Instructor:	Frank Secretain
Course:	Math 20
Date:	December 11/12, 2025
Assessment:	Final Test
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k = \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x)) \quad \text{(product rule)}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2} \quad \text{(quotient rule)}$$

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x)) \quad \text{(chain rule)}$$

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\} \quad \text{(polynomials)}$$

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax) \quad \text{(trigonometry)}$$

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$

$$\int ln(x)dx = xln(x) - x \quad \text{(exponentials)}$$

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

(2 marks) A charity group is assembling care packages. In the first hour, they complete 40 packages. Each hour after that, they increase their productivity by 8 packages (so 48 packages in the second hour, 56 in the third hour, and so on). If the goal is to assemble 10,000 packages, how many hours will it take them to reach the target?

(2 marks each) Take the derivative with respect to “x” of the following functions.

$$y = 3x^{\frac{2}{3}} - \sin(x)$$

$$y = 3x^{\frac{2}{3}} \sin(x)$$

$$y = \frac{b^2}{a}\cos(ax^2 - 2x + 1) + \frac{a}{x^a} + 1$$

$$y^2x - x + y = \sin(y)$$

(2 marks each) Integrate with respect to “x” the following functions.

$$\int ax^2 + \cos(2x) + 1 \; dx$$

$$\int \Gamma \sin(2^n x + 2) + C \; dx$$

$$\int \frac{2}{3} x^2 \cos(3x^3 + 2)\ dx$$

(4 marks) Find the area.