Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Marks allocated:	6 questions worth 25 marks
Percentage of final grade:	15% of final grade
Notes from instructor:	Be neat. Show your work where needed. Box final answers. Print your test and write answers in the space provided. If you can't print, then use blank paper and copy the question number as it is written on the test and answer in the space provided as if the test was printed.
Questions:	Give me a call on teams.
Submission:	At the end of your test: scan or take pictures of your test pages in order. Compile email and send it to: math20@franksecretain.ca by 10:30 am on October 16, 2020

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$

$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$

$$= a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$ (exponentials)

$$\int ln(x)dx = xln(x) - x$$

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

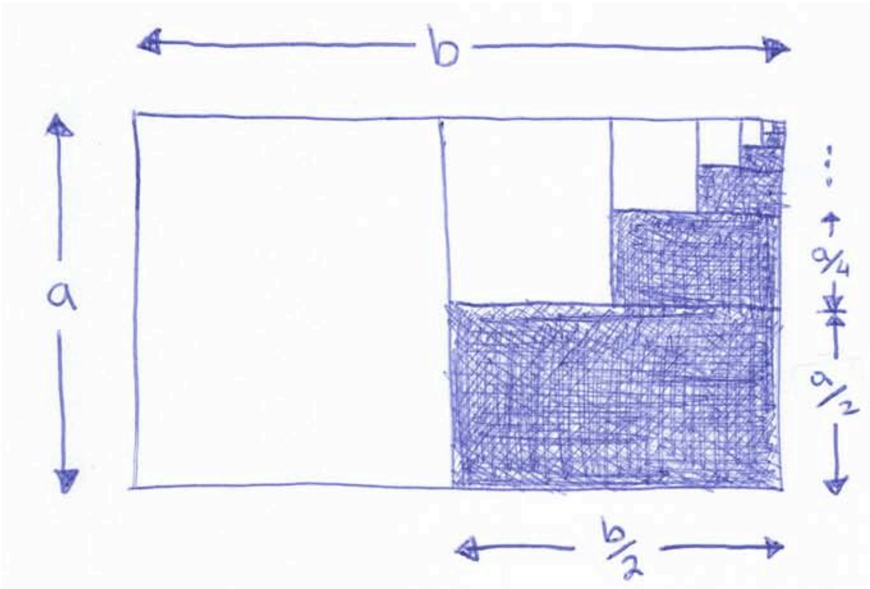
$$\int udv = uv - \int vdu$$

(2 marks each) Determine the 100th number in the sequence and the sum from the first number to the 100th number for each of the following series.

-25.2, -37.8, -56.7, …

-25.2, -37.8, -50.4, …

(4 marks) Use an infinite series to determine the area of the shaded area in terms of lengths of “a” and “b” given the below figure.   Set up an infinite series by adding the appropriate areas and then sum to infinity.



(2 marks each) Take the derivative with respect to “x” of the following y(x) functions.

$$y = \frac{4x^3}{7} + 2x^{\frac{3}{4}}$$

$$y = (2x^a - 1)^b \sin(x) + a\left(\frac{x}{z}\right)^2$$

(2 marks each) Take the derivative with respect to "z" of the following g(z) functions.

