimental data will be compared to theoretical models and discussed.

Groups of 1-4 people

Presentation (worth 5% of final grade) and Report (worth 10% of final grade) due week 14
Report out of 10 marks

Outline

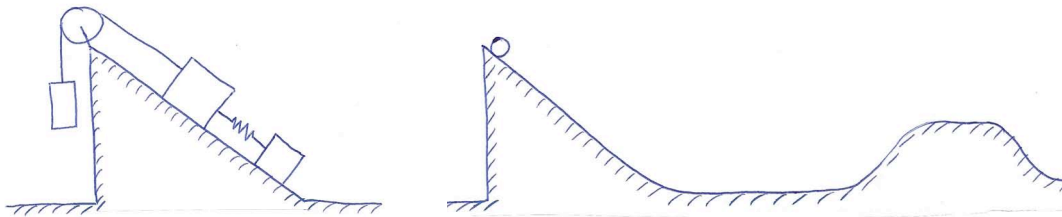
This report will outline the physics project by means of an example. It should only be used as a template and you should create your own report and excel file for your submission.

Instructions

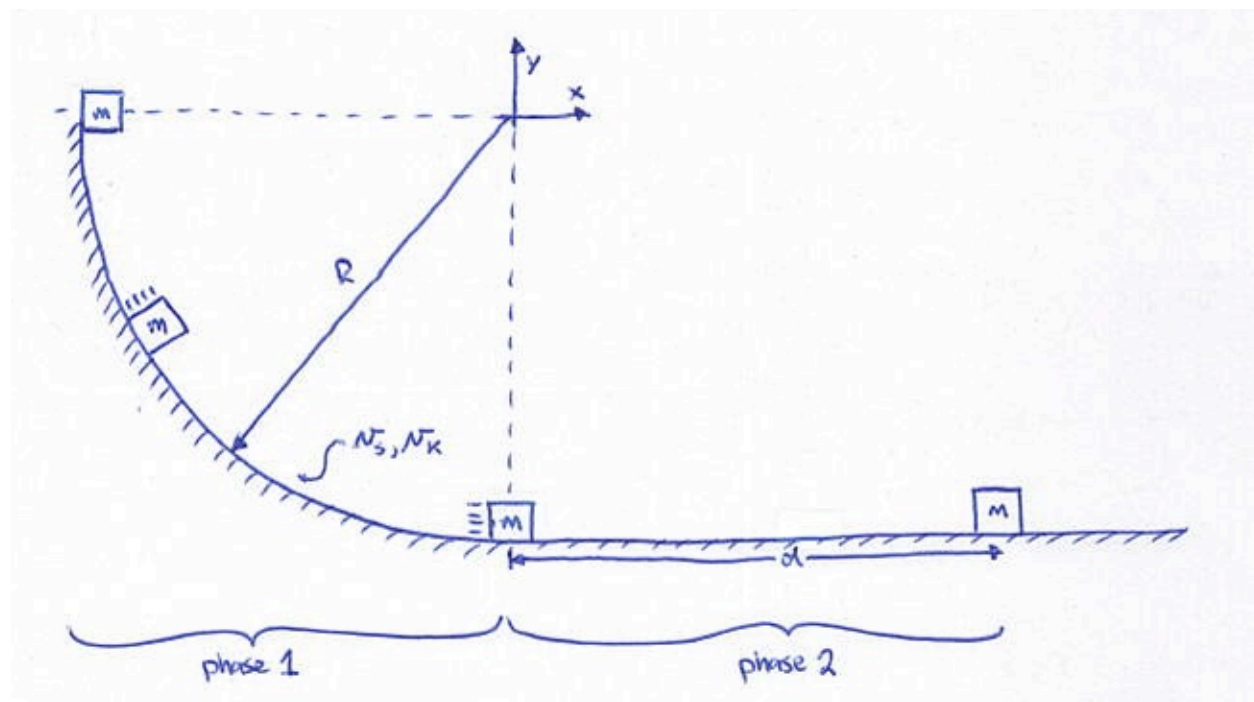
1. Choose a physics event, one page proposal of your event.

Predict analysis and construction.

Your event should be of the same complexity as the one presented. Some examples are shown below: (left) min/max mass to slide with different coefficients of friction and (right) distance ball rolls. You should try to predict all aspects of the project, such as, theoretical modelling of event, construction of event, materials of the event, measurements of the event and comparison to theory.

