Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k = \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x)) \quad \text{(product rule)}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2} \quad \text{(quotient rule)}$$

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x)) \quad \text{(chain rule)}$$

Derivatives of select functions

Integrals of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1} \qquad \int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\} \quad \text{(polynomials)}$$

$$\frac{d}{dx}(sin(x)) = cos(x) \qquad \int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\frac{d}{dx}(cos(x)) = -sin(x) \qquad \int cos(ax)dx = \frac{1}{a}sin(ax) \quad \text{(trigonometry)}$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2} \qquad \int tan(ax)dx = ln(|sec(x)|)$$

$$\frac{d}{dx}(a^x) = a^x ln(a) \qquad \int a^x dx = \frac{1}{ln(a)}a^x$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)} \qquad \int ln(x)dx = xln(x) - x \quad \text{(exponentials)}$$

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

(2 marks) Determine the derivative with respect to x of the following equation.

$$y = 4\sin(x) + 7$$

(3 marks each) Determine the derivative with respect to x of the following equations.

$$y = 4\sin(2x)\tan(x)$$

$$y = 8\sin(3x^2 + b) + C$$

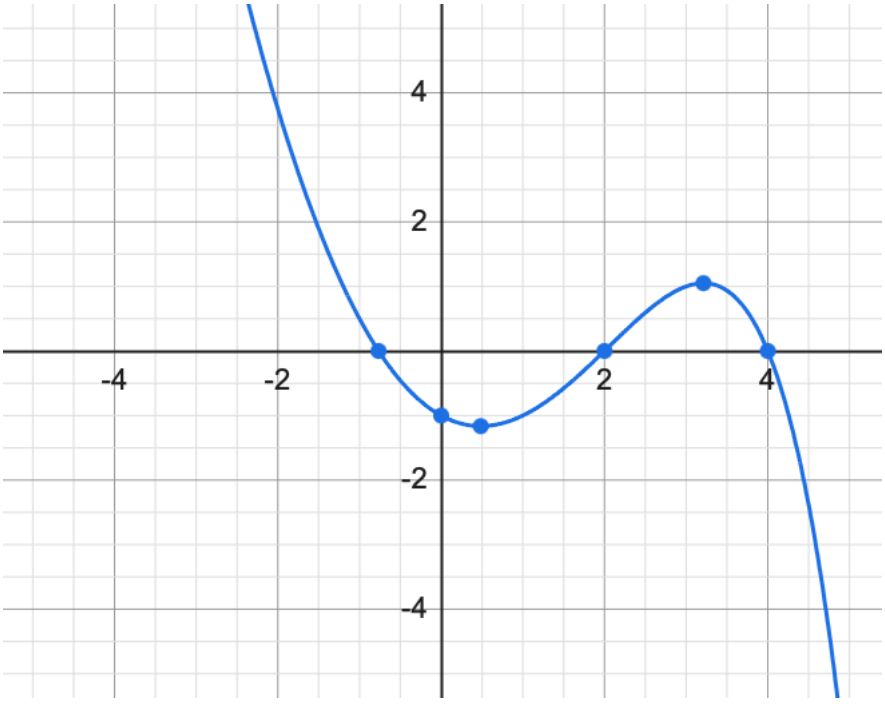
$$y = x^m \cos(x^m + 1) + 1$$

(4 marks) Determine the double derivative with respect to x of the following equation.

$$y = x\sin(x)$$

(5 marks) Determine the tangent line at x=3 for the following function. Plot the tangent line on the plot.

$$y = x^2 - 2^x$$



(2 marks) Determine the derivative with respect to x of the following equation.

$$y = 4\sin(x) + 7$$

$\frac{dy}{dx} = 4\cos(x) + 0$

(3 marks each) Determine the derivative with respect to x of the following equations.

$$y = 4\sin(2x)\tan(x)$$

$\frac{dy}{dx} = 4\cos(2x)(2)\tan(x) + 4\sin(2x)\frac{1}{\cos^2(x)}$

$\frac{dy}{dx} = 8\cos(2x)\tan(x) + \frac{4\sin(2x)}{\cos^2(x)}$

$$y = 8\sin(3x^2 + b) + C$$

$$\frac{dy}{dx} = 8\cos(3x^2 + b)(6x + 0) + 0$$

$$y = x^m \cos(x^m + 1) + 1$$

$$\frac{dy}{dx} = m x^{m-1} \cos(x^m + 1) - x^m \sin(x^m + 1)(m x^{m-1} + 0) + 0$$

(4 marks) Determine the double derivative with respect to x of the following equation.

$$y = x\sin(x)$$

$$\frac{dy}{dx} = \sin(x) + x\cos(x)$$

$$\frac{d^2y}{dx^2} = \cos(x) + \left[\cos(x) - x\sin(x)\right]$$

$$\frac{d^2y}{dx^2} = 2\cos(x) - x\sin(x)$$

(5 marks) Determine the tangent line at x=3 for the following function. Plot the tangent line on the plot.

$$y = x^2 - 2^x$$

$y\big|_{x=3} = (3)^2 - 2^3 = 1$

$\frac{dy}{dx} = 2x - 2^x \ln(2)$ $\quad \frac{dy}{dx}\Big|_{x=3} = 2(3) - 2^3 \ln(2) = 0.4548$

$y = ax + b$

$y = [0.455]x + b$

$[1] = 0.455[3] + b$

$b = 1 - (0.455)(3) = -0.364$

$$\boxed{y = 0.455x - 0.364}$$