Instructor:	Frank Secretain
Course:	Math 2000
Date:	November 27, 2025
Assessment:	Test 3
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 question worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a+b)$$

exponent rules

$$a^n a^m = a^{n+m} \qquad (a^n)^m = a^{nm} \qquad (ab)^n = a^n b^n$$

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n} \qquad a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

logarithmic rules

$$\log_b(b^x) = x \qquad b^{\log_b a} = a \qquad \log_b x = \frac{\log(x)}{\log(b)}$$

$$\log(xy) = \log(x) + \log(y) \qquad \log(x/y) = \log(x) - \log(y)$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$ (expanded form)

$$y = a(x-h)^2 + k$$ (vertex form)

$$y = a(x-m)(x-n)$$ (factored form)

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Rules of differentiation

$$\frac{\mathrm{d}}{\mathrm{dx}}(fg) = \frac{\mathrm{d}(f)}{\mathrm{dx}} g + f \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(product rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{f}{g}\right) = \frac{\frac{\mathrm{d}(f)}{\mathrm{dx}} g - f \frac{\mathrm{d}(g)}{\mathrm{dx}}}{g^2}$$

(quotient rule)

$$\frac{\mathrm{d}}{\mathrm{dx}}(f(g)) = \frac{\mathrm{d}(f)}{\mathrm{dx}} \frac{\mathrm{d}(g)}{\mathrm{dx}}$$

(chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a} cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a} sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)} a^x$$ (exponentials)

$$\int ln(x)dx = xln(x) - x$$

(2 marks) Convert to Polar form:

$$z = \frac{2i}{3} - 1 + i$$

(2 marks) Convert to Rectangular form:

$$z = 3e^{2i}e^{\pi}$$

(10 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2 + i) + 2$$

$$z = (2 - i)e^{\frac{\pi}{4}i}$$

$$z = \frac{e^{2i}}{2 - i}$$

$$z = \sqrt[3]{3 + i}$$

$$z = (2 + i - e^{\frac{\pi}{2} i})^3$$

(6 marks) Solve for z in either rectangular or polar form:

$$3z - iz + 3 = 1$$

$$z(z^2 - 27) + 2(i - 1) + 2 = 2i$$

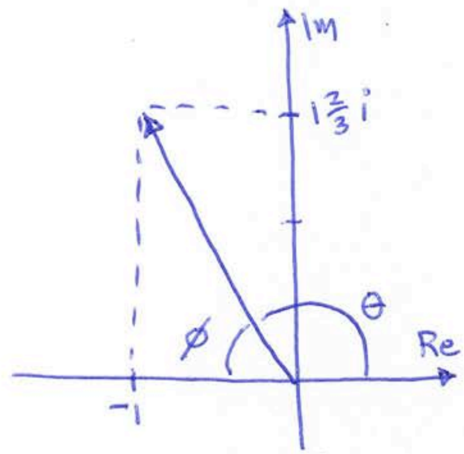
$$z + \frac{1}{z+1} = 0$$

(2 marks) Convert to Polar form:

$$z = \frac{2i}{3} - 1 + i$$

$$= -1 + 1\tfrac{2}{3}i$$

$$\boxed{z = \frac{\sqrt{34}}{3} e^{2.111i}}$$



$$\phi = \tan^{-1}\left(\frac{1\frac{2}{3}}{1}\right) = 1.0304$$

$$\theta = \pi - \phi = \pi - 1.0304$$
$$= 2.111$$

$$R = \sqrt{(1\tfrac{2}{3})^2 + (1)^2} = \frac{\sqrt{34}}{3}$$

(2 marks) Convert to Rectangular form:

$$z = 3e^{2i}e^{\pi}$$

$$= 3e^{\pi} e^{2i}$$

$$= (3e^{\pi})(\cos(2) + i\sin(2))$$

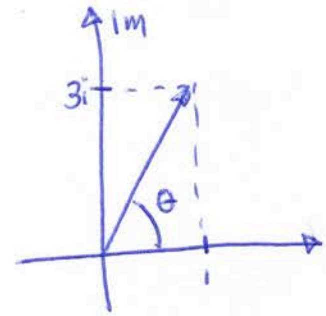
$$= (69.42)(-0.416 + 0.909i)$$

$$= -28.88 + 63.10i$$

$$\boxed{z = -28.9 + 63.1i}$$

(10 marks) Evaluate to either rectangular or polar form:

$$z = i + i(2+i) + 2$$



$= i + 2i + i^2 + 2$

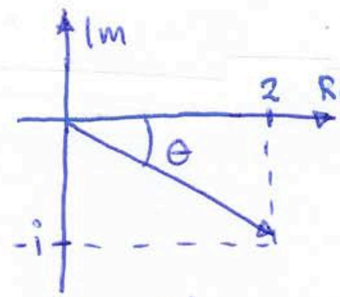
$= 1 + 3i$

$\theta = \tan^{-1}\left(\frac{3}{1}\right) = 1.249$

$R = \sqrt{(1)^2 + (3)^2} = \sqrt{10}$

$$\boxed{\begin{aligned} z = 1 + 3i &= \sqrt{10}\, e^{1.25i} \\ &= 3.16\, e^{1.25i} \end{aligned}}$$

$$z = (2-i)e^{\frac{\pi}{4}i}$$



$= \left(\sqrt{5}\, e^{-0.4636i}\right) e^{\frac{\pi}{4}i}$

$= \sqrt{5}\, e^{\left(-0.4636 + \frac{\pi}{4}\right)i}$

$= \sqrt{5}\, e^{0.3218i}$

$= 2.236\, e^{0.3218i}$

$= 2.236\left(\cos(0.3218) + i\sin(0.3218)\right)$

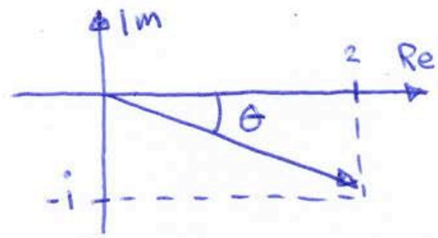
$= 2.121 + 0.7072i$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 0.4636$

$R = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

$$\boxed{\begin{aligned} z = 1.121 + 0.7072i &= \sqrt{5}\, e^{0.3218i} \\ &\doteq 2.236\, e^{0.3218i} \end{aligned}}$$

$$z = \frac{e^{2i}}{2-i}$$



$$= \frac{e^{2i}}{\left(\sqrt{5}\, e^{-0.4636i}\right)}$$

$$= \frac{1}{\sqrt{5}}\, e^{(2-(-0.4636))i}$$

$$= 0.447\, e^{2.4636i}$$

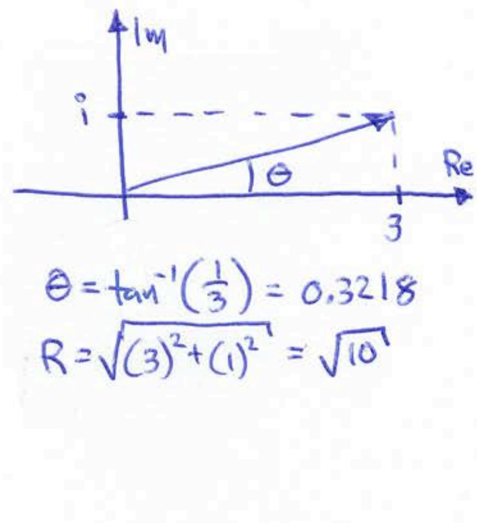
$$= 0.447\left(\cos(2.4636) + i\sin(2.4636)\right)$$

$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 0.4636$

$R = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

$$\boxed{Z = 0.447\, e^{2.4636i} = -0.348 + 0.281i}$$

$$z = \sqrt[3]{3+i}$$



$$= (3+i)^{\frac{1}{3}}$$

$$= \left(\sqrt{10}\, e^{0.3218i}\right)^{\frac{1}{3}}$$

$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 0.3218$

$R = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$

$\theta = 0.3218 + 0$ $\quad = \left(10^{\frac{1}{2}} e^{0.3218i}\right)^{\frac{1}{3}}$ $\quad = 10^{\frac{1}{6}} e^{0.107i}$

$\theta = 0.3218 + 2\pi$ $\quad = \left(10^{\frac{1}{2}} e^{6.605i}\right)^{\frac{1}{3}}$ $\quad = \sqrt[6]{10}\, e^{2.202i}$