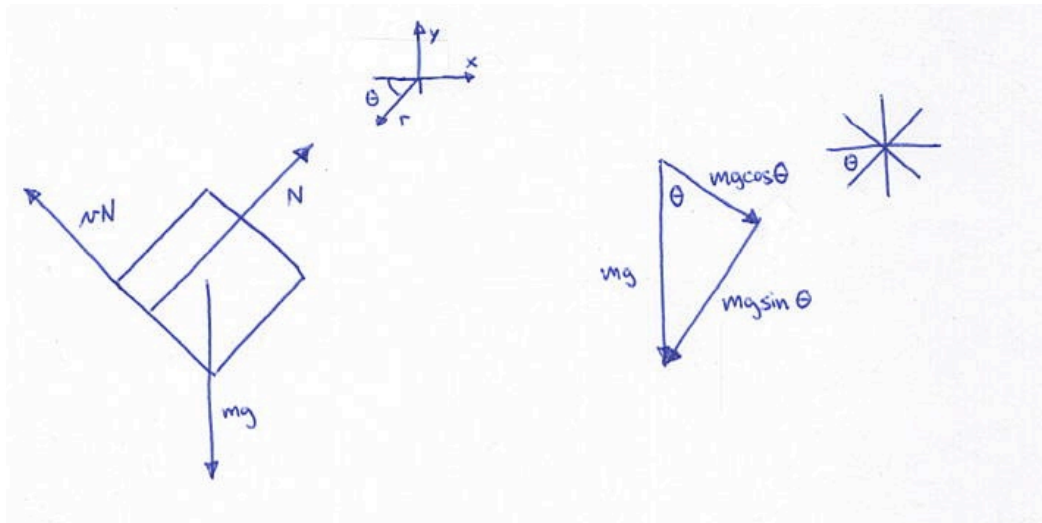


For this report a sliding block down a constant radius ramp was chosen (shown below). In phase 1, the block slides down a constant radius ramp. In phase 2, the block slides along a flat surface. The acceleration of the block in phase 1 could not be solved for directly as it was a function of time (not constant). Numerical models were implemented to analyze the block as it slid down the curve. However, in phase 2, the constant deceleration could be solved directly and kinematic equations were used to solve of the final distance (d).



2. (8 marks) Analyze the physics event. Derive final results.

Free Body Diagram (FBD) of the block on the curved ramp:



Newton's 2nd Laws applied to the block:

Newton's 2nd law

$$\sum F_r = ma_r = mg \sin \theta - N \quad (1)$$

$$\sum F_\theta = ma_\theta = mg \cos \theta - \mu N \quad (2)$$

solve for N in (1)

$$N = mg \sin \theta - ma_r \quad (1a)$$

sub (1a) into (2)

$$ma_\theta = mg \cos \theta - \mu [mg \sin \theta - ma_r] \quad (2a)$$

sub cylindrical accelerations (3) & (4) into (2a)

$$a_r = -\omega^2 r \quad (3)$$

$$a_\theta = \alpha r \quad (4)$$

$$\mu [\alpha r] = \mu g \cos \theta - \mu mg \sin \theta + \mu m [-\omega^2 r]$$

$$\alpha r = g (\cos \theta - \mu \sin \theta) - \mu \omega^2 r$$

switch to differential form

$$\boxed{\frac{d^2 \theta}{dt^2} = \frac{g}{r} (\cos \theta - \mu \sin \theta) - \mu \left(\frac{d\theta}{dt} \right)^2}$$

The acceleration (alpha) of the block depends on the velocity (omega) and therefore can not be solved directly. An iterative numerical method must be implemented to solve for the velocity and then the acceleration as a function of time. An excel routine that followed Euler method (shown below) was developed. In Euler's method, the definition of the derivative is used to estimate the solution at a given forward time step (delta t).

