| | |
|---|---|
| Instructor: | Frank Secretain |
| Course: | Math 20 |
| Date: | November 24/27, 2025 |
| Assessment: | Test 3 |
| Time allowed: | 110 minutes |
| Devices allowed: | Pencil, pen, eraser, calculator |
| Notes from instructor: | Be neat. Show your work where needed. Box final answers. |
| Marks allocated: | 3 questions worth 20 marks |
| Percentage of final grade: | 20% of final grade |

**DERIVATIVE RULES**

| | |
|---|---|
| Constant rule | $\frac{d}{dx}(C) = 0$ |
| Multiplier rule | $\frac{d}{dx}\left(C \cdot f(x)\right) = C \cdot f'(x)$ |
| Sum/difference rule | $\frac{d}{dx}\left(f(x) \pm g(x)\right) = f'(x) \pm g'(x)$ |
| Product rule | $\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f'(x)g(x) + f(x)g'(x)$ |
| Quotient rule | $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - g'(x)f(x)}{\left(g(x)\right)^2}$ |
| Chain rule | $\frac{d}{dx}f\left(g(x)\right) = f'\left(g(x)\right) \cdot g'(x)$ |
| Power rule | $\frac{d}{dx}(x^n) = nx^{n-1}$ |
| Exponent rule | $\frac{d}{dx}(e^x) = e^x$ $\quad$ $\frac{d}{dx}(b^x) = b^x \cdot \ln b$ |
| Logarithm rule | $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad$ $\frac{d}{dx}(\log_b x) = \frac{1}{x} \cdot \frac{1}{\ln b}$ |
| Trigonometric rules | $\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$ |

**INTEGRAL RULES**

| | |
|---|---|
| Multiplier rule | $\int C \cdot f(x)dx = C \cdot \int f(x)dx$ |
| Sum/difference rule | $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$ |
| Power rule | $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ $\quad$ $n \neq 1$ |
| $1/x$ rule | $\int x^{-1} \, dx = \ln\lvert x\rvert + C$ |
| Exponent rule | $\int e^x dx = e^x + C$ $\quad$ $\int b^x dx = \frac{b^x}{\ln b} + C$ |
| Trigonometric rules | $\int \cos x dx = \sin x + C$ $\quad$ $\int \sin x \, dx = -\cos x + C$ |
| Integration by parts | $\int u \, dv = uv - \int v du$ |
| Fundamental theorem | $\int_a^b f(x)dx = F(b) - F(a)$ $\quad$ $F'(x) = f(x)$ |
| Area rule | $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ |

# Formula Sheet

**Arithmetic Series**

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$

$$= \frac{n}{2}(a_1 + a_n)$$

**Geometric Series**

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$

$$= a_1 \frac{1-r^n}{1-r}$$

**Binomial Theorem**

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

**Line equation**

$$y = ax + b$$

**Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Definition of the derivative**

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

**Rules of differentiation**

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

**Derivatives of select functions**

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

**Integrals of select functions**

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$

$$\int ln(x)dx = xln(x) - x$$ (exponentials)

**Taylor series expansion**

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

**Integration by parts**

$$\int udv = uv - \int vdu$$

(2 marks each) Integrate the following:

