Instructor:	Frank Secretain
Course:	Math 101
Date:	December 11, 2024
Assessment:	Final Test
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

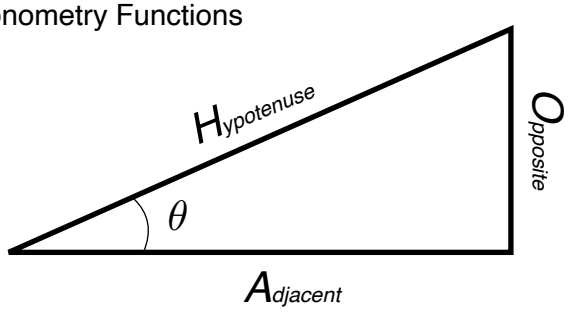
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

$$\vec{v}_{\frac{B}{A}} = -\vec{v}_{\frac{A}{B}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Unit Conversions

angles

$$2\pi = 6.28\,\text{rad} = 360^o$$

mass

$$1\,\text{kg} = 2.2\,\text{lbs.}$$

lengths

$$1\,\text{mile} = 1.6\,\text{km}$$

$$1\,\text{inch} = 2.54\,\text{cm}$$

$$1\,\text{m} = 3.3\,\text{ft}$$

volumes

$$1\,\text{gallon} = 3.78\,\text{Litres}$$

(1 mark each) Solve each expression and keep the correct number of significant digits.

(380)(3.2) + 120 =

(2.43)(3991.76) + 300 =

(1 mark each) Given a unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$37\ \frac{degrees}{s} \rightarrow\ \frac{radians}{hour}$$

$$3.18\ \frac{miles}{L^2} \rightarrow\ \frac{ft}{gal^2}$$

(3 marks) You run 150 m West, 120 m at 30^0 North of East and 70 m at 60^0 West of North. How far are you from where you started?

.(2 marks each) Solve for x in the following equations.

$$\frac{a(x-1)+2}{x-1}+b=3$$

$$\frac{3x(x-2)}{5}-x=1$$

$$1 - \frac{b x^{2} + c - 2}{d - s^{2}} - 3 b = c^{a}$$

$$\frac{b x - 1}{2 x} = 1 - \frac{1}{\sin(2 \theta)}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{3(x+y)-x}{y-1}+1=2$$

$$1-\frac{x+y}{x-y}=4$$

(1 mark each) Solve each expression and keep the correct number of significant digits.

(380)(3.2) + 120 =

1216 + 120 = 1336

= 1300

(2.43)(3991.76) + 300 =

9699.9768 + 300 = 9999.9768

= 10000

= 1.00×10^4

(1 mark each) Given a unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

37 $\frac{degrees}{s} \rightarrow \frac{radians}{hour}$

$37 \frac{deg}{s}\left(\frac{60\,s}{1\,min}\right)\left(\frac{60\,min}{1\,hour}\right)\left(\frac{2\pi\,rad}{360\,deg}\right)$ = 2324.78 $\frac{rad}{hour}$

= 2300 $\frac{rad}{hour}$

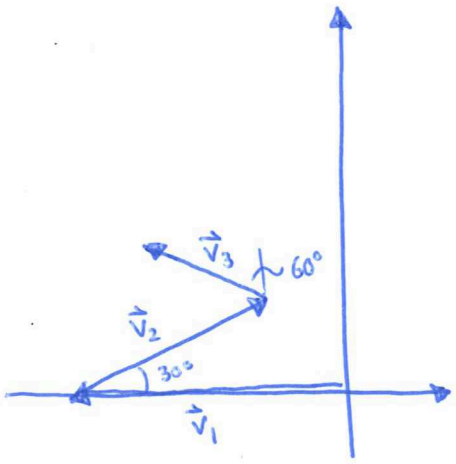
3.18 $\frac{miles}{L^2} \rightarrow \frac{ft}{gal^2}$

