

Instructor: Frank Secretain
 Course: Math 20
 Date: September 29, 2026

Assessment: Test 1
 Time allowed: 110 minutes
 Devices allowed: Pencil, pen, eraser, calculator
 Notes from instructor: Be neat. Show your work where needed. Box final answers.

Marks allocated: 5 questions worth 20 marks + 1 BONUS worth 1 mark
 Percentage of final grade: 20% of final grade

Constant rule	$\frac{d}{dx}(C) = 0$
Multiplier rule	$\frac{d}{dx}(C \cdot f(x)) = C \cdot f'(x)$
Sum/difference rule	$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
Product rule	$\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$
Quotient rule	$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$
Chain rule	$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$
Power rule	$\frac{d}{dx}(x^n) = nx^{n-1}$
Exponent rule	$\frac{d}{dx}(e^x) = e^x \quad \frac{d}{dx}(b^x) = b^x \cdot \ln b$
Logarithm rule	$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \frac{d}{dx}(\log_b x) = \frac{1}{x} \cdot \frac{1}{\ln b}$
Trigonometric rules	$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x$

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n - 1)k$$

$$S_n = \sum_{i=1}^n a_1 + (i - 1)k$$
$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^n a_1 r^{i-1}$$
$$= a_1 \frac{1 - r^n}{1 - r}$$

Binomial Theorem

$$(x + y)^n =$$

$$\sum_{k=0}^n \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x) \frac{d}{dx}(g(x)) + g(x) \frac{d}{dx}(f(x)) \quad (\text{product rule})$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx}(f(x)) - f(x) \frac{d}{dx}(g(x))}{(g(x))^2} \quad (\text{quotient rule})$$

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x))) \frac{d}{dx}(g(x)) \quad (\text{chain rule})$$

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \frac{1}{(\cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1} x^{n+1} & , n \neq -1 \\ \ln(|x|) & , n = -1 \end{array} \right\} \quad (\text{polynomials})$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) \quad (\text{trigonometry})$$

$$\int \tan(ax) dx = \ln(|\sec(x)|)$$

$$\int a^x dx = \frac{1}{\ln(a)} a^x$$

$$\int \ln(x) dx = x \ln(x) - x \quad (\text{exponentials})$$

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!} (x - x_o)^n$$

Integration by parts

$$\int u dv = uv - \int v du$$

(2 marks each) Determine the 85th number in the sequence and the sum from the first number to the 85th number for each of the following series.

5, 5.5, 6.05, ...

5, 5.5, 6.0, ...

(4 marks) A technician tests 12 circuit boards on the first day. Each day after that, she tests 1 more boards than the previous day. After how many days will she have tested 29337 circuit boards in total?

(1 mark each) Determine the limits of the following expressions:

$$\lim_{x \rightarrow 2} \frac{3x - 6}{x^2 - 5x + 6}$$

$$\lim_{x \rightarrow \infty} \frac{3x - 6}{x^2 - 5x + 6}$$

(2 marks) Take the derivative with respect to “x” of the following function **using the definition of the derivative**.

$$y(x) = 2x + b$$

(2 marks each) Determine the derivative with respect to x of the following equation.

$$y(x) = 2x^3 - \sin(x) + 7$$

$$y(x) = x \sin(x) - \frac{b^2}{x^2}$$

$$y(x) = a^2 x^2 \cos(x) + \frac{3x}{b-1}$$

$$y(x) = 4 \left[\sin \left(3x^2 \right) \right]^3$$

BONUS

(1 mark) Determine the derivative with respect to x of the following equation.

$$y^2 + \sin(y) = x^3$$

(2 marks each) Determine the 85th number in the sequence and the sum from the first number to the 85th number for each of the following series.

5, 5.5, 6.05, ...

$$a_1 = 5$$

$$a_n =$$

$$n = 85$$

$$K = \frac{5.5 - 5}{6.05 - 5.5} = 0.5 \quad \times$$

$$r = \frac{5.5/5}{6.05/5.5} = 1.1 \quad \checkmark \text{ (geometric)}$$

$$S_n =$$

$$a_n = a_1 r^{n-1} \\ = 5(1.1)^{85-1}$$

$$a_n = 14995$$

$$S_n = a_1 \frac{1 - r^n}{1 - r} \\ = 5 \frac{1 - (1.1)^{85}}{1 - 1.1}$$

$$S_n = 164898$$

5, 5.5, 6.0, ...