$$y = \int 3x^2 + \cos(3x) + 1 \quad dx$$

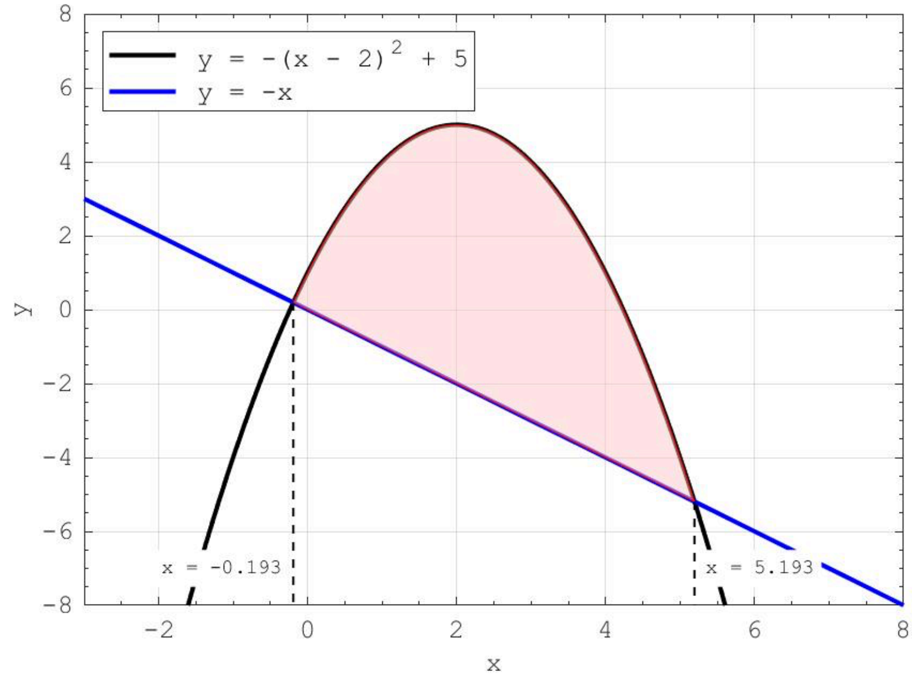
$$y = \int \frac{2x}{(3x^2 - 2)^3} \quad dx$$

$$y = \int x + \sin(x + b) - 4 \quad dx$$

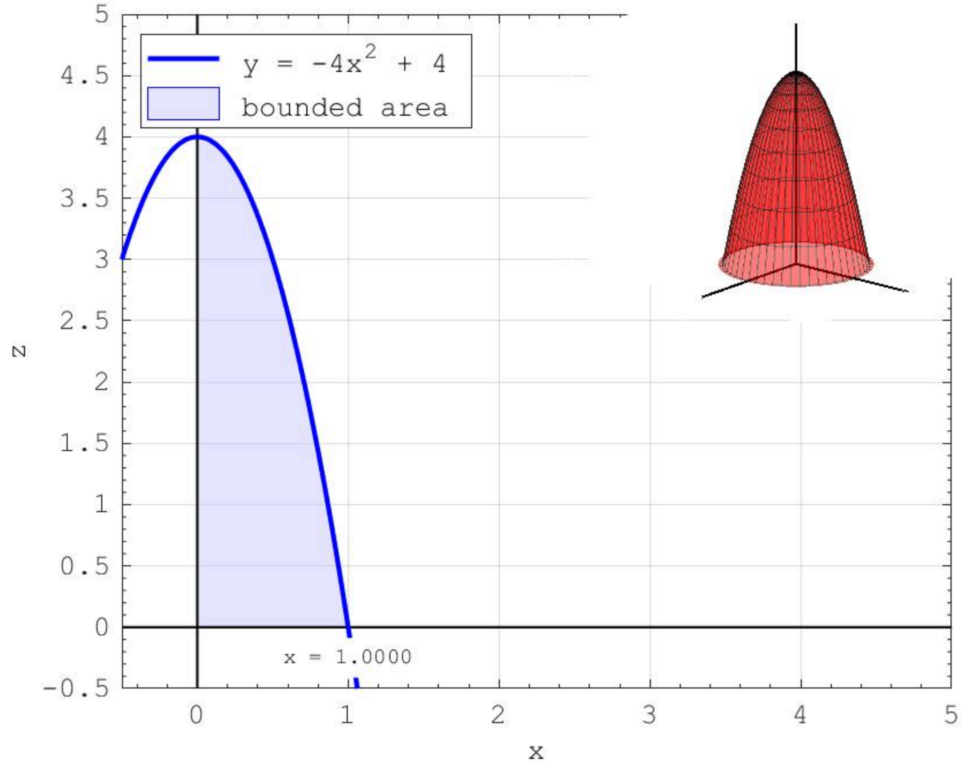
$$y = \int \frac{3b^2}{4+n} \sin(4x+1) + C \quad dx$$

$$y = \int 2x \cos(2x) \quad dx$$

(5 marks) Determine the shaded area of the two functions given in the legend.