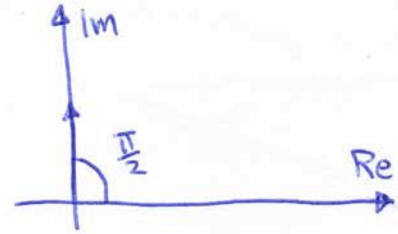
$\theta = 0.3218 + 4\pi$ $\quad = \left(10^{\frac{1}{2}} e^{12.888i}\right)^{\frac{1}{3}}$ $\quad = 1.468\, e^{4.296i}$

$$z = (2 + i - e^{\frac{\pi}{2}i})^3$$

$$= \left(2 + i - (i)\right)^3$$

$$= (2)^3$$

$$= 8$$

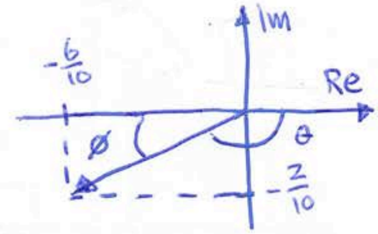


$$e^{\frac{\pi}{2}i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)$$

$$= 0 + i$$

$$\boxed{Z = 8 + 0i}$$

(6 marks) Solve for z in either rectangular or polar form:



$$3z - iz + \overset{-3}{3} = \overset{-3}{1}$$

$$3z - iz = -2$$

$$z(3-i) = -2$$

$$z = \frac{(-2)}{(3-i)}\frac{(3+i)}{(3+i)}$$

$$= \frac{-6-2i}{9+3i-3i-i^2}$$

$$\phi = \tan^{-1}\left(\frac{2/10}{6/10}\right) = 0.3218$$

$$R = \sqrt{\left(\frac{6}{10}\right)^2 + \left(\frac{2}{10}\right)^2} = \sqrt{\frac{2}{5}}$$

$$\theta = \pi - 0.3218 = 2.820$$

$$\boxed{\begin{aligned} z &= -\frac{6}{10} - \frac{2}{10}i \\ &= \sqrt{\frac{2}{5}}\, e^{-2.82i} \end{aligned}}$$

$$z(z^2 - 27) + 2(i-1) + 2 = 2i$$

$$z^3 - 27z + 2i - 2 + 2 = 2i$$

$$z^3 - 27z = 0$$

$$z(z^2 - 27) = 0$$

$$\boxed{z = 0}$$

$$z^2 = 27$$

$$z = (27)^{\frac{1}{2}} = (2$$

$\theta = 0$ $$= (27e^{0i})^{\frac{1}{2}} = 27^{\frac{1}{2}} e^{0i}$$

$\theta = 0 + 2\pi$ $$= (27e^{2\pi i})^{\frac{1}{2}} = 27^{\frac{1}{2}} e^{\pi i}$$

$$\boxed{\begin{aligned} z &= 5.196 \\ z &= -5.196 \end{aligned}}$$

$$z + \frac{1}{z+1} = 0$$

$$z(z+1) + 1 = 0(z+1)$$

$$z^2 + z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{-3}}{2}$$

$$= -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$\boxed{\begin{aligned} z_1 &= -0.5 + 0.866i \\ z_2 &= -0.5 - 0.866i \end{aligned}}$$

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$$\int cos(ax)dx = \frac{1}{a} sin(ax)$$

$$\int tan(ax)dx = ln(|sec(x)|)$$

(trigonometry)

$$\int a^x dx = \frac{1}{ln(a)} a^x$$

$$\int ln(x)dx = xln(x) - x$$

(exponentials)

(2 marks) Convert to Polar form:

$$z = \frac{i}{2} - \frac{3}{2} + 2i$$

(2 marks) Convert to Rectangular form:

$$z = 3e^{i\pi}e^{2+i}$$

(10 marks) Evaluate to either rectangular or polar form:

$$z = 2i + i(3 - 2i) - 1$$

$$z = (1 - i)e^{\frac{3i}{2}}$$

$$z = \frac{3}{1+i} - i$$

$$z = \sqrt{2+i}$$

$$z = (2 + 1)^{2i}$$

(6 marks) Solve for z in either rectangular or polar form:

$$2z + zi - 4 = i$$

$$z^3 + 1 + i = 9 + i$$

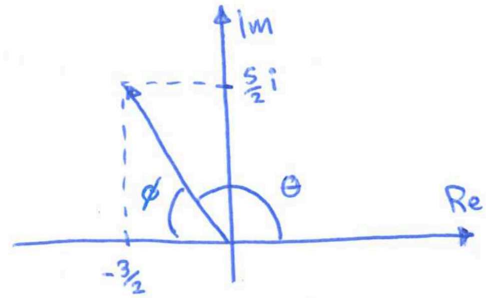
$$\frac{1}{z} + 2z = 1$$

(2 marks) Convert to Polar form:

$$z = \frac{i}{2} - \frac{3}{2} + 2i$$

$$= -\frac{3}{2} + \frac{5}{2}i$$

$$\boxed{= \frac{1}{2}\sqrt{34}\, e^{2.11i} = 2.92\, e^{2.11i}}$$



$$\phi = \tan^{-1}\left(\frac{5}{2}\Big/\frac{3}{2}\right) = 1.030$$

$$\theta = \pi - 1.030 = 2.111$$

$$R = \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{\frac{34}{4}} = \frac{1}{2}\sqrt{34}$$

(2 marks) Convert to Rectangular form:

$$z = 3e^{i\pi}e^{2+i}$$

$$= 3e^{i\pi + 2 + i}$$

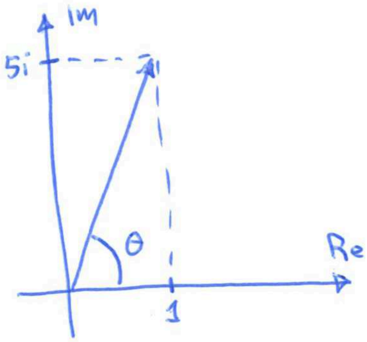
$$= 3e^{2 + i(1+\pi)}$$

$$= 3e^{2}e^{i(1+\pi)}$$

$$= 22.17\, e^{4.141i}$$

$$\boxed{= -11.98 - 18.65i}$$

(10 marks) Evaluate to either rectangular or polar form:



$$z = 2i + i(3 - 2i) - 1$$

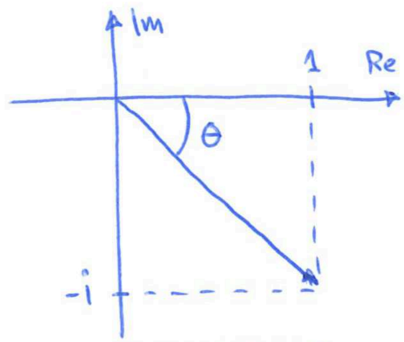
$= 2i + 3i - 2i^2 - 1$

$= 1 + 5i$

$\theta = \tan^{-1}\left(\frac{5}{1}\right) = 1.373$

$R = \sqrt{1^2 + 5^2} = \sqrt{26}$

$$\boxed{\begin{aligned} Z = 1 + 5i &= \sqrt{26}\, e^{1.373i} \\ &= 5.10\, e^{1.373i} \end{aligned}}$$



$$z = (1 - i)e^{\frac{3i}{2}}$$

$= \left(\sqrt{2}\, e^{-\frac{\pi}{4}i}\right) e^{\frac{3i}{2}}$

$= \sqrt{2}\, e^{-\frac{\pi}{4}i}\, e^{\frac{3}{2}i}$

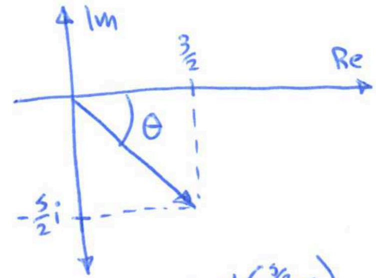
$= \sqrt{2}\, e^{\left(\frac{3}{2} - \frac{\pi}{4}\right)i}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$R = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$$\boxed{\begin{aligned} &= \sqrt{2}\, e^{\left(\frac{3}{2} - \frac{\pi}{4}\right)i} \\ &= 1.414\, e^{0.715i} = 1.068 + 0.927i \end{aligned}}$$

$$z = \frac{3}{1+i} - i$$



$$\theta = \tan^{-1}\left(\frac{5/2}{3/2}\right) = 1.030$$
$$R = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{5}{2}\right)^2} = \frac{1}{2}\sqrt{34}$$

