Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Test 3
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	3 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k = \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \begin{Bmatrix} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{Bmatrix}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$

$$\int ln(x)dx = xln(x) - x$$ (exponentials)

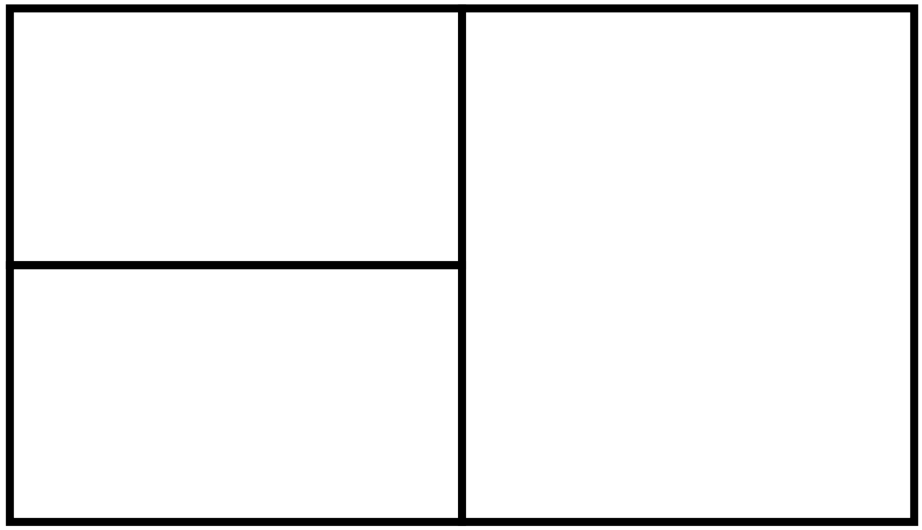
Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

(5 marks) Given 100 meters of fencing determine the maximum area of the proposed enclosure..



(2 marks each) Integrate the following:

$$\int 3x^2 - x \; dx$$

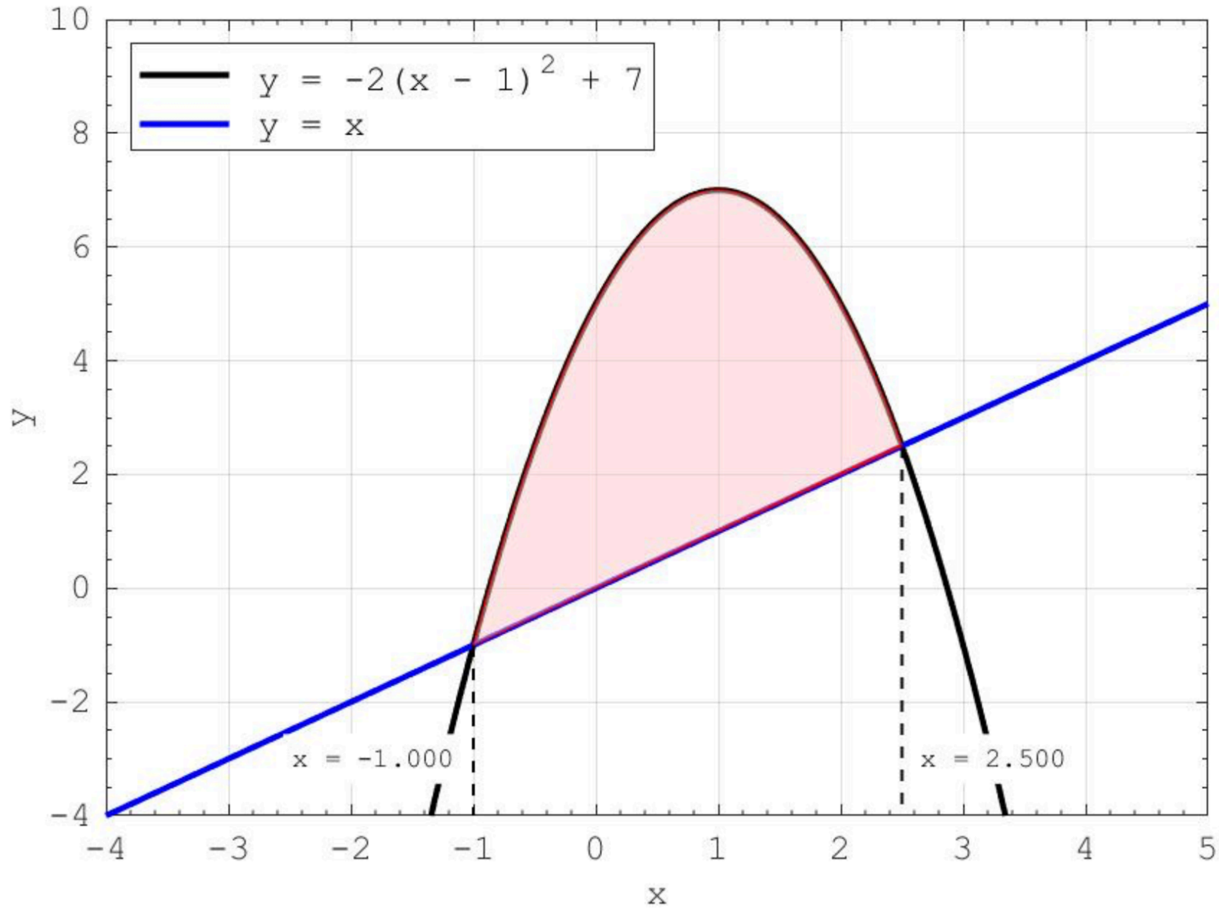
$$\int 4\cos(3x - a) + b \; dx$$

$$\int 3x \sin(5x^2 - 1)\ dx$$

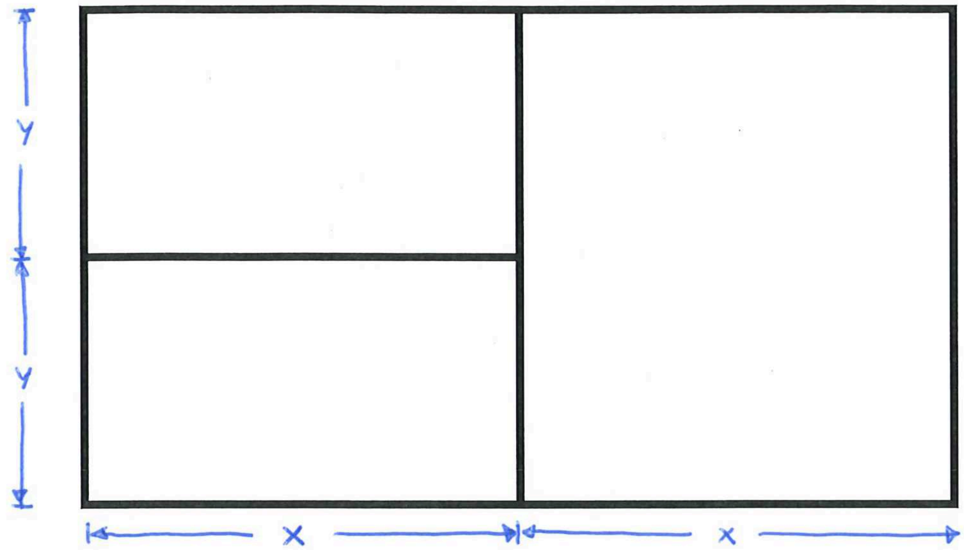
$$\int \alpha x^{3n} - \beta x\ dx$$

$$\int x\cos(x)\ dx$$

(5 marks) Determine the shaded area of the two functions given in the legend.