(5 marks) Calculate the volume of the bounded area of the function given in the legend (shaded in blue) revolved around the z-axis (as shown in the top left corner shaded in red).



(2 marks each) Integrate the following:

$$y = \int 3x^2 + \cos(3x) + 1 \;\; dx$$

$$\boxed{= x^3 + \frac{1}{3}\sin(3x) + x + C}$$

$$y = \int \frac{2x}{(3x^2 - 2)^3} \;\; dx$$

let $u = 3x^2 - 2$

$\frac{du}{dx} = 6x$

and $dx = \frac{du}{6x}$

$$= \int \frac{2x}{u^3} \frac{du}{6x}$$

$$= \frac{1}{3} \int u^{-3} \, du$$

$$= \frac{1}{3}\left[\frac{1}{-2} u^{-2}\right] + C$$

$$\boxed{= -\frac{1}{6(3x^2-2)^2} + C}$$

$$y = \int x + \sin(x + b) - 4 \;\; dx$$

$$\boxed{= \frac{1}{2}x^2 - \cos(x+b) - 4x + C}$$

$$y = \int \frac{3b^2}{4+n} \sin(4x+1) + C \;\; dx$$

$$\boxed{= \frac{-3b^2}{4(4+n)} \cos(4x+1) + Cx + D}$$

$$y = \int 2x \cos(2x) \;\; dx$$

let $u = 2x$ $\qquad \int dv = \int \cos(2x)\, dx$

$\frac{du}{dx} = 2$ $\qquad v = \frac{1}{2}\sin(2x)$

$du = 2\,dx$