$$a_1 = 5$$

$$a_n =$$

$$n = 85$$

$$K = \frac{5.5 - 5}{6.0 - 5.5} = 0.5 \quad \checkmark \text{ (arithmetic)}$$

$$r = \frac{5.5/5}{6.0/5.5} = 1.1 \quad \times$$

$$S_n =$$

$$a_n = a_1 + (n-1)K \\ = 5 + (85-1)(0.5)$$

$$a_n = 47$$

$$S_n = \frac{n}{2} (a_1 + a_n) \\ = \frac{85}{2} (5 + 47)$$

$$S_n = 2210$$

(4 marks) A technician tests 12 circuit boards on the first day. Each day after that, she tests 1 more board than the previous day. After how many days will she have tested 29337 circuit boards in total?

$$12, 13, 14, \dots$$

$$a_1 = 12$$

$$a_n =$$

$$n =$$

$$k = \frac{13-12}{14-13} = 1 \quad \checkmark \quad (\text{arithmetic})$$

$$r = \frac{13}{12} = 1.083 \quad \times$$

$$r = \frac{14}{13} = 1.077 \quad \times$$

$$S_n = 29337$$

$$a_n = a_1 + (n-1)k \quad (1)$$

$$S_n = \frac{n}{2}(a_1 + a_n) \quad (2)$$

Sub (1) into (2) and rearrange:

$$S_n = \frac{n}{2}(a_1 + [a_1 + (n-1)k])$$

$$2S_n = n(a_1 + a_1 + kn - k)$$

$$2S_n = 2a_1n + kn^2 - kn$$

$$0 = kn^2 + 2a_1n - kn - 2S_n$$

sub numbers and solve quadratic:

$$0 = n^2 + 2[12]n - n - 2[29337]$$

$$0 = n^2 + 23n - 58674$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-23 \pm \sqrt{(23)^2 + (4)(58674)}}{2}$$

$$n = 231, -254$$

$$n = 231 \text{ days}$$

(1 mark each) Determine the limits of the following expressions:

$$\lim_{x \rightarrow 2} \frac{3x - 6}{x^2 - 5x + 6} = \frac{2(2) - 6}{2^2 - 5(2) + 6} = \frac{0}{0}$$

factor $x^2 - 5x + 6 = (x - 2)(x - 3)$

$$\lim_{x \rightarrow 2} \frac{3(\cancel{x-2})}{(\cancel{x-2})(x-3)} = \frac{3}{2-3} = -3$$

$$\boxed{\lim_{x \rightarrow 2} \frac{3x-6}{x^2-5x+6} = -3}$$

$$\lim_{x \rightarrow \infty} \frac{3x - 6}{x^2 - 5x + 6} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{(3x-6) \left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x} - \frac{6}{x^2}}{1 - \frac{5}{x} + \frac{6}{x^2}} = 0$$

$$\boxed{\lim_{x \rightarrow \infty} \frac{3x-6}{x^2-5x+6} = 0}$$

(2 marks) Take the derivative with respect to "x" of the following function **using the definition of the derivative**.

$$y(x) = 2x + b$$

$$\begin{aligned} y(x) &= 2x + b \\ y(x+\Delta x) &= 2(x+\Delta x) + b \\ &= 2x + 2\Delta x + b \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{y(x+\Delta x) - y(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{[2x + 2\Delta x + b] - [2x + b]}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\cancel{2x} + 2\Delta x + \cancel{b} - \cancel{2x} - \cancel{b}}{\Delta x} \end{aligned}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} = 2$$

$$\boxed{\frac{dy}{dx} = 2}$$

(2 marks each) Determine the derivative with respect to x of the following equation.

$$y(x) = 2x^3 - \sin(x) + 7$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = 6x^2 - \cos(x)$$

$$y(x) = x \sin(x) - \frac{b^2}{x^2}$$

$$y(x) = x \sin(x) - b^2 x^{-2}$$

$\frac{d}{dx}$

$$\begin{aligned} \frac{dy}{dx} &= \sin(x) + x \cos(x) + 2b^2 x^{-3} \\ &= \sin(x) + x \cos(x) + \frac{2b^2}{x^3} \end{aligned}$$

$$y(x) = a^2 x^2 \cos(x) + \frac{3x}{b-1}$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = 2a^2 x \cos(x) - a^2 x^2 \sin(x) + \frac{3}{b-1}$$

$$y(x) = 4 \left[\sin(3x^2) \right]^3$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = 12 \left[\sin(3x^2) \right]^2 \left[\cos(3x^2) \right] [6x]$$

$$= 72x \cos(3x^2) \sin^2(3x^2)$$

BONUS

(1 mark) Determine the derivative with respect to x of the following equation.

$$y^2 + \sin(y) = x^3$$

$\frac{d}{dx}$

$$2y \frac{dy}{dx} + \cos(y) \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} (2y + \cos(y)) = 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2}{2y + \cos(y)}}$$