Instructor:	Frank Secretain
Course:	Math 20
Date:	October 16/17, 2025
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$

$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$

$$= a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{cases} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{cases}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$ (exponentials)

$$\int ln(x)dx = xln(x) - x$$

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

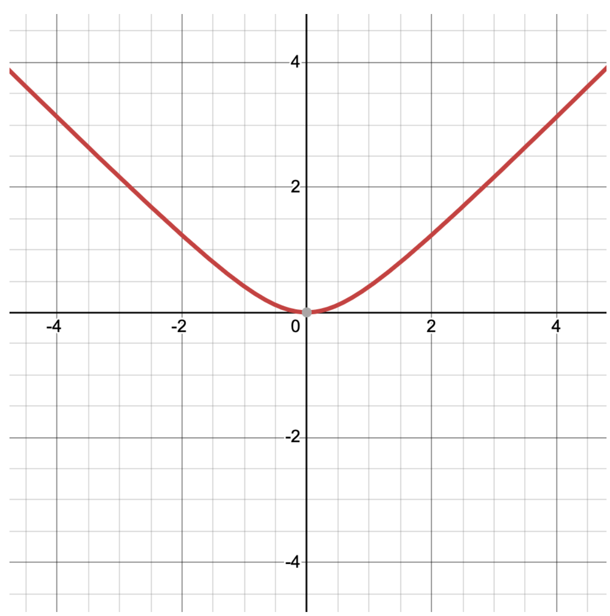
(2 marks each) Determine the derivative with respect to x of the following equation.

$$y(x) = x^2 \sin\left(2x^3 + \frac{1}{x}\right)$$

$$y^2 + xy - a = \sin(x^2)$$

(5 marks) Determine the tangent line at x=1.0 for the following function. Plot the tangent line on the plot.

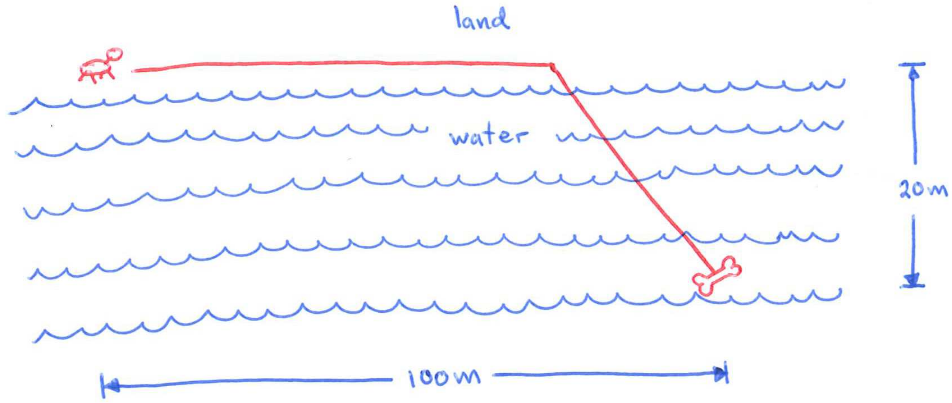
$$y = \sqrt{x^2 + 1} - 1$$



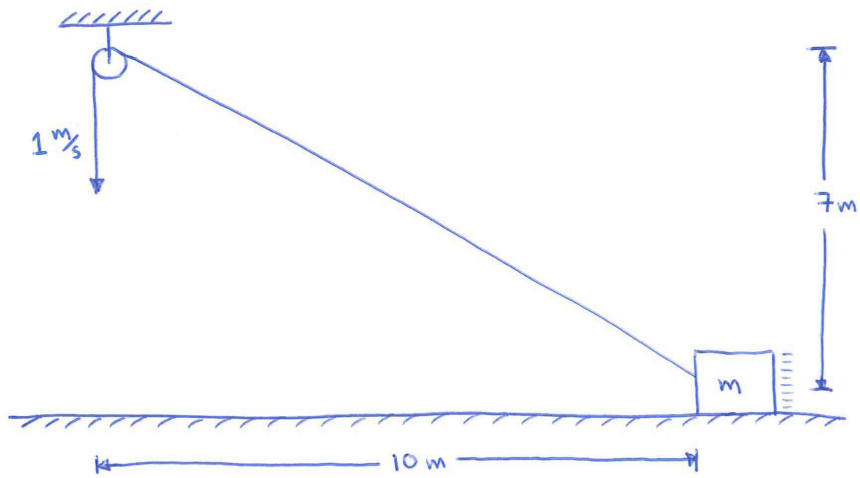
(5 marks) A dog is on the shoreline (on land) and wants to retrieve its bone, which is in the water, as shown below. The dog can run on land at a speed of 12 m/s but can only swim at 7 m/s. What path should the dog take (i.e., how far should it travel on land versus in the water) to minimize the time it takes to reach the bone?

Remember: $v = \frac{x}{t}$

where v is velocity,
x is distance travelled
and t is time.