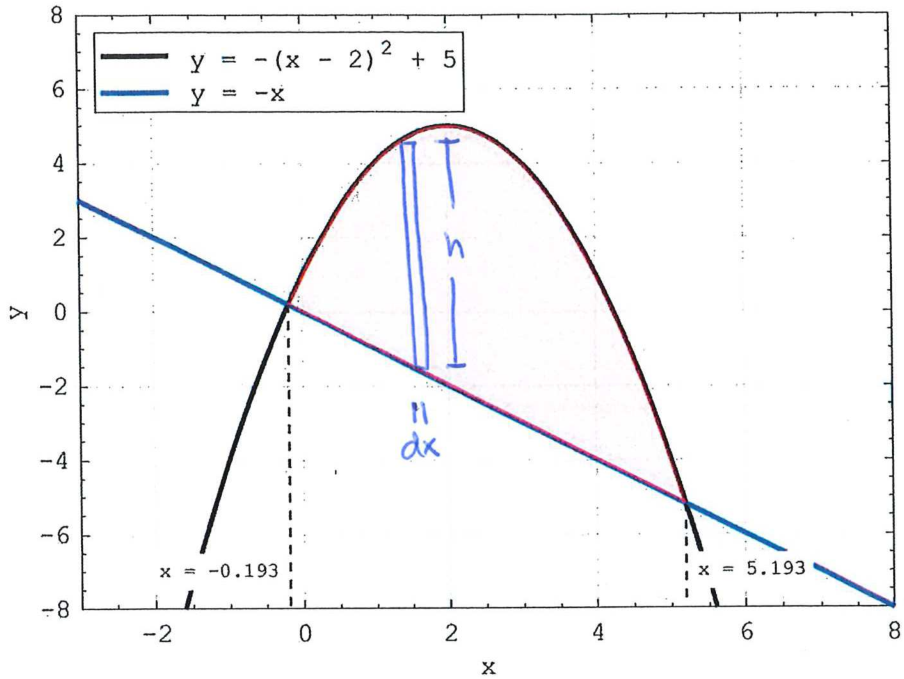
$$= uv - \int v\, du$$

$$= (2x)\left(\tfrac{1}{2}\sin(2x)\right) - \int \tfrac{1}{2}\sin(2x)\, 2dx$$

$$= x\sin(2x) - \int \sin(2x)\, dx$$

$$\boxed{= x\sin(2x) + \tfrac{1}{2}\cos(2x) + C}$$

(5 marks) Determine the shaded area of the two functions given in the legend.




using columns:

$$A = \int h \, dx \qquad h = y_{top} - y_{bottom} = (-(x-2)^2+5) - (-x)$$

$$= -(x-2)^2 + 5 + x$$

$$= \int_{-0.193}^{5.193} -(x-2)^2 + 5 + x \, dx$$

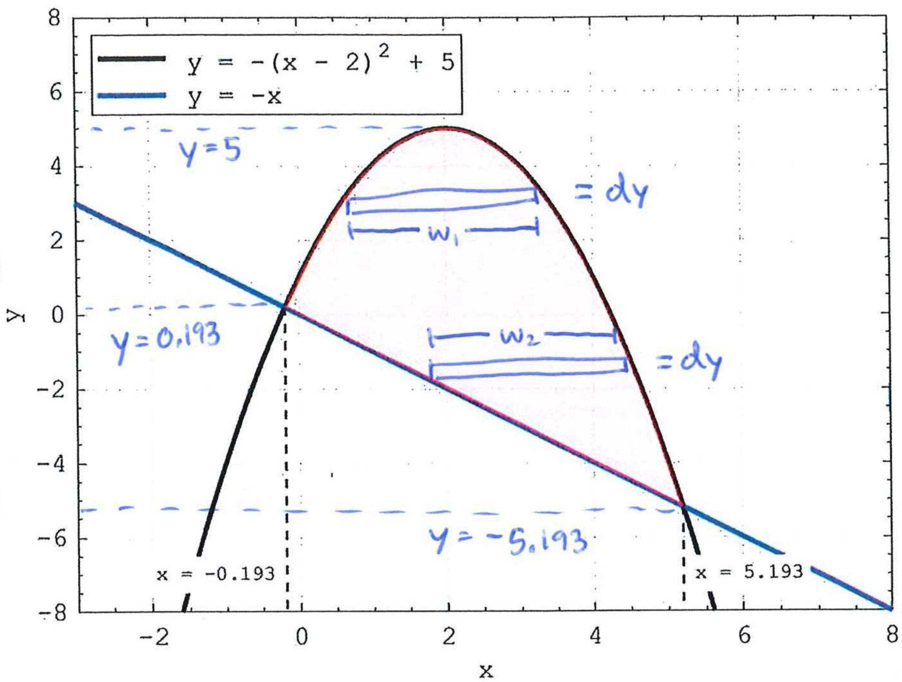
$$= \left[ -\frac{1}{3}(x-2)^3 + 5x + \frac{1}{2}x^2 \right]_{-0.193}^{5.193}$$

$$= \left[ -\frac{1}{3}([5.193]-2)^3 + 5[5.193] + \frac{1}{2}[5.193]^2 \right] - \left[ -\frac{1}{3}([-0.193]-2)^3 + 5[-0.193] + \frac{1}{2}[-0.193]^2 \right]$$

$$= \left[ -10.851 + 25.965 + 13.484 \right] - \left[ 3.5156 - 0.965 + 0.0186 \right]$$

$$\boxed{= 26.0 \text{ units}^2}$$

(5 marks) Determine the shaded area of the two functions given in the legend.