Definition of the Derivative

$$\frac{dy}{dt} = \lim_{\Delta t \rightarrow 0} \frac{y(t+\Delta t) - y(t)}{\Delta t}$$

$$\approx \frac{y(t+\Delta t) - y(t)}{\Delta t} \quad (\text{For small } \Delta t's)$$

so

$$y(t+\Delta t) \approx y(t) + \left(\frac{dy}{dt}\right)(\Delta t)$$

or

$$\boxed{y_{n+1} = y_n + \left(\frac{dy}{dt}\right)_n (\Delta t)} \quad \text{Euler's Method}$$

This routine (or methodology) can be applied to all unknown variables in the differential equation. The last differential term should be solved using the governing equation.

given

$$\frac{d^2 y}{dt^2} = f(t, y(t), \frac{dy}{dt}(t)) \quad \text{and} \quad y_0, \left.\frac{dy}{dt}\right|_0$$

solve for $y(t)$ using Euler's Method

n	t	$y(t)$	$\frac{dy}{dt}$	$\frac{d^2 y}{dt^2}$
0	t_0	y_0	$\left.\frac{dy}{dt}\right _0$	$F(t, y, \frac{dy}{dt})$
1	$t_1 = t_0 + \Delta t$	$y_1 = y_0 + \left(\frac{dy}{dt}\right)_0 \Delta t$	$\frac{dy}{dt}_1 = \left.\frac{dy}{dt}\right _0 + \left(\frac{d^2 y}{dt^2}\right)_0 \Delta t$	$F(t_1, y_1, \frac{dy}{dt}_1)$
2	$t_2 = t_1 + \Delta t$	$y_2 = y_1 + \left(\frac{dy}{dt}\right)_1 \Delta t$	$\frac{dy}{dt}_2 = \frac{dy}{dt}_1 + \left(\frac{d^2 y}{dt^2}\right)_1 \Delta t$	$F(t_2, y_2, \frac{dy}{dt}_2)$
3	$\vdots$	$\vdots$	$\vdots$	$\vdots$

The excel routine was constructed with inputs of the physical properties of the system, initial conditions and the numerical parameters, as:

:

Block sliding down constant radius curve

Phase 1

Constants

g	9.81 m/s ²
m	1 kg
R	1 m
mu_k	0.1

Initial conditions

t_o	0 s
theta(t_o)	0 degrees
dtheta/dt(t_o)	0 degrees/s

Numerical parameters

dt	0.05 s
t_max	2 s
N =	40 points
theta_max	90 degrees
N_theta_max =	13 points
N_min =	13 points

The excel routine then used Euler's Method to solve for the distance (theta), velocity (omega) and acceleration (alpha) of the block, as:

Numerical analysis

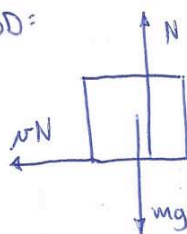
n	t [s]	theta [rads]	omega [rads/s]	alpha [rads/s ²]	N [N]	x [m]	y [m]	v_theta [m/s]	a_theta [m/s ²]	a_r [m/s ²]	a [m/s ²]	phi_a [degrees]
0	0.0	0.0	0.0	9.8	0.0	-1.0	1.0	0.0	9.8	0.0	9.8	0.0
1	0.05	0.0	0.5	9.8	0.2	-1.0	1.0	0.5	9.8	-0.2	9.8	1.4
2	0.10	0.0	1.0	9.7	1.2	-1.0	1.0	1.0	9.7	-1.0	9.7	5.7
3	0.15	0.1	1.5	9.5	2.9	-1.0	0.9	1.5	9.5	-2.1	9.7	12.7
4	0.20	0.1	1.9	9.2	5.2	-1.0	0.9	1.9	9.2	-3.8	9.9	22.3
5	0.25	0.2	2.4	8.7	8.1	-1.0	0.8	2.4	8.7	-5.8	10.4	33.4
6	0.30	0.4	2.8	8.0	11.5	-0.9	0.6	2.8	8.0	-8.0	11.3	45.0
7	0.35	0.5	3.2	7.1	15.2	-0.9	0.5	3.2	7.1	-10.5	12.6	56.0
8	0.40	0.7	3.6	5.8	18.9	-0.8	0.4	3.6	5.8	-12.9	14.1	65.7
9	0.45	0.8	3.9	4.3	22.4	-0.7	0.3	3.9	4.3	-15.0	15.6	74.2
10	0.50	1.0	4.1	2.4	25.2	-0.5	0.1	4.1	2.4	-16.7	16.9	81.7
11	0.55	1.2	4.2	0.4	27.0	-0.3	0.1	4.2	0.4	-17.8	17.8	88.6
12	0.60	1.5	4.2	-1.6	27.7	-0.1	0.0	4.2	-1.6	-17.9	18.0	95.2
13	0.65	1.7	4.2	-3.6	27.0	0.1	0.0	4.2	-3.6	-17.3	17.6	101.9

The output of phase 1 was the final velocity of the block as it transitioned to the flat section (phase 2).

