Instructor:	Frank Secretain
Course:	Math 101
Assessment:	Final Test
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 20 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Order of Operations

$$ac + bc = c(a + b)$$

exponents

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

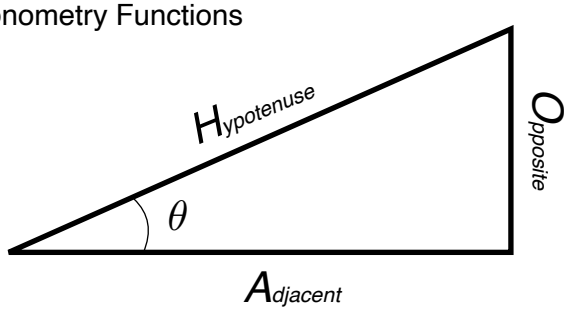
$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

radicals

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

Trigonometry Functions



$$sin(\theta) = \frac{O}{H} \qquad sin^{-1}\left(\frac{O}{H}\right) = \theta$$

$$cos(\theta) = \frac{A}{H} \qquad cos^{-1}\left(\frac{A}{H}\right) = \theta$$

$$tan(\theta) = \frac{O}{A} \qquad tan^{-1}\left(\frac{O}{A}\right) = \theta$$

Pythagoras Theorem

$$H^2 = O^2 + A^2$$

Relative Velocity

$$\vec{v}_{\frac{A}{C}} = \vec{v}_{\frac{A}{B}} + \vec{v}_{\frac{B}{C}}$$

$$\vec{v}_{\frac{B}{A}} = -\vec{v}_{\frac{A}{B}}$$

Linear equations (Cramer's rule)

$$x_i = \frac{det(A_i)}{det(A)}$$

Forms of a 1st order polynomial

$$y = ax + b$$

Forms of a 2nd order polynomial

$$y = ax^2 + bx + c$$

$$y = a(x - h)^2 + k$$

$$y = (x - m)(x - n)$$

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Unit Conversions

angles

$$2\pi = 6.28\,\text{rad} = 360^o$$

mass

$$1\,\text{kg} = 2.2\,\text{lbs.}$$

lengths

$$1\,\text{mile} = 1.6\,\text{km}$$

$$1\,\text{inch} = 2.54\,\text{cm}$$

$$1\,\text{m} = 3.3\,\text{ft}$$

volumes

$$1\,\text{gallon} = 3.78\,\text{Litres}$$

(1 mark) Solve the expression to the correct number of significant digits.

1.419 + (230)(0.020) =

(1 mark each) Given a unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$37\ \frac{degrees}{s} \rightarrow\ \frac{radians}{hour}$$

$$3.18\ \frac{miles}{L^2} \rightarrow\ \frac{ft}{gal^2}$$

(3 marks) You run 100 m East, 150 m at 70^0 North of East and 30 m at 60^0 West of North. How far are you from where you started?

.(2 marks each) Solve for x in the following equations.

$$\frac{a(x-1)+2}{x-1}+b=3$$

$$\frac{a+1}{3\cos(\theta)}-\frac{1}{x}=1$$

$$1 - x = \frac{4(x-3)}{2x}$$

$$\frac{2\sin(3x-1) + a^n}{b} - 1 = 3$$

$$1 = \frac{2}{b}(\alpha - x^2) + 2$$

(4 marks) Solve for x and y in the following equation.

$$\frac{x - y}{x + y} - 1 = 3$$

$$1 - \frac{3(x + 1)}{y} = 2$$

(1 mark) Solve the expression to the correct number of significant digits.

$1.419 + (230)(0.020) =$

$1.419 + 4.6 = 6.019$

$= 6.0$

(1 mark each) Given a unit conversion table on the formula sheet (1st page), convert each of the numbers to the stated units.

$$37 \frac{degrees}{s} \rightarrow \frac{radians}{hour}$$

$$37 \frac{deg}{s}\left(\frac{2\pi\ rad}{360^{\circ}}\right)\left(\frac{60\ sec}{1\ min}\right)\left(\frac{60\ min}{1\ hour}\right) = 2324.78$$

