Instructor:	Frank Secretain
Course:	Math 20
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	6 questions worth 25 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Arithmetic Series

$$a_n = a_1 + (n-1)k$$

$$S_n = \sum_{i=1}^{n} a_1 + (i-1)k$$

$$= \frac{n}{2}(a_1 + a_n)$$

Geometric Series

$$a_n = a_1 r^{n-1}$$

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1}$$

$$= a_1 \frac{1-r^n}{1-r}$$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!k!} x^{n-k} y^k$$

Line equation

$$y = ax + b$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Definition of the derivative

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Rules of differentiation

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$ (product rule)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$ (quotient rule)

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}(f(g(x)))\frac{d}{dx}(g(x))$$ (chain rule)

Derivatives of select functions

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(sin(x)) = cos(x)$$

$$\frac{d}{dx}(cos(x)) = -sin(x)$$

$$\frac{d}{dx}(tan(x)) = \frac{1}{(cos(x))^2}$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(log_a x) = \frac{1}{xln(a)}$$

Integrals of select functions

$$\int ax^n dx = \left\{ \begin{array}{ll} \frac{a}{n+1}x^{n+1} & , n \neq -1 \\ ln(|x|) & , n = -1 \end{array} \right\}$$ (polynomials)

$$\int sin(ax)dx = -\frac{1}{a}cos(ax)$$

$$\int cos(ax)dx = \frac{1}{a}sin(ax)$$ (trigonometry)

$$\int tan(ax)dx = ln(|sec(x)|)$$

$$\int a^x dx = \frac{1}{ln(a)}a^x$$

$$\int ln(x)dx = xln(x) - x$$ (exponentials)

Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!}(x - x_o)^n$$

Integration by parts

$$\int udv = uv - \int vdu$$

Integrate (3 marks)

$$\int 2x^5 + 2\cos(x+2) + 1 \ dx$$

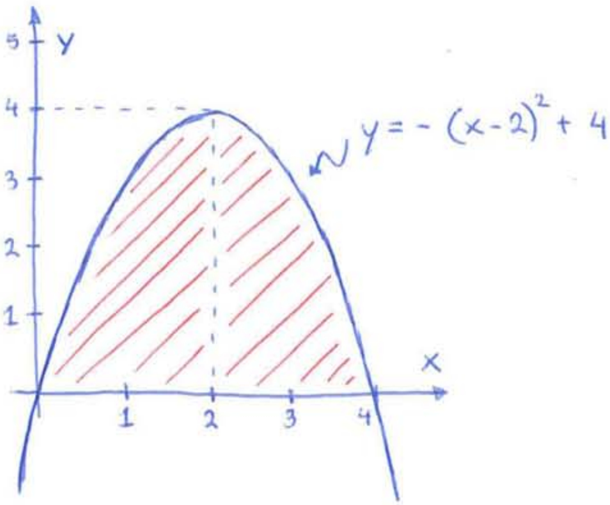
Integrate (3 marks)

$$\int \frac{x}{(x^2-1)^{\frac{3}{2}}} + 1 \ dx$$

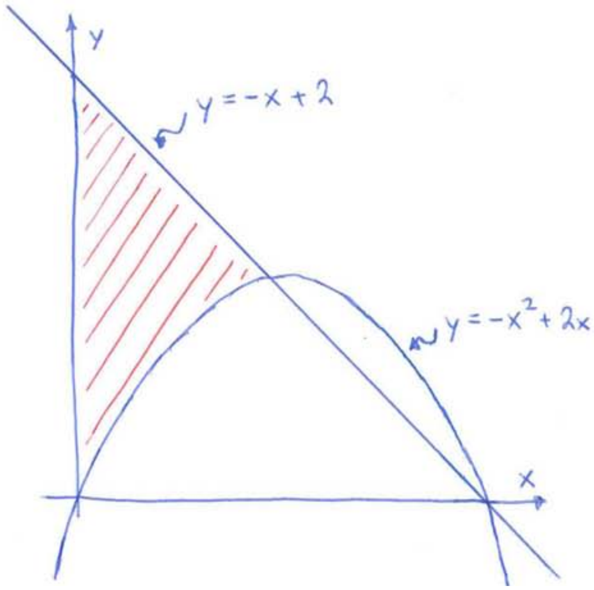
Integrate (4 marks)

$$\int \frac{x\sqrt{x^2+1} + 2x\sin(x^2+1)}{3}\ dx$$

Find the area of the shaded section (5 marks)

