

Instructor: Frank Secretain
 Course: Civil 46
 Date: October 21, 2026

Assessment: Test 2 (practice)
 Time allowed: 110 minutes
 Devices allowed: Pencil, pen, eraser, calculator
 Notes from instructor: Be neat. Show your work where needed. Box final answers.

Marks allocated: 3 questions worth 20 marks
 Percentage of final grade: 20% of final grade

Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + l^2)$ $I_{yy} = \frac{1}{12} m(b^2 + l^2)$ $I_{zz} = \frac{1}{12} m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12} mb^2 + \frac{1}{3} ml^2$
Circular Cylinder		—	$I_{xx} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Sphere		—	$I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Uniform Slender Rod		—	$I_{yy} = \frac{1}{12} ml^2$ $I_{y_1y_1} = \frac{1}{3} ml^2$

Formula Sheet

Line equation

$$y = ax + b$$

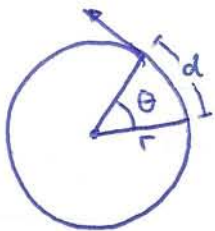
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a}$$

$$\vec{F}_g = \frac{Gm_1 m_2}{r^2} = mg$$

$$\vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

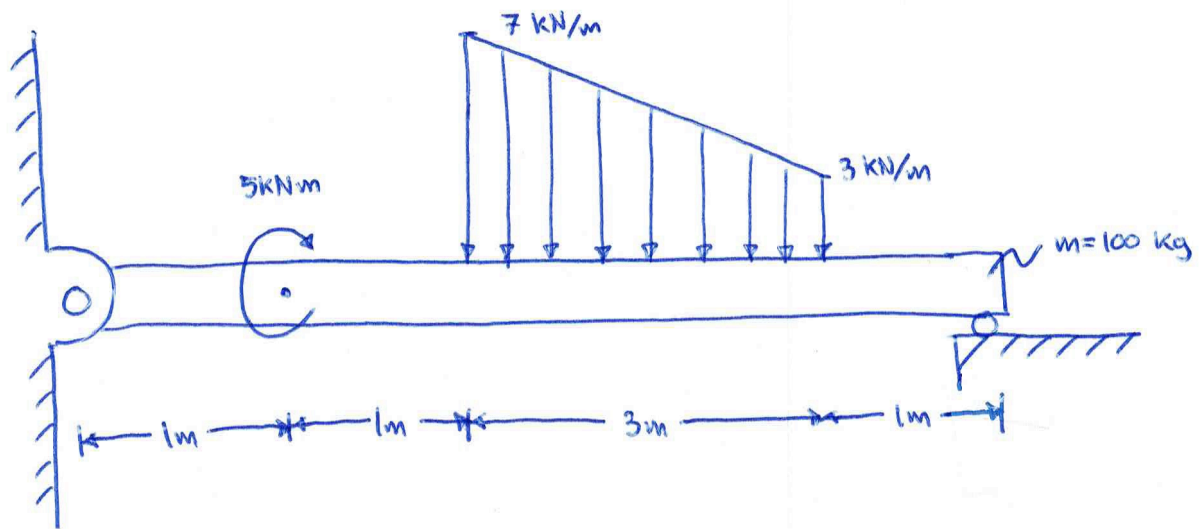
$$E_f - E_o = Q - W$$

$$E_{PE} = mg(y - y_o)$$

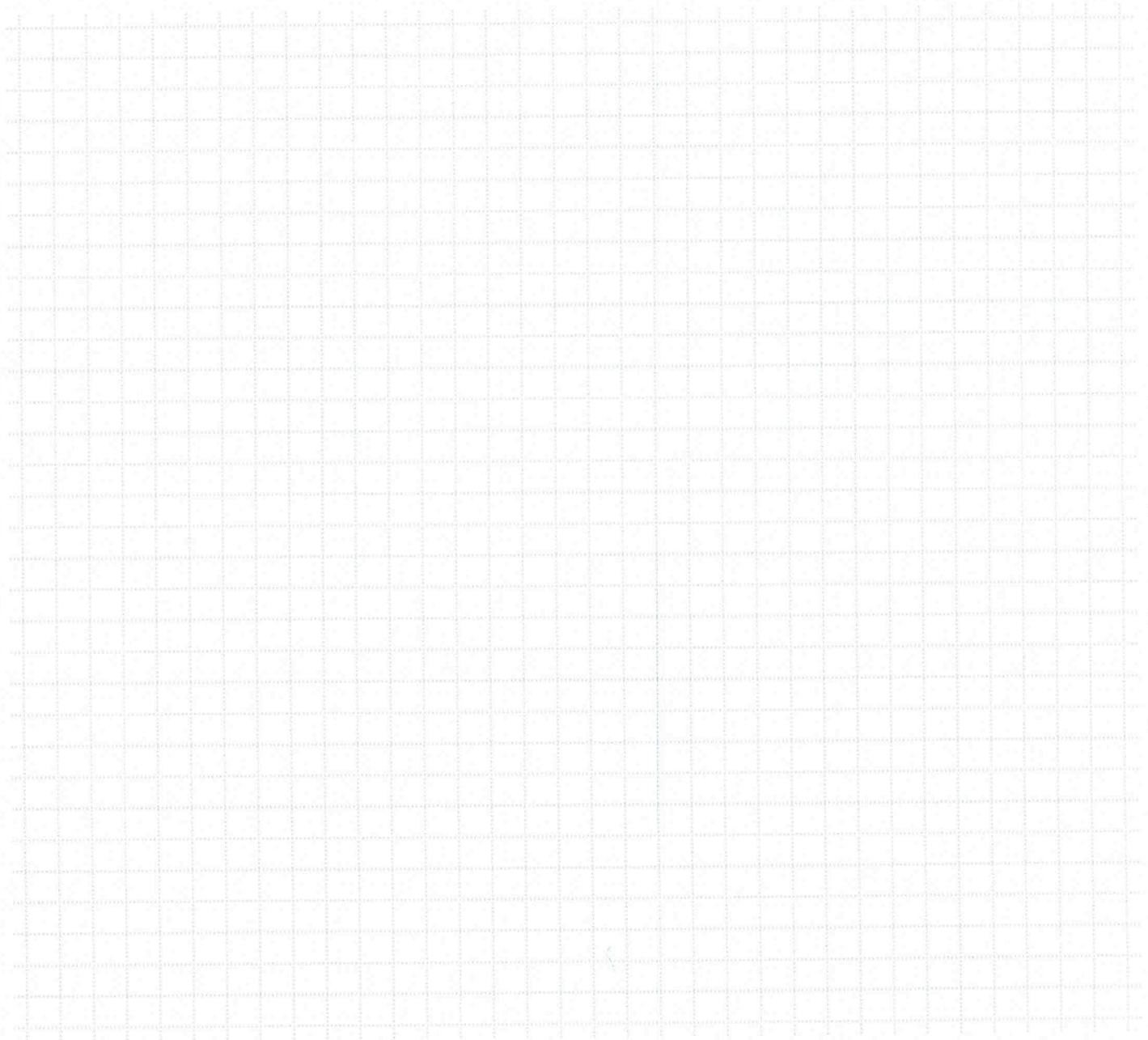
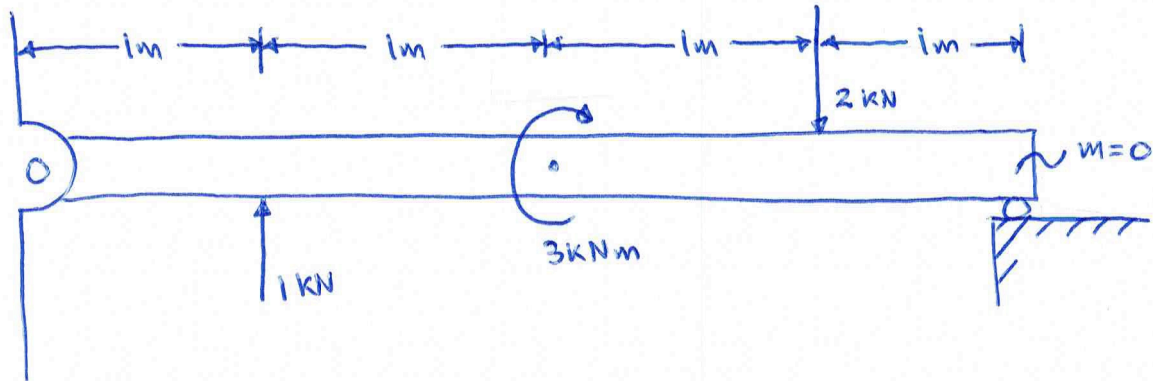
$$E_{KE} = \frac{1}{2} m v^2$$