$$g = (2z^3 + x)^a + 3sin(x)$$

$$g = (sin((2z^3 + x)^a + 3sin(x)))^2$$

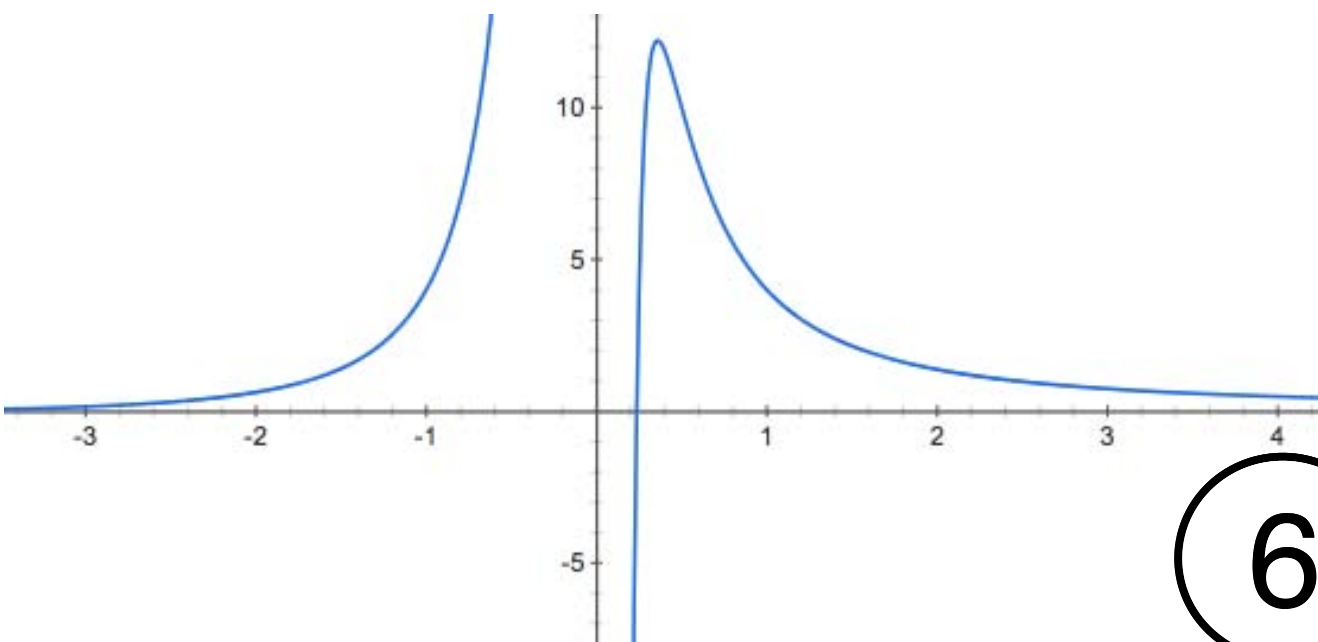
(2 marks each) Take the derivative with respect to “z” of the following h(z) functions.

$$\frac{2h}{3} + hz^2 = \sin(h + z - x^2) + 1$$

$$\frac{h}{z^2} = (\sin(h^2 + z - x^2) + 1)^{a-1}$$

(5 marks) Determine the tangent line at x=1 for the following function. Plot the tangent line on the plot.

$$yx^2 - 4 = x - \frac{1}{x}$$



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(2 marks each) Determine the 100th number in the sequence and the sum from the first number to the 100th number for each of the following series.

-25.2, -37.8, -56.7, ...

$$r = \frac{-56.7}{-37.8} = 1.5$$

$$= \frac{-37.8}{-25.2} = 1.5 \text{ (geometric series)}$$

$a_1 = -25.2$

$n = 100$

$$a_n = a_1 r^{n-1}$$

$$= (-25.2)(1.5^{100-1})$$

$$\boxed{a_{100} = -6.83 \times 10^{18}}$$

$$S_n = a_1 \frac{1-r^n}{1-r}$$

$$= (-25.2)\frac{1-1.5^{100}}{1-1.5}$$

$$\boxed{S_{100} = -2.05 \times 10^{19}}$$

-25.2, -37.8, -50.4, ...

$$k = -50.4 - (-37.8) = -12.6$$

$$= -37.8 - (-25.2) = -12.6 \text{ (arithmetic series)}$$

$a_1 = -25.2$

$n = 100$

$$a_n = a_1 + (n-1)k$$

$$= -25.2 + (100-1)(-12.6)$$

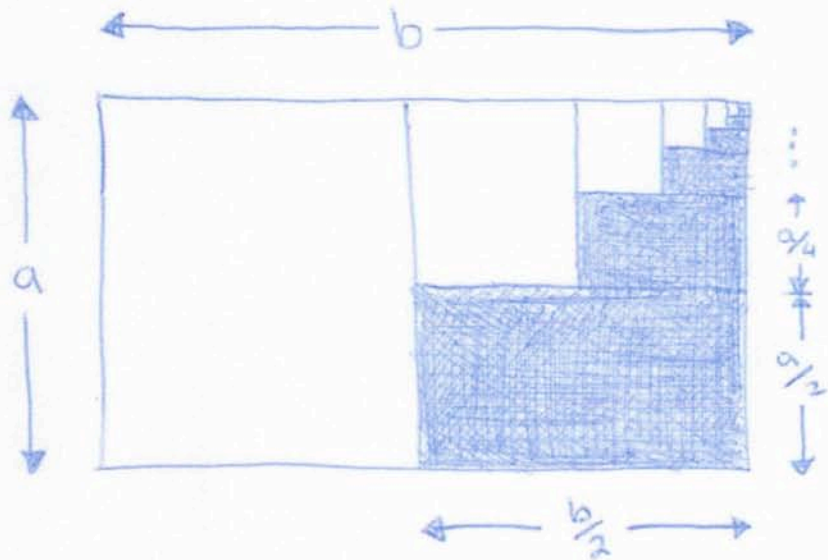
$$\boxed{a_{100} = -1272.6}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{100}{2}(-25.2 + -1272.6)$$

$$\boxed{S_{100} = -64890}$$

(4 marks) Use an infinite series to determine the area of the shaded area in terms of lengths of "a" and "b" given the below figure. Set up an infinite series by adding the appropriate areas and then sum to infinity.