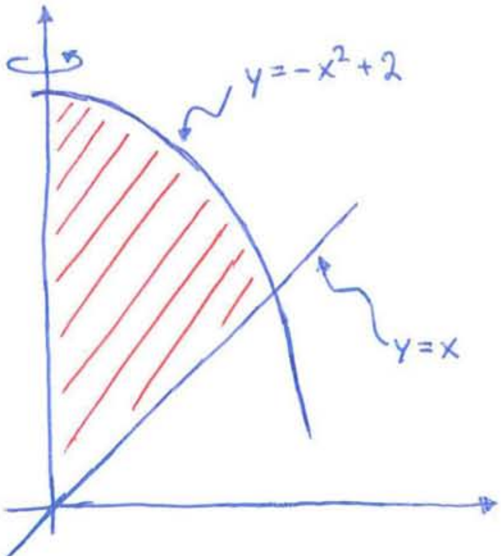


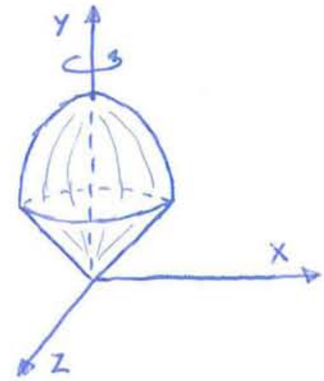
Find the area of the shaded section (5 marks)



Find the volume between the y-axis and the two curves revolved around the y-axis (5 marks).
Remember that:

area of a disk = πr^2
area of a shell = $2\pi rh$





Integrate (3 marks)

$$\int 2x^5 + 2\cos(x+2) + 1 \; dx$$

$$\frac{2}{6}x^6 + 2\sin(x+2) + x + c$$

$$\boxed{\frac{1}{3}x^6 + 2\sin(x+2) + x + c}$$

Integrate (3 marks)

$$\int \frac{x}{(x^2-1)^{\frac{3}{2}}} + 1\ dx$$

$$\int \frac{x}{(x^2-1)^{3/2}}\,dx + \int 1\,dx$$

let $u = x^2 - 1$

$$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$$

so

$$\int \frac{\cancel{x}}{u^{3/2}} \frac{du}{2\cancel{x}} + \int 1\,dx$$

$$\frac{1}{2}\int u^{-\frac{3}{2}}\,du + \int 1\,dx$$

$$\frac{1}{2}\left[-2u^{-\frac{1}{2}}\right] + x + c$$

sub back

$$-(x^2-1)^{-\frac{1}{2}} + x + c$$

$$\boxed{\frac{-1}{\sqrt{x^2-1}} + x + c}$$

Integrate (4 marks)

$$\int \frac{x\sqrt{x^2+1}+2x\sin(x^2+1)}{3}\,dx$$

let $u = x^2 + 1$

$$\frac{du}{dx} = 2x \quad \Rightarrow \quad dx = \frac{du}{2x}$$

so

$$\frac{1}{3}\int \cancel{x}\, u^{\frac{1}{2}} + 2\cancel{x}\sin(u)\ \frac{du}{2\cancel{x}}$$

$$\frac{1}{6}\int u^{\frac{1}{2}} + 2\sin(u)\ du$$

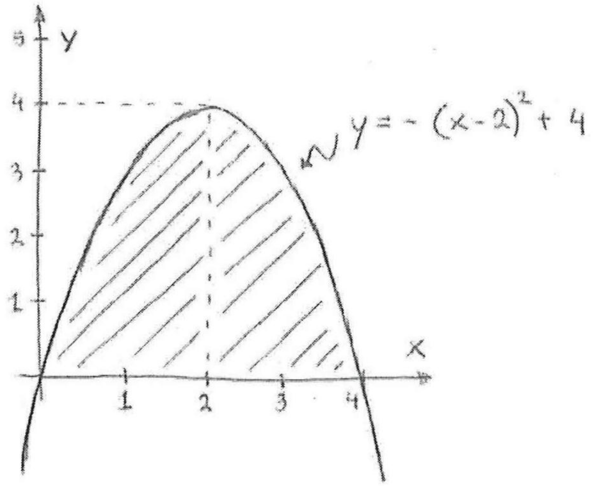
$$\frac{1}{6}\left[\frac{2}{3}u^{\frac{3}{2}} - 2\cos(u)\right] + C$$