Solve for x:

$y_{top} = -(x-2)^2 + 5$

$x = \pm\sqrt{5-y} + 2$ (1)

$y_{bottom} = -x$

$x = -y$ (2)

using rows:

$A = \int w_1\,dy + \int w_2\,dy$

$= \int_{0.193}^{5} \sqrt{5-y}+2 - -\sqrt{5-y}-2\,dy + \int_{-5.193}^{0.193} \sqrt{5-y}+2-(-x)\,dx$

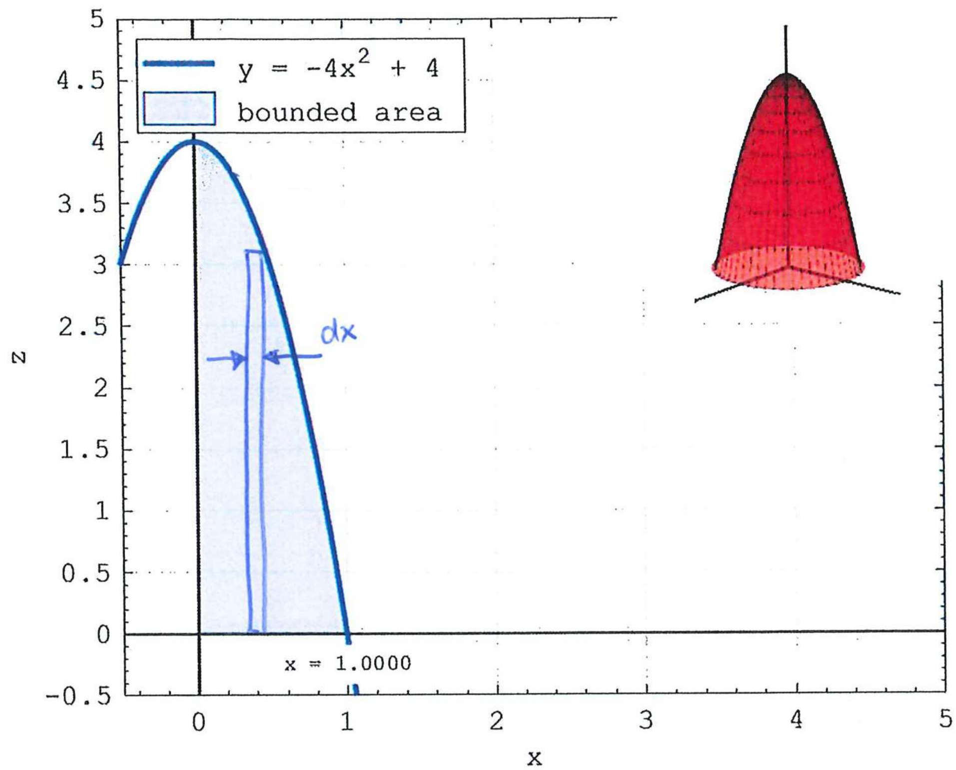
$= \left[-\frac{2}{3}[5-y]^{\frac{3}{2}} - \frac{2}{3}[5-y]^{\frac{3}{2}}\right]_{0.193}^{5} + \left[-\frac{2}{3}[5-y]^{\frac{3}{2}} + 2x + \frac{1}{2}x^2\right]_{-5.193}^{0.193}$

$= \left[-\frac{4}{3}[5-[5]]^{\frac{3}{2}}\right] - \left[-\frac{4}{3}[5-[0.193]]^{\frac{3}{2}}\right] + \left[-\frac{2}{3}[5-[0.193]]^{\frac{3}{2}} + 2[0.193] + \frac{1}{2}[0.193]^2\right] -$
$\left[-\frac{2}{3}[5-[-5.193]]^{\frac{3}{2}} + 2[-5.193] + \frac{1}{2}[-5.193]^2\right]$

$= [0] - [-14.052] + [-6.622] - [-18.597]$

$\boxed{= 26.0 \text{ units}^2}$

(5 marks) Calculate the volume of the bounded area of the function given in the legend (shaded in blue) revolved around the z-axis (as shown in the top left corner shaded in red).