When the block starts sliding on the flat section (phase 2), the acceleration was determined using Newton's 2nd Law. Then kinematic equations were used to solve for the final distance travelled. The final distance travelled was the value compared to experimental results.



FBD:



Newton's 2nd Law

$$\sum F_x = ma_x = -\mu N \quad (1)$$

$$\sum F_y = ma_y = N - mg \quad (2)$$

Solve for N in eqn. (2) and sub into (1):

$$\mu a_x = -\mu [mg]$$

$$a_x = -\mu g$$

Kinematics

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{xf}^2 = v_{x0}^2 + 2a_x(x_f - x_0) \quad l$$

$$0 = v_{x0}^2 + 2[-\mu g]l$$

$$l = \frac{v_{x0}^2}{2\mu g}$$

3. (3 marks) Build the physics event.

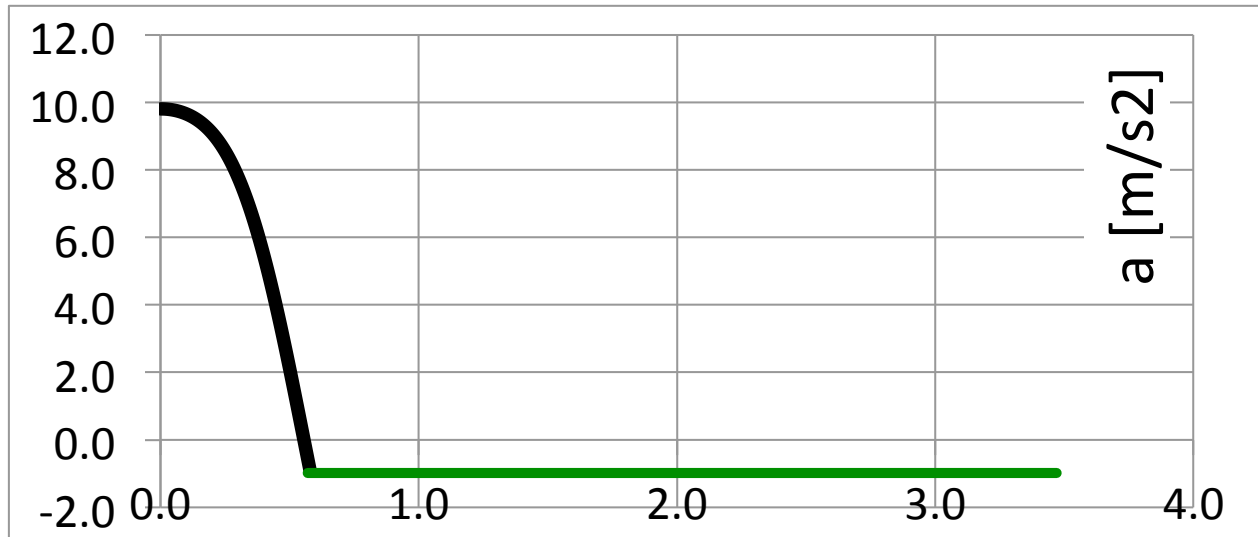
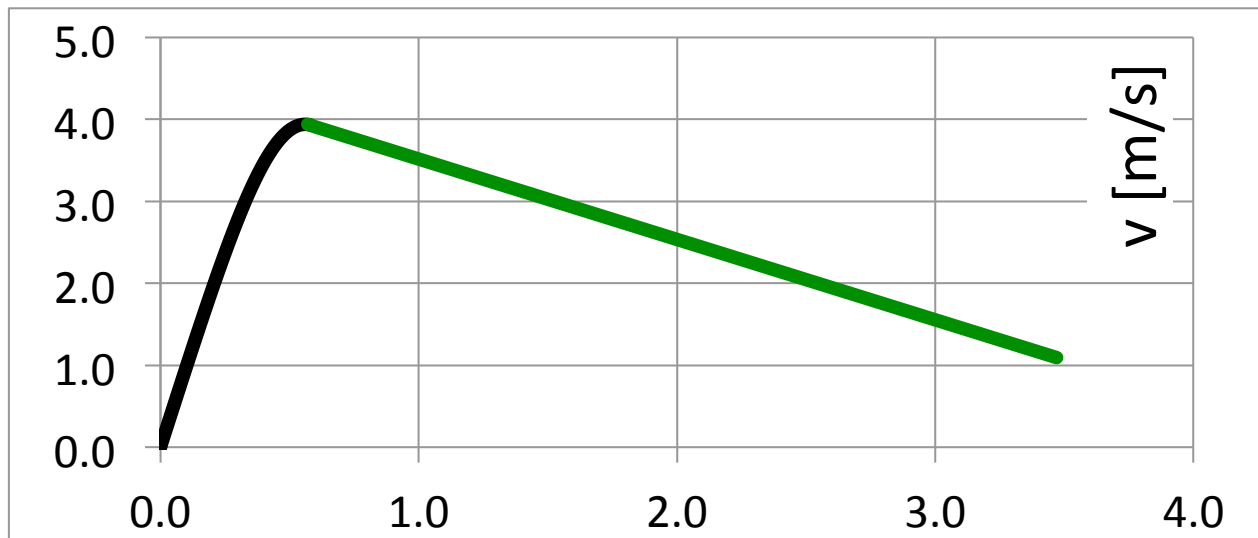
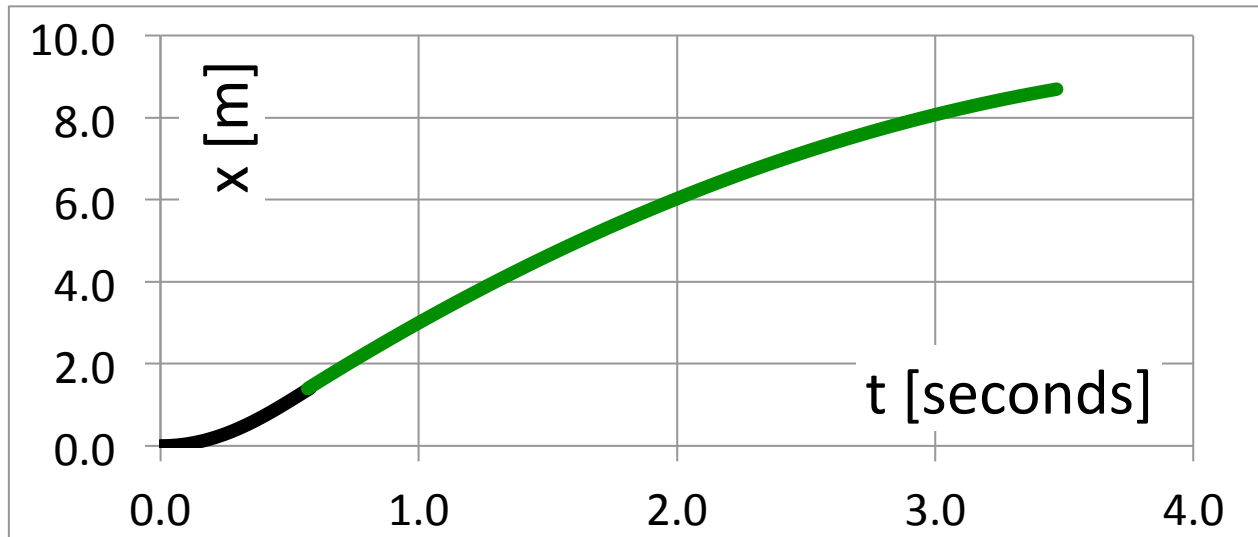
The construction of your physics event should be as close as possible to your theoretical model. This will reduce the number of unknowns and assumptions that have to be made.