sub back

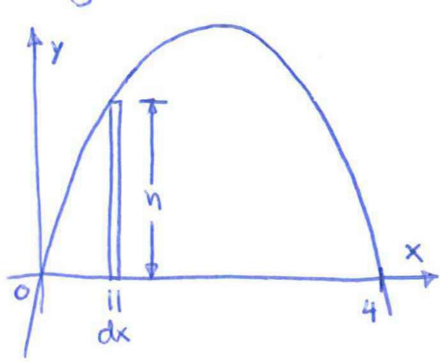
$$\frac{2}{18}\left(x^2+1\right)^{\frac{3}{2}} - \frac{2}{6}\cos\left(x^2+1\right) + C$$

$$\boxed{\frac{1}{9}\left(x^2+1\right)^{\frac{3}{2}} - \frac{1}{3}\cos\left(x^2+1\right) + C}$$

Find the area of the shaded section (5 marks)



using column's



so

$$A = \int_0^4 \left[-(x-2)^2+4\right] dx$$

$$= \left[-\frac{1}{3}(x-2)^3 + 4x\right]_0^4$$

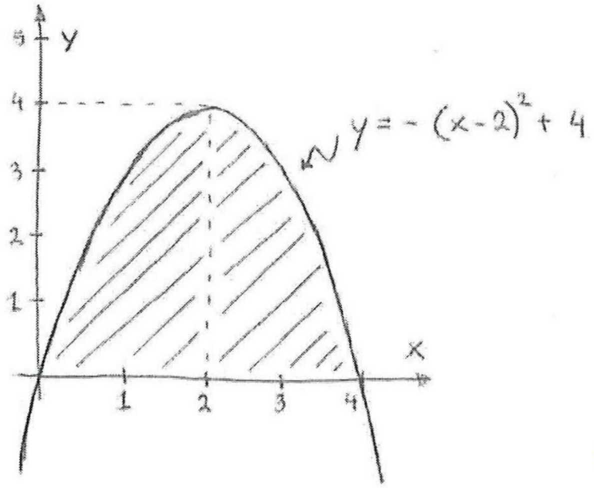
$$= \left[-\frac{1}{3}(4-2)^3 + 4(4)\right] - \left[-\frac{1}{3}(0-2)^3 + \cancel{4(0)}^{\,0}\right]$$

$$= \left[-\frac{8}{3} + 16\right] - \left[\frac{8}{3}\right]$$

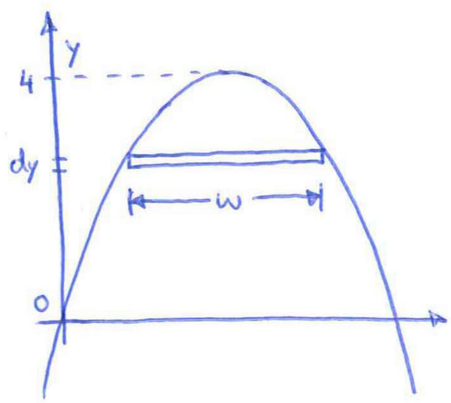
$$= 16 - \frac{16}{3} = \frac{32}{3} \cong 10.67$$

$$\boxed{A = \frac{32}{3} \cong 10.67}$$

Find the area of the shaded section (5 marks)



using row's



solve for x:

$$y = -(x-2)^2 + 4$$

$$(x-2)^2 = 4 - y$$

$$x - 2 = \pm\sqrt{4-y}$$

$$x = \pm\sqrt{4-y} + 2$$

so

$$A = \int_0^4 \left[\left(\sqrt{4-y} + 2\right) - \left(-\sqrt{4-y} + 2\right)\right] dy$$