(5 marks) Suppose you are dragging a mass (m) across the floor by the use of a rope and pulley, as shown in the below figure. If you are pulling the rope at a rate of 1 m/s how fast is the block sliding across the floor when at the position shown in the figure?



(2 marks each) Determine the derivative with respect to x of the following equation.

$$y(x) = x^2 \sin\left(2x^3 + \frac{1}{x}\right)$$

$y(x) = x^2 \sin\left(2x^3 + x^{-1}\right)$

$\frac{d}{dx}$

$$\frac{dy}{dx} = 2x \sin\left(2x^3 + x^{-1}\right) + x^2 \cos\left(2x^3 + x^{-1}\right)\left(6x^2 - x^{-2}\right)$$

$$y^2 + xy - a = \sin(x^2)$$

$\frac{d}{dx}$

$$2y\frac{dy}{dx} + y + x\frac{dy}{dx} = \cos\left(x^2\right)\left(2x\right)$$

(5 marks) Determine the tangent line at x=1.0 for the following function. Plot the tangent line on the plot.

$$y = \sqrt{x^2 + 1} - 1$$

$$y\big|_{x=1} = \sqrt{[1]^2 + 1} - 1 = \sqrt{2} - 1 = 0.4142$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}(2x)$$

$$\frac{dy}{dx}\bigg|_{x=1} = \frac{1}{2}\left([1]^2+1\right)^{-\frac{1}{2}}(2[1]) = 0.7071$$

Equation of the line:

$$y = ax + b$$

sub. slope:

$$y = [0.7071]x + b$$

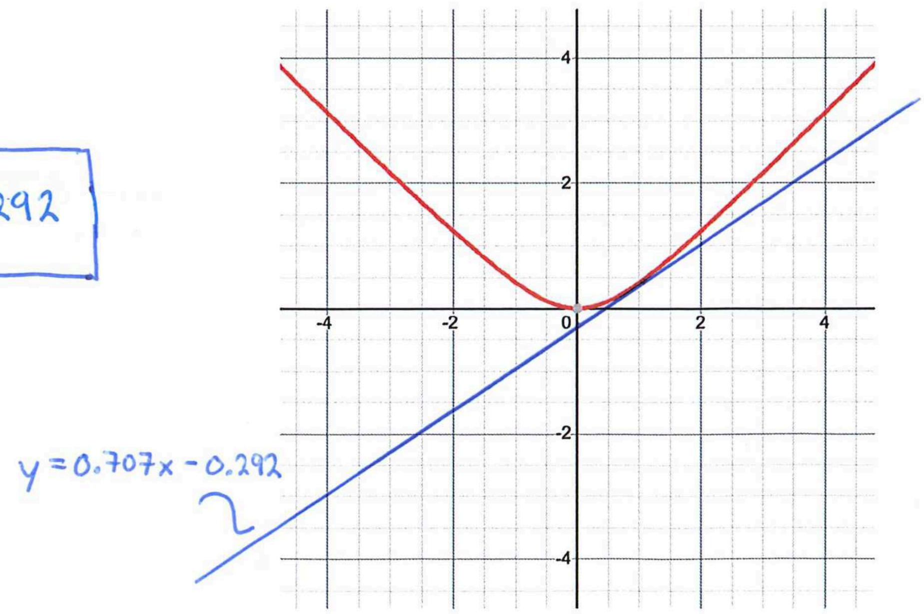
sub. (1, 0.4142) point:

$$[0.4142] = 0.7071[1] + b$$

$$b = -0.2929$$

so

$$\boxed{y = (0.707)x - 0.292}$$



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Remember: $v = \dfrac{x}{t}$

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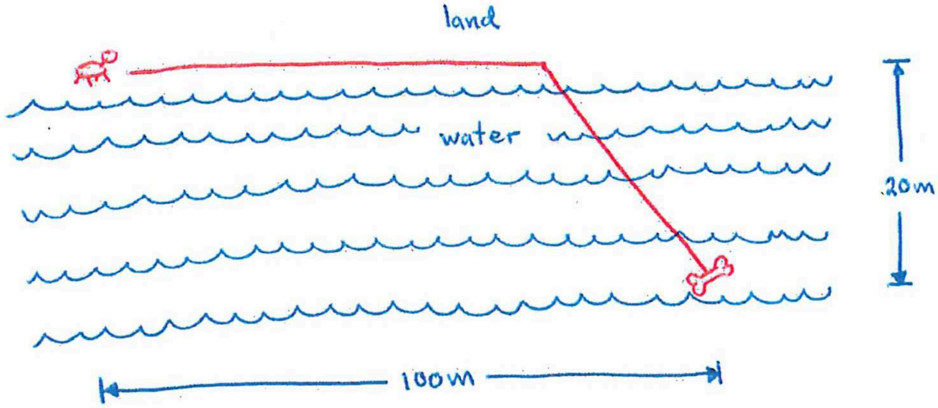
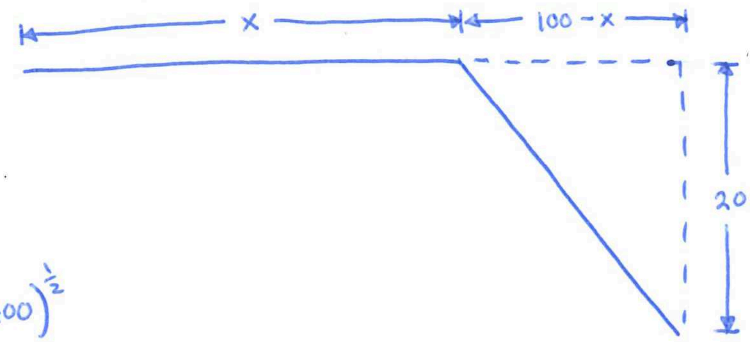


