

Instructor: Frank Secretain
Course: Math 101
Date: November 29, 2024

Assessment: Test 3
Time allowed: 110 minutes
Devices allowed: Pencil, pen, eraser, calculator
Notes from instructor: Be neat. Show your work where needed. Box final answers.

Marks allocated: 3 questions worth 20 marks
Percentage of final grade: 20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

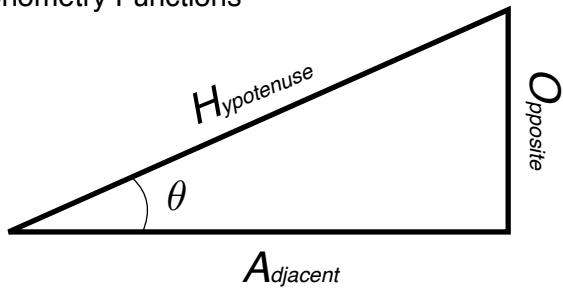
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$\sin(\theta) = \frac{O}{H} \quad \sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$\cos(\theta) = \frac{A}{H} \quad \cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$\tan(\theta) = \frac{O}{A} \quad \tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

$$\vec{v}_{\frac{B}{A}} = -\vec{v}_{\frac{A}{B}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{\det(A_i)}{\det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Unit Conversions

angles

$$2\pi = 6.28 \text{ rad} = 360^\circ$$

mass

$$1 \text{ kg} = 2.2 \text{ lbs.}$$

lengths

$$1 \text{ mile} = 1.6 \text{ km}$$

$$1 \text{ inch} = 2.54 \text{ cm}$$

$$1 \text{ m} = 3.3 \text{ ft}$$

volumes

$$1 \text{ gallon} = 3.78 \text{ Litres}$$

.(3 marks each) Solve for x in the following equations.

$$3(x + 1) - b = 4x + 2$$

$$\frac{4b \sin(\theta) - x}{2} + b = \alpha^2$$

$$\frac{x-1}{x+1}+x=2$$

$$\frac{x-1}{x+1}=a$$

(4 marks) Solve for x and y in the following equation.

$$\frac{6(x + y) - 3x}{y - 4} = 1$$

$$\frac{x - 5(y - x)}{x - 1} + 1 = 3$$

(4 marks) 8 buckets and 13 cups can hold 80 litres of water. However, 7 buckets minus 5 cups of water holds 10 litres of water. What is the volume of the bucket and cup.

(3 marks each) Solve for x in the following equations.

$$3(x + 1) - b = 4x + 2$$

$$3x + 3 - b = 4x + 2$$

$$-x = b - 1$$

$$x = 1 - b$$

$$\frac{4b \sin(\theta) - x}{2} + b = \alpha^2$$

$$\frac{4b \sin \theta - x}{2} = \alpha^2 - b$$

$$4b \sin \theta - x = 2(\alpha^2 - b)$$

$$-x = 2(\alpha^2 - b) - 4b \sin \theta$$

$$x = 4b \sin \theta - 2(\alpha^2 - b)$$

$$\frac{x-1}{x+1} + x = 2$$

$$x-1 + x(x+1) = 2(x+1)$$

$$x-1 + x^2 + x = 2x + 2$$

$$x^2 = 3$$

$$x = \pm \sqrt{3}$$

$$\frac{x-1}{x+1} = a$$

$$x-1 = a(x+1)$$

$$x-1 = ax + a$$

$$x - ax = 1 + a$$

$$x(1-a) = 1 + a$$

$$x = \frac{1+a}{1-a}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{6(x+y) - 3x}{y-4} = 1 \quad (1)$$

$$\frac{x - 5(y-x)}{x-1} + 1 = 3 \quad (2)$$

simplify (1)

$$6x + 6y - 3x = y - 4$$

$$3x + 5y = -4 \quad (1a)$$

simplify (2)

$$x - 5y + 5x = 2(x-1)$$

$$6x - 5y = 2x - 2$$

$$4x - 5y = -2 \quad (2a)$$

solve for y in (1a)

$$y = \frac{-4 - 3x}{5} \quad (1b)$$

sub (1b) into (2a)

$$4x - 5\left[\frac{-4 - 3x}{5}\right] = -2$$

$$4x + 4 + 3x = -2$$

$$7x = -6$$

$$x = \frac{-6}{7} = -0.857$$

sub (2b) into (1b)

$$y = \frac{-4 - 3[-0.857]}{5}$$

$$y = -\frac{2}{7} = -0.286$$

(1b)

(4 marks) 8 buckets and 13 cups can hold 80 litres of water. However, 7 buckets minus 5 cups of water holds 10 litres of water. What is the volume of the bucket and cup.

let b = volume of a bucket

c = " " " cup

so

$$8b + 13c = 80 \quad (1)$$

$$7b - 5c = 10 \quad (2)$$

solve for b in (2):

$$b = \frac{10 + 5c}{7} \quad (2a)$$

sub (2a) into (1):

$$8 \left[\frac{10 + 5c}{7} \right] + 13c = 80$$

$$8(10 + 5c) + 91c = 560$$

$$80 + 40c + 91c = 560$$

$$131c = 480$$

$$c = \frac{480}{131} = 3.66 \text{ litres} \quad (1a)$$

sub (1a) into (2a)

$$b = \frac{10 + 5[3.66]}{7} = 4.05 \text{ litres}$$