$$= \int_0^4 2(4-y)^{\frac{1}{2}}\, dy$$

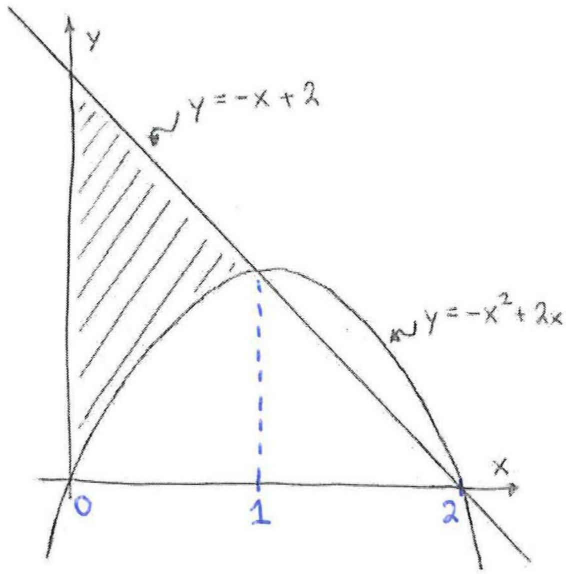
$$= \left[-\frac{4}{3}(4-y)^{\frac{3}{2}}\right]_0^4$$

$$= \left[\cancel{-\frac{4}{3}(4-4)^{\frac{3}{2}}}^{\,0}\right] - \left[-\frac{4}{3}(4-0)^{\frac{3}{2}}\right]$$

$$= \frac{4}{3}(8) = \frac{32}{3} \cong 10.67$$

$$\boxed{A = \frac{32}{3} \cong 10.67}$$

Find the area of the shaded section (5 marks)



Find intercepts:

$$-x+2=-x^2+2x$$

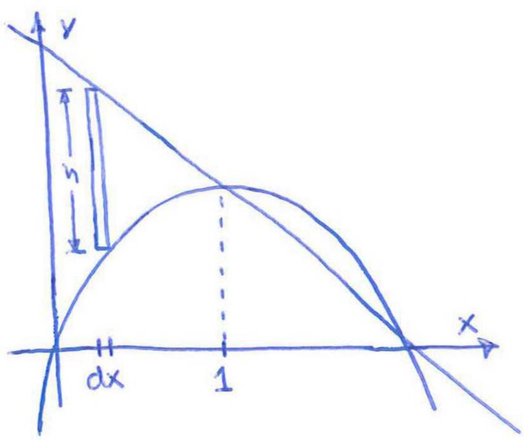
$$x^2-3x+2=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{3\pm\sqrt{9-(4)(2)}}{2}$$

$$=\frac{3\pm1}{2}=1 \text{ or } 2$$

so

using column's



$$A=\int_0^1\left[-x+2\right]-\left[-x^2+2x\right]dx$$

$$=\int_0^1 x^2-3x+2\ dx$$

$$=\left[\frac{1}{3}x^3-\frac{3}{2}x^2+2x\right]_0^1$$

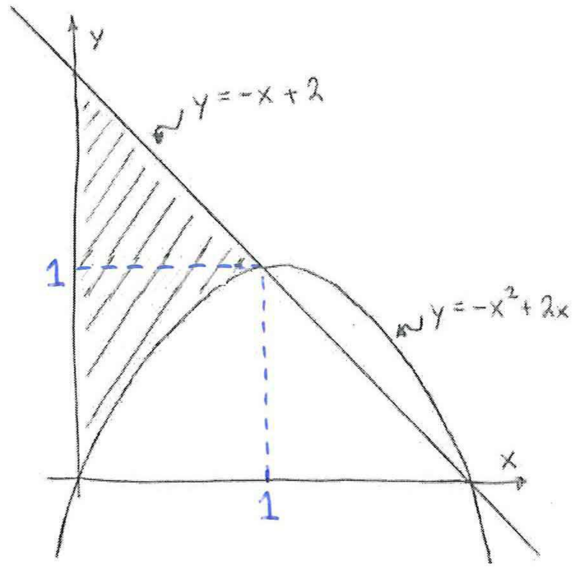
$$=\left[\frac{1}{3}(1)^3-\frac{3}{2}(1)^2+2(1)\right]-\left[0\right]$$