$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

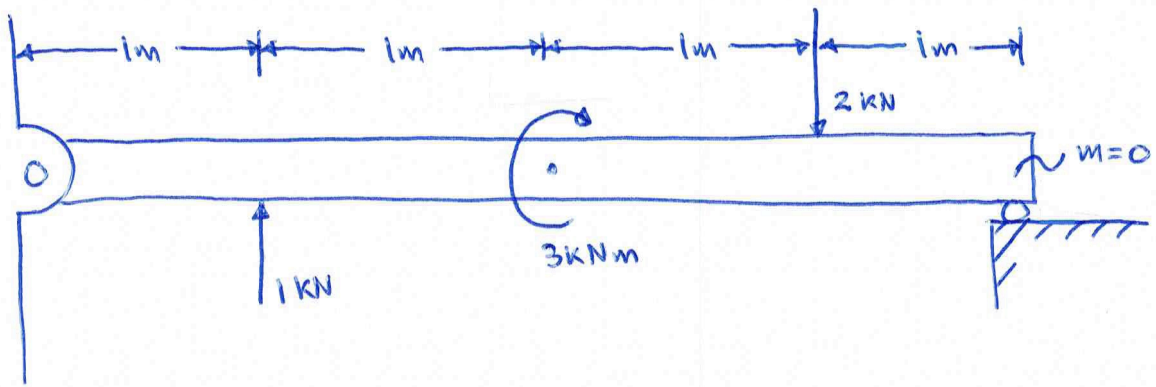
(7 marks) Determine all the reaction forces on the beam member.



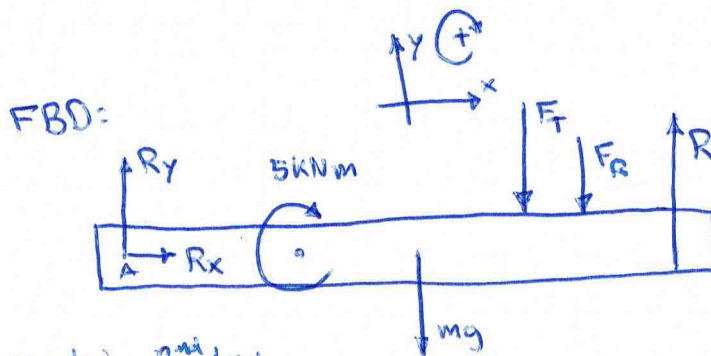
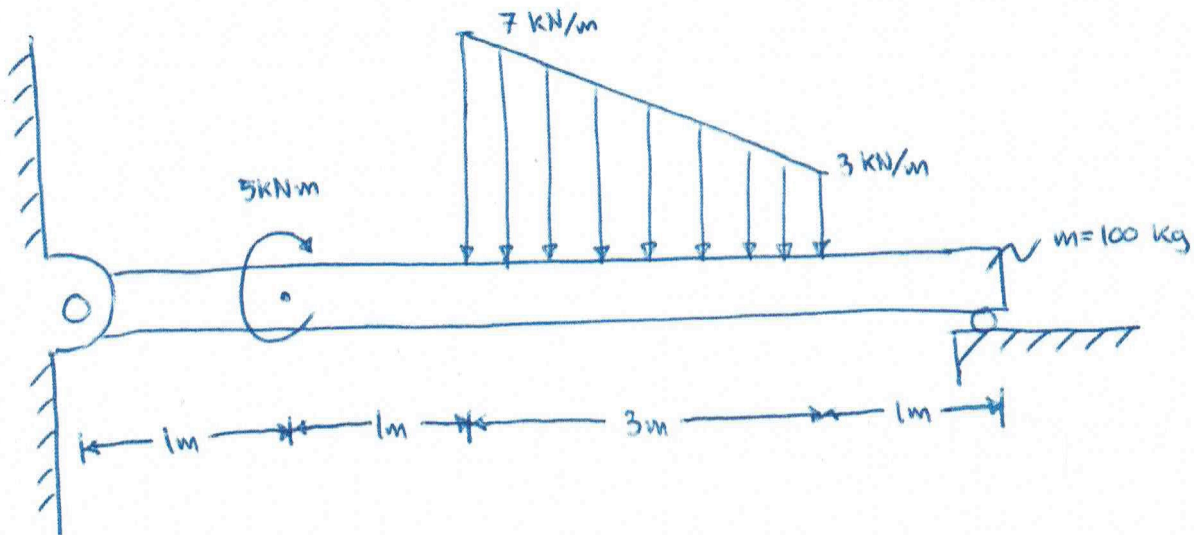
(5 marks) Draw the shear and bending moment diagrams for:



(8 marks) Determine the shear, axial and bending moment equations for



(7 marks) Determine all the reaction forces on the beam member.



Distributed loads:

Triangle: $F_T = \frac{1}{2}(3)(7-3) = 6 \text{ kN}$

$x_T = 2 + \frac{1}{3}(3) = 3 \text{ m}$

Rectangle: $F_R = (3)(3) = 9 \text{ kN}$

$x_R = 2 + \frac{1}{2}(3) = 3.5 \text{ m}$

Newton's 2nd Law

$$\sum F_x = m a_x = 0 = R_x \quad (1)$$

$$\sum F_y = m a_y = 0 = R_y - mg - F_T - F_R + R \quad (2)$$

$$\sum M = \cancel{I \alpha} = 5000 + 3mg + x_T F_T + x_R F_R - 6R \quad (3)$$

Solve (1) for R_x :

$$\boxed{R_x = 0}$$

Solve (3) for R :

$$R = \frac{1}{6} (5000 + (3)(100)(9.81) + (3)(6000) + (3.5)(9000))$$

$$\boxed{R = 9574 \text{ N}}$$

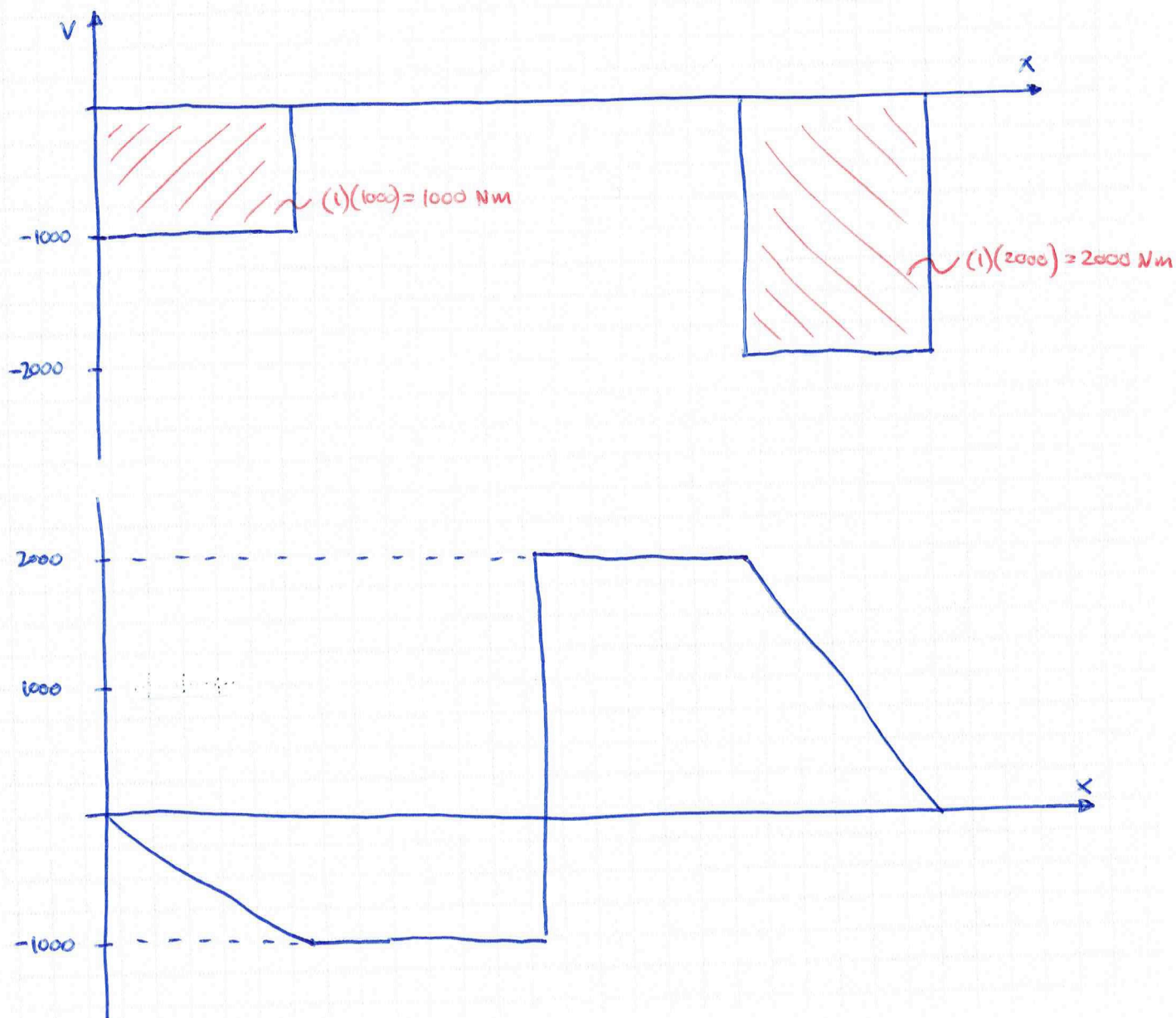
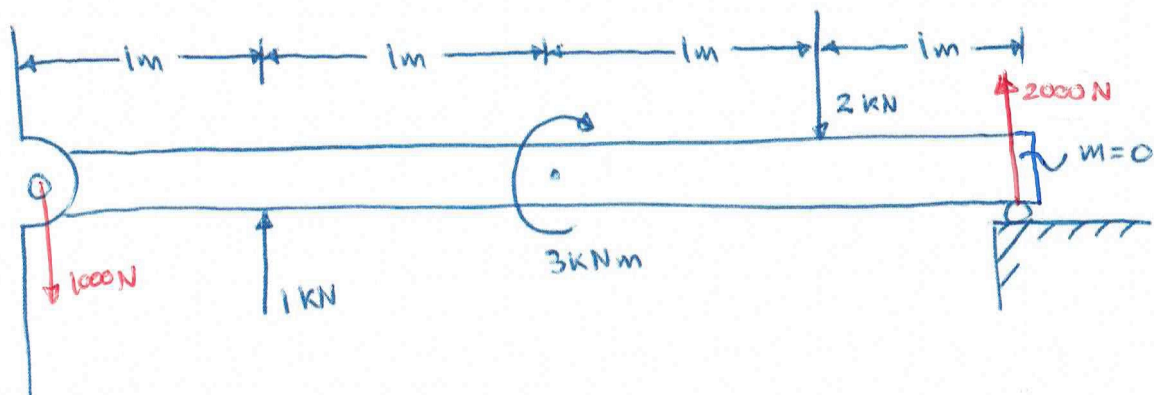
Solve (2) for R_y :