$$\text{shaded area} = \left(\frac{a}{2}\right)\left(\frac{b}{2}\right) + \left(\frac{a}{4}\right)\left(\frac{b}{4}\right) + \left(\frac{a}{8}\right)\left(\frac{b}{8}\right) + \dots$$

$$= ab\left[\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right]$$

the sequence $= \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \dots$

$$r = \frac{\frac{1}{64}}{\frac{1}{16}} = 0.25$$

$$= \frac{1/16}{1/4} = 0.25 \quad \text{(geometric series)}$$

$a_1 = \frac{1}{4}$

$n \to \infty$

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$= \left(\frac{1}{4}\right)\frac{1 - 0.25^{\infty}}{1 - 0.25} \quad (0.25^{\infty} \to 0)$$

$$\boxed{S_\infty = \frac{1}{3}}$$

so

$$\text{shaded area} = ab\left[\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots\right]$$

$$= ab\left[\frac{1}{3}\right]$$

$$\boxed{\text{shaded area} = \frac{ab}{3}}$$

(2 marks each) Take the derivative with respect to "x" of the following y(x) functions.

$$y = \frac{4x^3}{7} + 2x^{\frac{3}{4}}$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = \frac{12x^2}{7} + \frac{6}{4}x^{-\frac{1}{4}}$$

$$y = (2x^a - 1)^b \sin(x) + a\left(\frac{x}{z}\right)^2$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = b(2x^a - 1)^{b-1}(2ax^{a-1})\sin(x) + (2x^a - 1)^b \cos(x) + 2a\left(\frac{x}{z}\right)\left(\frac{1}{z}\right)$$

(2 marks each) Take the derivative with respect to "z" of the following g(z) functions.

$$g = (2z^3 + x)^a + 3sin(x)$$

$\frac{d}{dz}$

$$\frac{dg}{dz} = a(2z^3+x)^{a-1}(6z^2)$$

$$g = (sin((2z^3 + x)^a + 3sin(x)))^2$$

$\frac{d}{dz}$

$$\frac{dg}{dz} = 2\left(\sin\left((2z^3+x)^a + 3\sin(x)\right)\right)\left(\cos\left((2z^3+x)^a + 3\sin(x)\right)\right)\left(a(2z^3+x)^{a-1}(6z^2)\right)$$

(2 marks each) Take the derivative with respect to "z" of the following h(z) functions.

$$\frac{2h}{3} + hz^2 = \sin(h + z - x^2) + 1$$

$\frac{d}{dz}$

$$\frac{2}{3}\frac{dh}{dz} + \frac{dh}{dz}z^2 + (h)(2z) = \cos(h + z - x^2)\left(\frac{dh}{dz} + 1\right)$$

$$\frac{h}{z^2} = (\sin(h^2 + z - x^2) + 1)^{a-1}$$

$$\frac{\frac{dh}{dz}z^2 - (h)(2z)}{z^4} = (a-1)(\sin(h^2+z-x^2)+1)^{a-2}$$
$$(\cos(h^2+z-x^2)(2h\frac{dh}{dz}+1)$$

(5 marks) Determine the tangent line at x=1 for the following function. Plot the tangent line on the plot.

$$yx^2 - 4 = x - \frac{1}{x}$$

$$yx^2 = x - x^{-1} + 4$$

$$\boxed{y = x^{-1} - x^{-3} + 4x^{-2}}$$

$\frac{d}{dx}$

$$\frac{dy}{dx} = -x^{-2} + 3x^{-4} - 8x^{-3}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=1} = -1 + 3 - 8 = -6} \quad \Rightarrow \quad y = -6x + b$$

$$y(1) = 1 - 1 + 4 = 4$$

sub into line equation

$$4 = (-6)(1) + b \quad \Rightarrow \quad b = 10$$

so

$$\boxed{y = -6x + 10}$$