(5 marks) Given 100 meters of fencing determine the maximum area of the proposed enclosure..



so

$$\ell = 6y + 5x = 100 \qquad (1)$$

$$A = (2y)(2x) \qquad (2)$$

solve (1) for y:

$$y = \frac{100 - 5x}{6} \qquad (1a)$$

sub (1a) into (2)

$$A = 4\left[\frac{100-5x}{6}\right]x$$

$$= \frac{400x - 20x^2}{6}$$

$$= \frac{1}{6}\left(400x - 20x^2\right) \qquad (2a)$$

differeniate (2a) wrt x:

$$\frac{dA}{dx} = \frac{400}{6} - \frac{40x}{6} \qquad (2b)$$

set (2b) to 0 for max/min

$$\frac{dA}{dx} = \frac{400}{6} - \frac{40x}{6} = 0$$

$$400 = 40x$$

$$\boxed{x = 10} \qquad (2c)$$

sub (2c) into (1a)

$$y = \frac{100 - 5[10]}{6}$$

$$\boxed{y = \frac{50}{6} = \frac{25}{3} = 8\tfrac{1}{3}}$$

$$A = 4xy$$

$$= 4(10)(8\tfrac{1}{3})$$

$$\boxed{A = 333\tfrac{1}{3} = 333.\overline{3}}$$

(2 marks each) Integrate the following:

$$\int 3x^2 - x\ dx \qquad \boxed{= x^3 - \frac{1}{2}x^2 + c}$$

$$\int 4\cos(3x - a) + b\ dx \qquad \boxed{= \frac{4}{3}\sin(3x - a) + bx + c}$$

let $u = 3x - a$

$dx = \dfrac{du}{3}$

so

$$\int 4\cos(u)\frac{du}{3} + \int b\,dx$$

$$\frac{4}{3}\sin(u) + bx + c$$

$$\frac{4}{3}\sin(3x - a) + bx + c$$

$$\int 3x\sin(5x^2-1)\ dx$$

let $u = 5x^2 - 1$

$$\frac{du}{dx} = 10x$$

$$dx = \frac{du}{10x}$$

sub.

$$\int 3x \sin(u) \frac{du}{10x}$$

$$\int \frac{3}{10} \sin(u)\ du$$

$$= -\frac{3}{10}\cos(u) + C$$

$$\boxed{= -\frac{3}{10}\cos(5x^2-1) + C}$$

$$\int \alpha x^{3n} - \beta x\ dx \quad \boxed{= \frac{\alpha}{3n+1} x^{3n+1} - \frac{\beta}{2}x^2 + C}$$