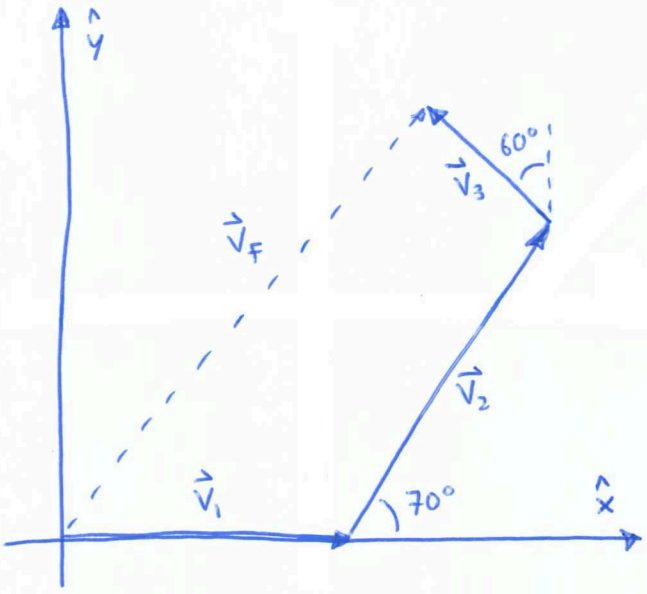
$$= 2300 \frac{rad}{hour}$$

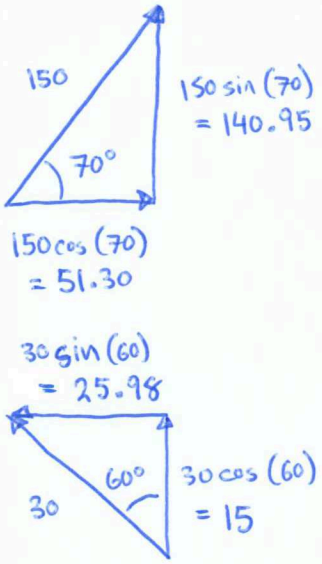
$$3.18 \frac{miles}{L^2} \rightarrow \frac{ft}{gal^2}$$

$$3.18 \frac{miles}{L^2}\left(\frac{1.6\ Km}{1\ mile}\right)\left(\frac{1000\ m}{1\ Km}\right)\left(\frac{3.3\ ft}{1\ m}\right)\left(\frac{3.78\ L}{1\ gal}\right)\left(\frac{3.78\ L}{1\ gal}\right) = 239908$$

$$= 240000 \frac{ft}{gal^2}$$

(3 marks) You run 100 m East, 150 m at 70º North of East and 30 m at 60º West of North. How far are you from where you started?





$\vec{V}_1 = 100\,\hat{x} + 0\,\hat{y}$

$\vec{V}_2 = 51.3\,\hat{x} + 140.95\,\hat{y}$

$\vec{V}_3 = -25.98\,\hat{x} + 15\,\hat{y}$

$\vec{V}_F = 125.32\,\hat{x} + 155.95\,\hat{y}$

$$|\vec{V}_F| = \sqrt{(125.32)^2 + (155.95)^2}$$

$$\boxed{|\vec{V}_F| = 200\,m}$$

.(2 marks each) Solve for x in the following equations.

$$\frac{a(x-1)+2}{x-1}+b=3$$

$a(x-1)+2+b(x-1)=3(x-1)$

$ax-a+2+bx-b=3x-3$

$ax+bx-3x=a+b-5$

$x(a+b-3)=a+b-5$

$$x=\frac{a+b-5}{a+b-3}$$

$$\frac{a+1}{3\cos(\theta)}-\frac{1}{x}=1$$

$a+1-\frac{3\cos\theta}{x}=3\cos\theta$

$ax+x-3\cos\theta=3x\cos\theta$

$ax+x-3x\cos\theta=3\cos\theta$

$x(a+1-3\cos\theta)=3\cos\theta$

$$x=\frac{3\cos\theta}{a+1-3\cos\theta}$$

$$1 - x = \frac{4(x-3)}{2x}$$

$2x - 2x^2 = 4(x-3)$

$2x - 2x^2 = 4x - 12$

$2x^2 + 2x - 12 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(2)(-12)}}{2(2)}$$

$= -\frac{1}{2} \pm \frac{5}{2} = 2, -3$

$\boxed{x = 2, -3}$

$$\frac{2\sin(3x-1) + a^n}{b} - 1 = 3$$

$2\sin(3x-1) + a^n - b = 3b$

$2\sin(3x-1) = 4b - a^n$

$\sin(3x-1) = \frac{4b - a^n}{2}$

$3x - 1 = \sin^{-1}\left(\frac{4b-a^n}{2}\right)$

$3x = \sin^{-1}\left(\frac{4b-a^n}{2}\right) + 1$

$\boxed{x = \frac{1}{3}\left(\sin^{-1}\left(\frac{4b-a^n}{2}\right) + 1\right)}$

$$1 = \frac{2}{b}(\alpha - x^2) + 2$$

$$b = 2(\alpha - x^2) + 2b$$

$$2(\alpha - x^2) = b - 2b$$

$$\alpha - x^2 = \frac{-b}{2}$$

$$-x^2 = -\frac{b}{2} - \alpha$$

$$x^2 = \frac{b}{2} + \alpha$$

$$\boxed{x = \sqrt{\frac{b}{2} + \alpha}}$$

(4 marks) Solve for x and y in the following equation.

$$\frac{x-y}{x+y} - 1 = 3 \qquad (1)$$

$$1 - \frac{3(x+1)}{y} = 2 \qquad (2)$$

simplify (1)

$$x - y - (x + y) = 3(x + y)$$

$$x - y - x - y = 3x + 3y$$

$$0 = 3x + 5y \qquad (1a)$$

simplify (2)

$$y - 3(x+1) = 2y$$

$$-y - 3x - 3 = 0$$

$$3x + y = -3 \qquad (2a)$$

solve (1a) for x or y:

$$x = \frac{-5y}{3}$$

$$y = \frac{-3x}{5}$$

solve (2a) for x or y:

$$x = \frac{-3-y}{3}$$

$$y = -3 - 3x \qquad (2b)$$

sub (2b) into (1a)

$$0 = 3x + 5[-3 - 3x]$$

$$0 = 3x - 15 - 15x$$

$$15 = -12x$$

$$\boxed{x = -\frac{15}{12} = -\frac{5}{4} = -1.25} \qquad (1b)$$

sub (1b) into (2b)

$$\boxed{y = -3 - 3[-1.25] = \frac{3}{4} = 0.75}$$