$$L = x + \sqrt{(100-x)^2 + 20^2}$$



$$T = \frac{x}{v}$$

$$= \frac{x}{12} + \frac{\sqrt{(100-x)^2 + 20^2}}{7}$$

$$T = \frac{1}{12}x + \frac{1}{7}\left(x^2 - 200x + 10400\right)^{\frac{1}{2}}$$

$\frac{d}{dx}$

$$\frac{dT}{dx} = \frac{1}{12} + \frac{1}{14}\left(x^2 - 200x + 10400\right)^{-\frac{1}{2}}(2x - 200) = 0 \quad \text{max/min}$$

$$\left(\frac{2x - 200}{\sqrt{x^2 - 200x + 10400}}\right)^2 = \left(-\frac{14}{12}\right)^2$$

$$\frac{4x^2 - 800x + 40000}{x^2 - 200x + 10400} = \frac{196}{144}$$

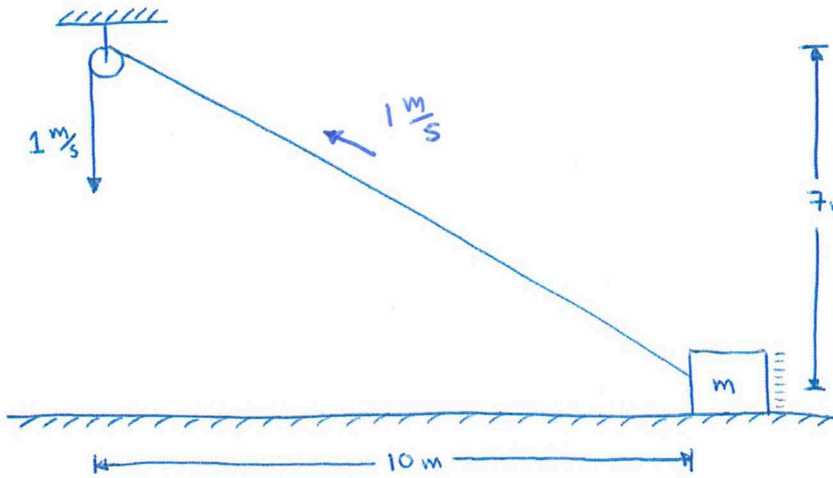
$$2.64x^2 - 527.8x + 25844 = 0$$

$$x = 114.2,\ 85.7 \text{ m}$$

$$T = 13.0,\ 10.7 \text{ s}$$

$$x = 85.7 \text{ m}$$
$$t = 10.7 \text{ s}$$

(5 marks) Suppose you are dragging a mass (m) across the floor by the use of a rope and pulley, as shown in the below figure. If you are pulling the rope at a rate of 1 m/s how fast is the block sliding across the floor when at the position shown in the figure?



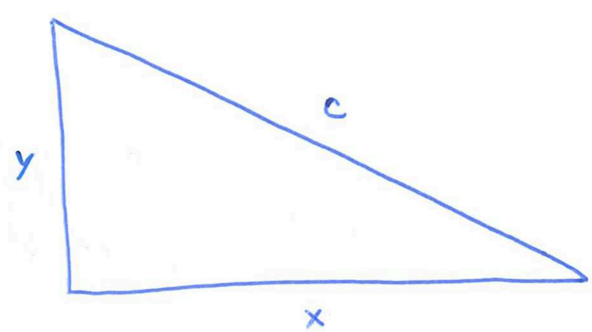
$$x^2 + y^2 = c^2$$

$\frac{d}{dt}$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt}^{\,0} = 2c\frac{dc}{dt}$$

$$\frac{dx}{dt} = \frac{c}{x}\frac{dc}{dt}$$

sub $c = \sqrt{x^2+y^2}$



$$\frac{dx}{dt} = \frac{\sqrt{x^2+y^2}}{x}\frac{dc}{dt}$$

$$= \frac{\sqrt{(10)^2+(7)^2}}{10}(-1)$$

$$= -1.22 \text{ m/s}$$

$$\boxed{\frac{dx}{dt} = -1.22 \text{ m/s}}$$