$$\int x\cos(x)\ dx$$

let $\quad u = x \qquad dv = \cos(x)\,dx$

$\qquad du = dx \qquad v = \sin(x)$

by parts

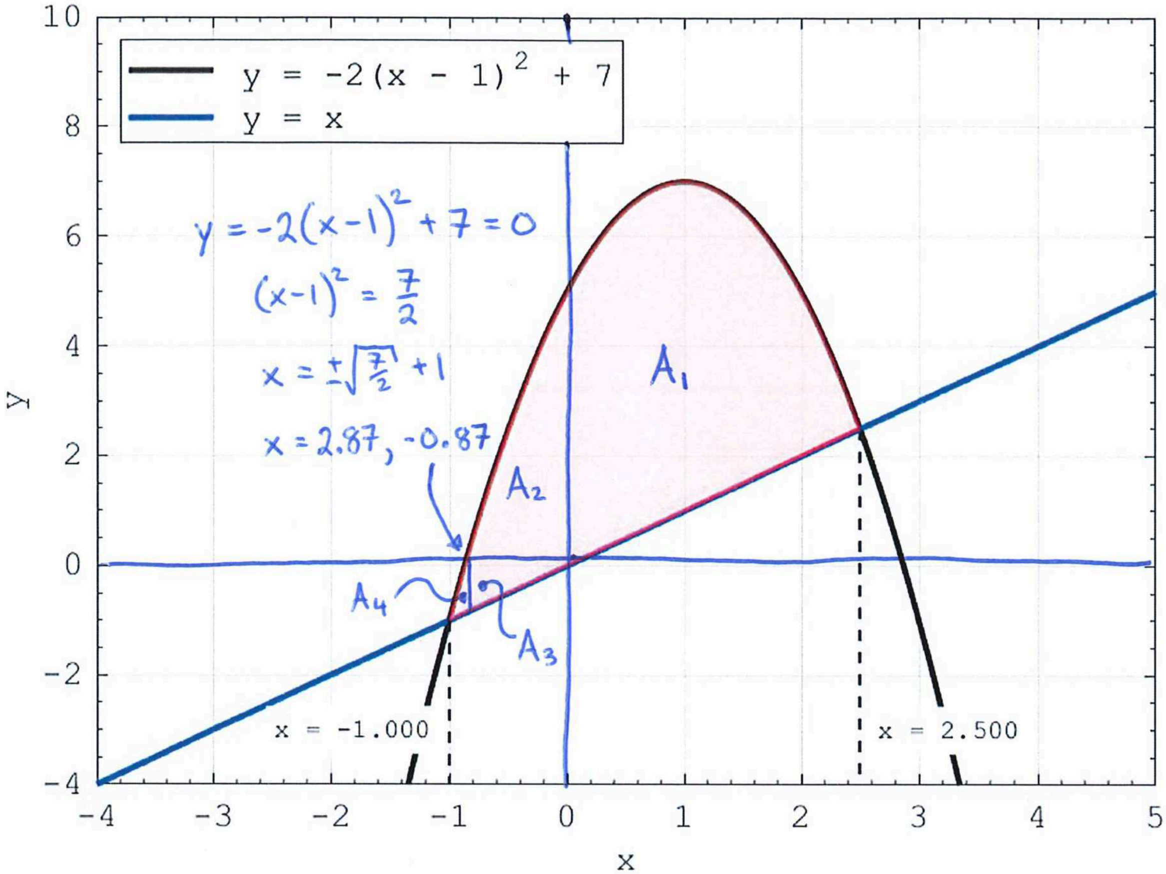
$$\int u\,dv = uv - \int v\,du$$

$$= (x)(\sin(x)) - \int \sin(x)\,dx$$

$$= x\sin(x) - -\cos(x) + c$$

$$\boxed{\int x\cos(x)\,dx = x\sin(x) + \cos(x) + c}$$

(5 marks) Determine the shaded area of the two functions given in the legend.



$$A_1 = \int_0^{2.5} \left[-2(x-1)^2+7\right] - \left[x\right] dx = \left[-\tfrac{2}{3}(x-1)^3 + 7x - \tfrac{1}{2}x^2\right]_0^{2.5} = \left[12\tfrac{1}{8}\right] - \left[\tfrac{2}{3}\right] = 11\tfrac{11}{24} = 11.46$$

$$A_2 = \int_{-0.87}^{0} \left[-2(x-1)^2+7\right] dx = \left[-\tfrac{2}{3}(x-1)^3 + 7x\right]_{-0.87}^{0} = \left[\tfrac{2}{3}\right] - \left[-1.73\right] = 2.40$$