using shells:

$$V = \int 2\pi r h \, dx$$

$$= \int_0^1 2\pi x\left(-4x^2+4\right) dx$$

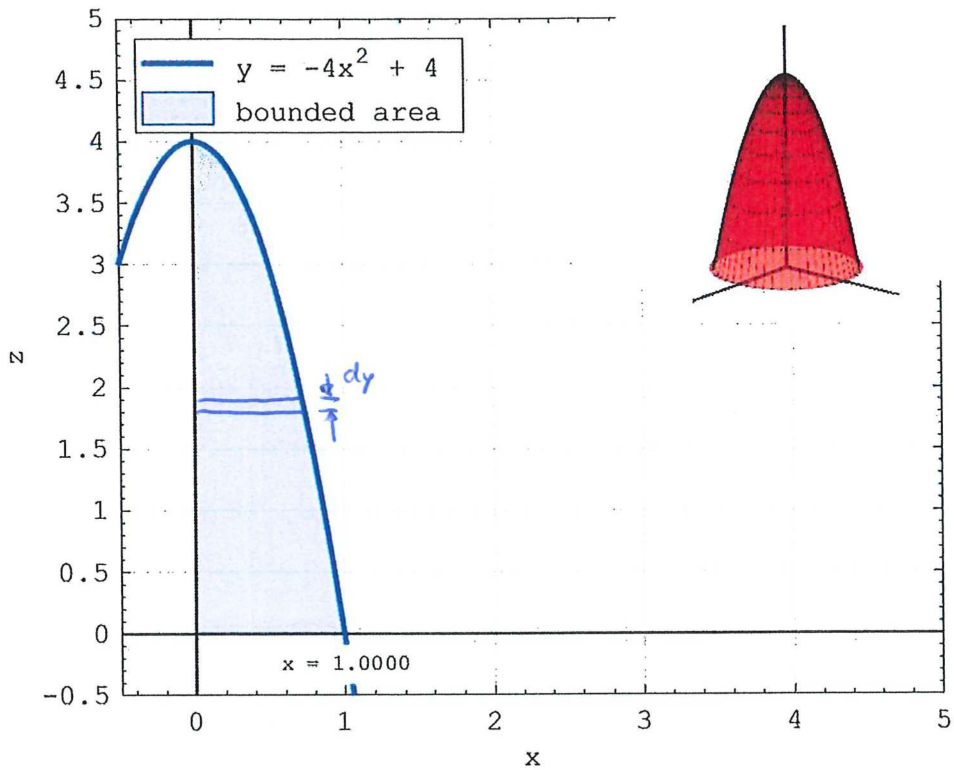
$$= \int_0^1 -8\pi x^3 + 8\pi x \; dx$$

$$= \left[-2\pi x^4 + 4\pi x^2\right]_0^1$$

$$= \left[-2\pi[1]^4 + 4\pi[1]^2\right] - \left[-2\pi[0]^4 + 4\pi[0]^2\right]$$

$$\boxed{= 2\pi = 6.28 \text{ units}^3}$$

(5 marks) Calculate the volume of the bounded area of the function given in the legend (shaded in blue) revolved around the z-axis (as shown in the top left corner shaded in red).




solve for x:

$$y = -4x^2 + 4$$

$$x = \pm\sqrt{1 - \frac{y}{4}}$$

using disks:

$$\forall = \int \pi r^2 \, dy$$

$$= \int_0^4 \pi \left[\sqrt{1-\frac{y}{4}}\right]^2 dy$$

$$= \int_0^4 \pi - \frac{\pi y}{4} \, dy$$

$$= \left[\pi x - \frac{\pi y^2}{8}\right]_0^4$$

$$= \left[\pi[4] - \frac{\pi[4]^2}{8}\right] - \left[\pi[0] - \frac{\pi[0]^2}{8}\right]$$

$$\boxed{= 2\pi = 6.28 \text{ units}^2}$$