4. Test, record, analyze and compare experiments to theory.

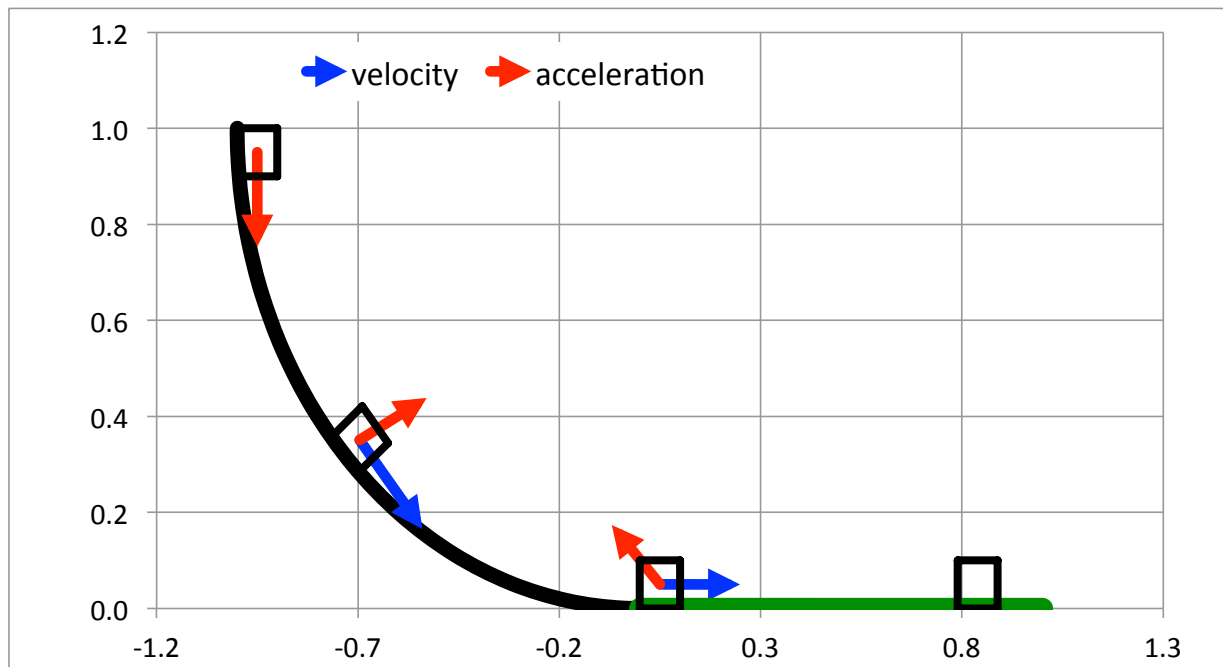
Record trials ensuring that you record how easy the results can change with the initial conditions. Determine the average of your events along with the approximate error. Compare to your theoretical model.

5. (2 marks) Plot relevant theoretical and experimental figures of the system.

Below are the distance-velocity-acceleration time graphs of the system.



Below is the curved surface along with the velocity and acceleration vectors.



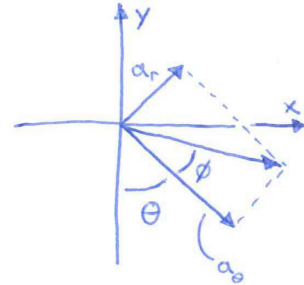
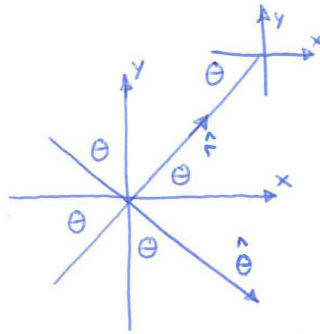
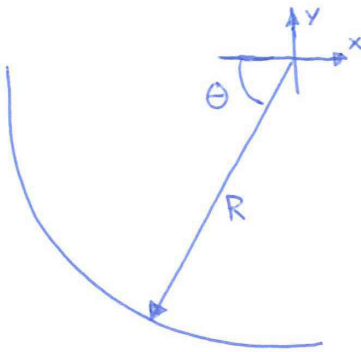
Report

Write up a report (such as this) that includes:

- a picture or drawing of the physics event.
- (2 marks) FBD and angle calculations of the physics event.
- (2 marks) derivation of the acceleration using Newton's 2nd Law.
- (2 marks) analytical or numerical solution of the event.
- (3 marks) construction of the physics event (images).
- (2 marks) a dis-vel-acc graph of the physics event.
- (2 marks) effective communication or physics and results in presentation..
- (2 marks) neatly laid out and formatted document.
- appropriate appendixes with additional data and derivations (Appendix A).
- signed workload distribution (Appendix B).

Appendix A

Angle calculations of tangential and radial components.



Appendix B

Workload Distribution

Name

Signature

Percentage of load

Contribution to Project