$$=\frac{1}{3}-\frac{3}{2}+2=\frac{2}{6}-\frac{9}{6}+\frac{12}{6}$$

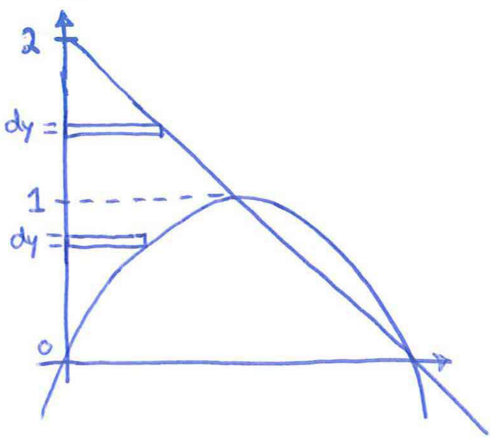
$$=\frac{5}{6}\cong 0.833$$

$$\boxed{A=\frac{5}{6}\cong 0.83}$$

Find the area of the shaded section (5 marks)



using rows



solve for x:

$y = -x + 2$

$\boxed{x = 2 - y}$

$y = -x^2 + 2x$

$x^2 - 2x + y = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{2 \pm \sqrt{4 - 4y}}{2}$

$\boxed{x = \dfrac{2 \pm 2\sqrt{1-y}}{2} = 1 \pm \sqrt{1-y}}$

Find intercepts (x-values)

$-x + 2 = -x^2 + 2x$

$x^2 - 3x + 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{3 \pm \sqrt{9 - (4)(2)}}{2}$

$= \dfrac{3 \pm 1}{2} = 1$ or 2

Find intercepts (y - values)

$y = -x + 2$

$= -(1) + 2 = 1$

$y = -x + 2$

$= -(0) + 2 = 2$

so:

$$A = \int_0^1 \left(1 - \sqrt{1-y}\right) dy + \int_1^2 (2 - y)\, dy$$

$$= \left[y + \tfrac{2}{3}(1-y)^{\frac{3}{2}}\right]_0^1 + \left[2y - \tfrac{1}{2}y^2\right]_1^2$$

$$= \left[1 + \cancel{\tfrac{2}{3}(1-1)^{\frac{3}{2}}}^{\,0}\right] - \left[\cancel{0}^{\,0} + \tfrac{2}{3}(1-0)^{\frac{3}{2}}\right] +$$

$$\left[2(2) - \tfrac{1}{2}(2)^2\right] - \left[2(1) - \tfrac{1}{2}(1)^2\right]$$

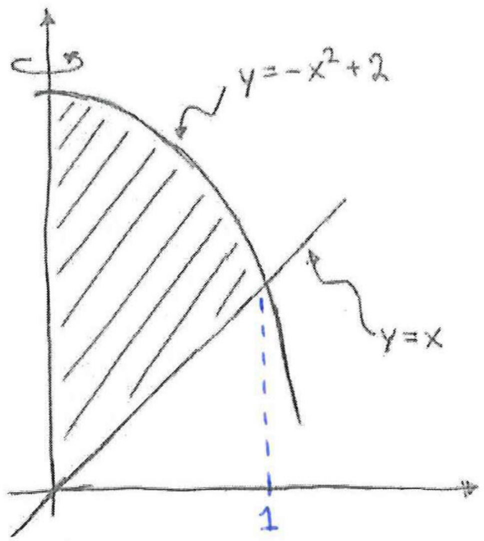
$$= \left[1 - \tfrac{2}{3}\right] + \left[(4 - 2) - (2 - \tfrac{1}{2})\right]$$

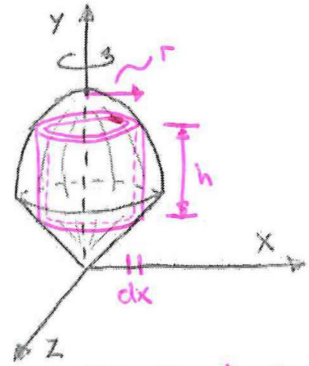
$$= \left[\tfrac{1}{3}\right] + \left[\tfrac{1}{2}\right] = \tfrac{2}{6} + \tfrac{3}{6} = \tfrac{5}{6}$$

$$\boxed{A = \tfrac{5}{6} \cong 0.83}$$

Find the volume between the y-axis and the two curves revolved around the y-axis (5 marks).
Remember that:

area of a disk = πr^2
area of a shell = $2\pi rh$





$dV = 2\pi r h\, dx$

Find intercepts

$-x^2+2 = x$

$x^2+x-2=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1+8}}{2}$

$= \frac{-1 \pm 3}{2} = 1, -2$

using shell's

$$dV = 2\pi r h\, dx$$

$$= 2\pi(x)\left[(-x^2+2)-(x)\right] dx$$

so

$$V = \int_0^1 2\pi x\left[-x^2+2-x\right] dx$$

$$= 2\pi \int_0^1 -x^3 - x^2 + 2x \; dx$$

$$= 2\pi\left[-\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3 + x^2\right]_0^1$$

$$= 2\pi\left[-\tfrac{1}{4}(1)^4 - \tfrac{1}{3}(1)^3 + (1)^2\right] - 2\pi\left[\cancel{-\tfrac{1}{4}(0)^4}^{\,0} - \cancel{\tfrac{1}{3}(0)^3}^{\,0} + \cancel{(0)^2}^{\,0}\right]$$

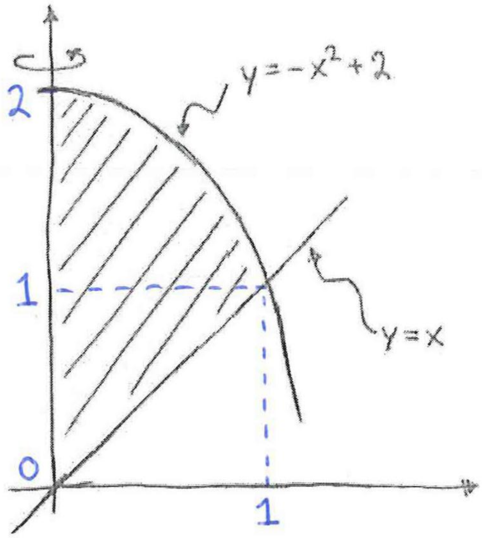
$$V = 2\pi\left[-\tfrac{1}{4} - \tfrac{1}{3} + 1\right]$$

$$= 2\pi\left[\tfrac{5}{12}\right]$$

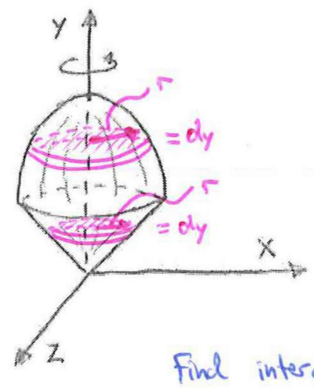
$$\boxed{V = \frac{5\pi}{6} \cong 2.62 \cong 0.83\pi}$$

Find the volume between the y-axis and the two curves revolved around the y-axis (5 marks).
Remember that:

area of a disk = πr^2
area of a shell = $2\pi rh$



$dV = \pi r^2 dy$



Find intercepts (x-values)

$-x^2 + 2 = x$

$x^2 + x - 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{1+8}}{2}$

$= \dfrac{-1 \pm 3}{2} = 1, -2$

Find intercepts (y-values)

$y = x = 1$ & -2

solve for x:

$y = -x^2 + 2$

$x^2 = 2 - y$

$\boxed{x = \pm\sqrt{2-y}}$

$y = x$

$\boxed{x = y}$

using disk's

$dV = \pi r^2 dy$

$= \pi (y)^2 dy$ &

$\pi\left(\pm\sqrt{2-y}\right)^2 dy$

so

$$V = \int_0^1 \pi (y)^2 dy + \int_1^2 \pi\left(\pm\sqrt{2-y}\right)^2 dy$$

$$= \left[\frac{\pi}{3} y^3\right]_0^1 + \int_1^2 2\pi - \pi y \, dy$$

$$= \left[\frac{\pi}{3}(1)^3 - \cancel{\frac{\pi}{3}(0)^3}^{\,0}\right] + \left[2\pi y - \frac{1}{2}\pi y^2\right]_1^2$$

$$= \frac{\pi}{3} + \left[2\pi(2) - \frac{1}{2}\pi(2)^2\right] - \left[2\pi(1) - \frac{1}{2}\pi(1)^2\right]$$

$$= \frac{\pi}{3} + 4\pi - 2\pi - 2\pi + \frac{\pi}{2}$$

$$= \frac{5\pi}{6} \cong 0.83\pi \cong 2.62$$

$$\boxed{V = \frac{5\pi}{6} \cong 2.62}$$