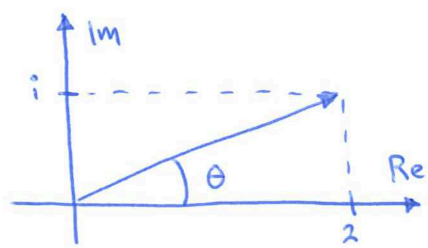
$$= \left(\frac{3}{1+i}\right)\left(\frac{1-i}{1-i}\right) - i$$

$$= \frac{3-3i}{1-\cancel{i}+\cancel{i}-i^2} - i$$

$$= \frac{3}{2} - \frac{3}{2}i - i$$

$$\boxed{\begin{aligned} Z = \frac{3}{2} - \frac{5}{2}i &= \frac{1}{2}\sqrt{34}\, e^{-1.030 i} \\ &= 2.915\, e^{-1.030 i} \end{aligned}}$$

$$z = \sqrt{2+i}$$



$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 0.4636$$
$$R = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$= (2+i)^{\frac{1}{2}}$$

$$= \left(\sqrt{5}\, e^{0.4636 i}\right)^{\frac{1}{2}}$$

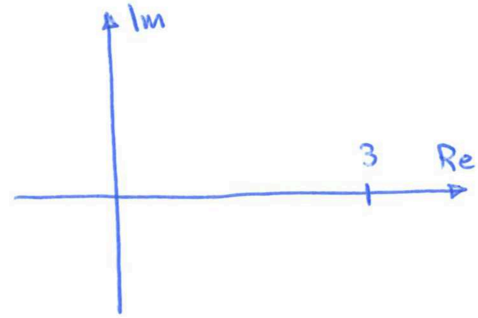
$\theta = 0.4636$

$$\boxed{Z = \sqrt[4]{5}\, e^{0.232 i} = 1.46 + 0.344 i}$$

$\theta = 0.4636 + 2\pi$

$$\boxed{Z = \sqrt[4]{5}\, e^{3.373 i} = -1.46 - 0.344 i}$$

$z = (2+1)^{2i}$



$= 3^{2i}$

$= \left(3e^{i0}\right)^{2i}$

$= 3^{2i} e^{0}$

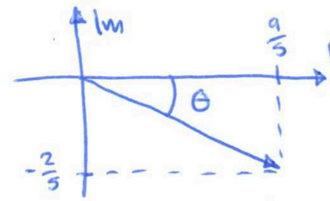
$= 3^{2i}$

$= e^{\ln\left(3^{2i}\right)}$

$= e^{2i \ln(3)}$

$$\boxed{\begin{aligned} Z &= e^{2\ln(3)\, i} \\ &= e^{2.197\, i} \end{aligned}}$$

(6 marks) Solve for z in either rectangular or polar form:



$\theta = \tan^{-1}\left(\frac{2/5}{9/5}\right) = 0.219$

$R = \sqrt{\left(\frac{9}{5}\right)^2 + \left(-\frac{2}{5}\right)^2} = \frac{1}{5}\sqrt{85}$

$$2z + zi - 4 = i$$

$$z(2+i) = i + 4$$

$$z = \frac{4+i}{2+i}\left(\frac{2-i}{2-i}\right) = \frac{8 - 4i + 2i - i^2}{4 - \cancel{2i} + \cancel{2i} - i^2}$$

$$\boxed{z = \frac{9}{5} - \frac{2}{5}i = \frac{1}{5}\sqrt{85}\, e^{-0.219i} = 1.844\, e^{-0.219i}}$$

$$z^3 + 1 + i = 9 + i$$

$\theta = 0 \ , \ R = 8$

$$z^3 = 8$$

$$z = (8)^{\frac{1}{3}}$$

$\theta = 0$ $\quad = \left(8e^{i0}\right)^{\frac{1}{3}} = 2e^{i0} = 2 + 0i$

$\theta = 0 + 2\pi$ $\quad = \left(8e^{2\pi i}\right)^{\frac{1}{3}} = 2e^{\frac{2\pi}{3}i} = -1 + 1.732i$

$\theta = 0 + 4\pi$ $\quad = \left(8e^{4\pi i}\right)^{\frac{1}{3}} = 2e^{\frac{4\pi}{3}i} = -1 - 1.732i$

$$\frac{1}{z} + 2z = 1$$

$$1 + 2z^2 = z$$

$$2z^2 - z + 1 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 8}}{4}$$

$$= \frac{1}{4} \pm \frac{1}{4}\sqrt{-7}$$

$$= \frac{1}{4} \pm \frac{\sqrt{7}}{4} i$$

$$\boxed{\begin{aligned} z &= \frac{1}{4} \pm \frac{\sqrt{7}}{4} i \\ &= 0.25 \pm 0.66 i \end{aligned}}$$