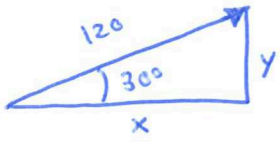
$3.18 \frac{miles}{L^2}\left(\frac{1.6\,km}{1\,mile}\right)\left(\frac{1000\,m}{1\,km}\right)\left(\frac{3.3\,ft}{1\,m}\right)\left(\frac{3.78^2\,L^2}{1^2\,gal^2}\right)$

= 239907.95 $\frac{ft}{gal^2}$

= 2.40×10^5 $\frac{ft}{gal^2}$

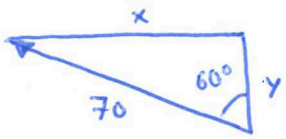
(3 marks) You run 150 m West, 120 m at 30° North of East and 70 m at 60° West of North. How far are you from where you started?





$x = 120 \cos(30) = 103.923$

$y = 120 \sin(30) = 60$



$x = 70 \sin(60) = 60.622$

$y = 70 \cos(60) = 35$

$$\vec{V}_1 = -150\,\hat{x} + 0\,\hat{y}$$

$$\vec{V}_2 = 103.923\,\hat{x} + 60\,\hat{y}$$

$$+\ \vec{V}_3 = -60.622\,\hat{x} + 35\,\hat{y}$$

$$\vec{V}_f = -106.699\,\hat{x} + 95\,\hat{y}$$

$$\left|\vec{V}_f\right| = \sqrt{(-106.699)^2 + (95)^2}$$

$$= 142.86$$

$$\boxed{\vec{V}_f = 143 \text{ m}}$$

.(2 marks each) Solve for x in the following equations.

$$\frac{a(x-1)+2}{x-1}+b=3$$

$ax - a + 2 + b(x-1) = 3(x-1)$

$ax - a + 2 + bx - b = 3x - 3$

$ax + bx - 3x = a + b - 5$

$$x = \frac{a+b-5}{a+b-3}$$

$$\frac{3x(x-2)}{5}-x=1$$

$3x^2 - 6x - 5x = 5$

$3x^2 - 11x - 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{11 \pm \sqrt{(-11)^2 - 4(3)(-5)}}{2(3)}$$

$$= \frac{11}{6} \pm 2.242$$

$= 4.08$ or -0.409

$x = 4.08$ or -0.41

$$1 - \frac{bx^2 + c - 2}{d - s^2} - 3b = c^a$$

$$\frac{bx^2 + c - 2}{d - s^2} = 1 - 3b - c^a$$

$$bx^2 + c - 2 = (1 - 3b - c^a)(d - s^2)$$

$$x = \sqrt{\frac{(1 - 3b - c^a)(d - s^2) + 2 - c}{b}}$$

$$\frac{bx - 1}{2x} = 1 - \frac{1}{\sin(2\theta)}$$

$$(bx - 1)\sin(2\theta) = 2x\sin(2\theta) - 2x$$

$$bx\sin(2\theta) - \sin(2\theta) = 2x\sin(2\theta) - 2x$$

$$x(b\sin(2\theta) - 2\sin(2\theta) + 2) = \sin(2\theta)$$

$$x = \frac{\sin(2\theta)}{b\sin(2\theta) - 2\sin(2\theta) + 2}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{3(x+y)-x}{y-1}+1=2 \qquad (1)$$

$$1-\frac{x+y}{x-y}=4 \qquad (2)$$

simplify (1):

$3x + 3y - x = y - 1$

$2x + 2y = -1$ (1a)

simplify (2):

$-3x + 3y = x + y$

$4x - 2y = 0$ (2a)

solve (2a) for y:

$y = 2x$ (2b)

sub (2b) into (1a)

$2x + 2[2x] = -1$

$6x = -1$

$x = -\frac{1}{6} = -0.1\overline{6}$ (1b)

sub (1b) into (2b)

$y = 2\left[-\frac{1}{6}\right]$

$y = -\frac{1}{3} = -0.\overline{3}$