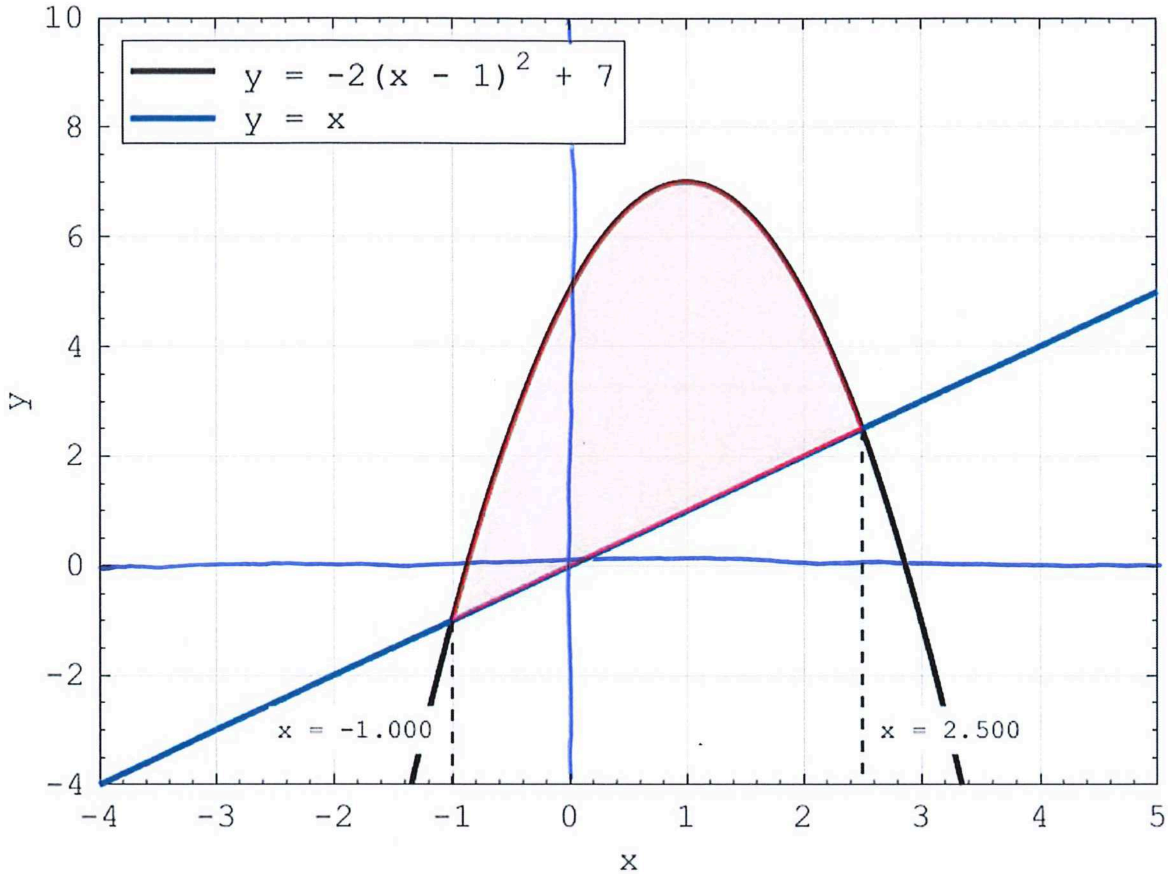
$$A_3 = -\int_{-0.87}^{0} x \, dx = -\left[\tfrac{1}{2}x^2\right]_{-0.87}^{0} = -\left[0\right] + \left[0.38\right] = 0.38$$

$$A_4 = -\int_{-1}^{-0.37} \left[x\right] - \left[-2(x-1)^2+7\right] dx = -\left[\tfrac{1}{2}x^2 + \tfrac{2}{3}(x-1)^3 - 7x\right]_{-1}^{-0.87} = -\left[2.11\right] + \left[2.16\right]$$

$$= 0.058$$

$$\boxed{A_{total} = A_{1-4} = 11.46 + 2.40 + 0.38 + 0.06 = 14.30}$$

(5 marks) Determine the shaded area of the two functions given in the legend.



Shift functions +1 up:

$$A = \int_{-1}^{2.5} \left[-2(x-1)^2 + 7 + 1\right] - \left[x+1\right] dx$$

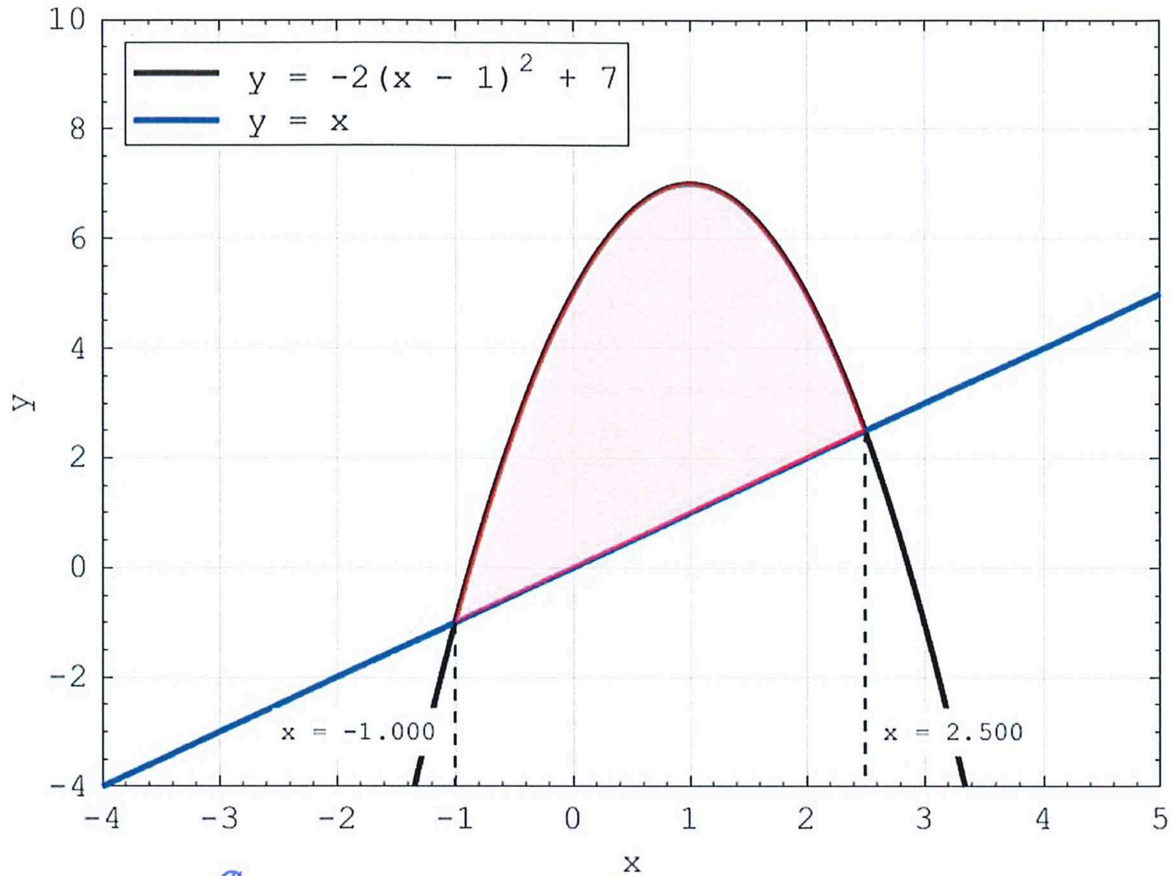
$$= \int_{-1}^{2.5} -2(x-1)^2 - x + 7 \, dx$$

$$= \left[-\frac{2}{3}(x-1)^3 - \frac{1}{2}x^2 + 7x \right]_{-1}^{2.5} = \left[-\frac{2}{3}(2.5-1)^3 - \frac{1}{2}(2.5)^2 + 7(2.5)\right] - \left[-\frac{2}{3}(-1-1)^3 - \frac{1}{2}(-1)^2 + 7(-1)\right]$$

$$= [12.125] - [-2.1\overline{6}]$$

$$\boxed{A = 14.29 = 14\tfrac{7}{24}}$$

(5 marks) Determine the shaded area of the two functions given in the legend.



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$$A = \int_{-1}^{2.5} -2(x-1)^2 + 7 \, dx - \int_{-1}^{2.5} x \, dx = \int_{-1}^{2.5} -2(x-1)^2 + 7 - x \, dx$$

$$= \left[-\tfrac{2}{3}(x-1)^3 - \tfrac{1}{2}x^2 + 7x \right]_{-1}^{2.5}$$

$$= \left[-\tfrac{2}{3}(2.5-1)^3 - \tfrac{1}{2}(2.5)^2 + 7(2.5) \right] - \left[-\tfrac{2}{3}(-1-1)^3 - \tfrac{1}{2}(-1)^2 + 7(-1) \right]$$

$$= \left[12.125 \right] - \left[-2.1\bar{6} \right]$$

$$\boxed{A = 14.29 = 14\tfrac{7}{24}}$$