$$R_y = F_T + F_R - R + mg$$

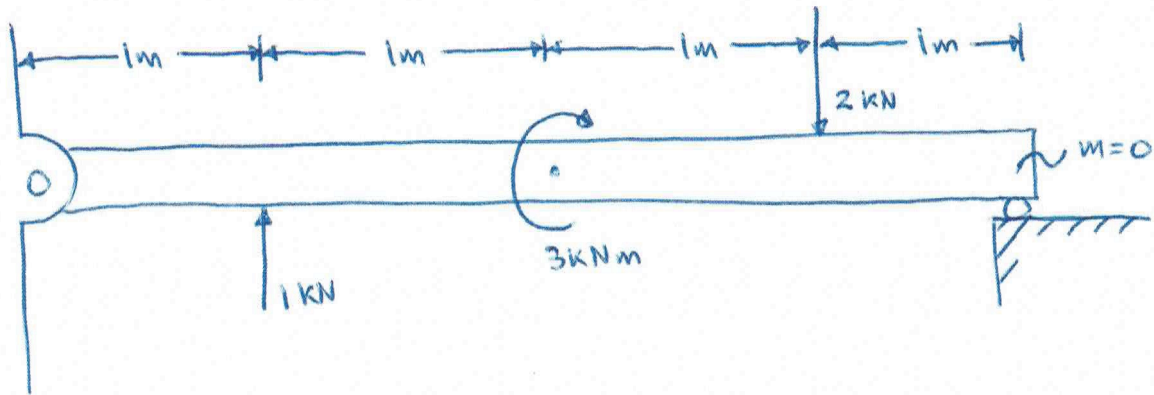
$$= [6000] + [9000] - [9574] + (100)(9.81)$$

$$\boxed{R_y = 6407 \text{ N}}$$

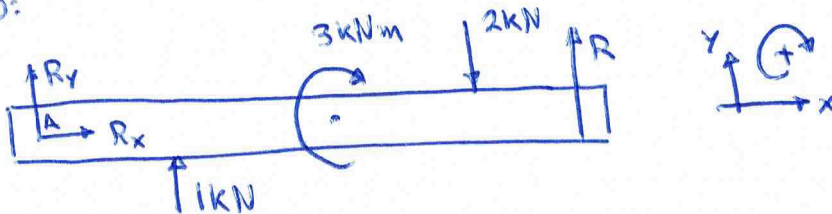
(5 marks) Draw the shear and bending moment diagrams for:



(8 marks) Determine the shear, axial and bending moment equations for



FBD:



Newton's 2nd Law:

$$\sum F_x = m a_x = R_x \quad (1)$$

$$\sum F_y = m a_y = R_y + 1000 - 2000 + R \quad (2)$$

$$\sum M = I_{xx} \alpha = -(1)(1000) + 3000 + (3)(2000) - 4R \quad (3)$$

Solve (1) for R_x :

$$\boxed{R_x = 0}$$

Solve (3) for R :

$$R = \frac{1}{4}(-1000 + 3000 + 6000)$$

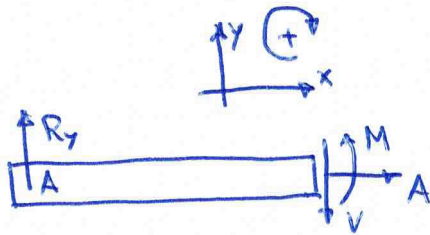
$$\boxed{R = 2000 \text{ N}}$$

Solve (2) for R_y :

$$\begin{aligned} R_y &= 1000 - R \\ &= 1000 - [2000] \end{aligned}$$

$$\boxed{R_y = -1000 \text{ N}}$$

Section cut ($0 < x < 1$)



$$\sum F_x = \cancel{w} \cancel{dx} = A \quad (4)$$

$$\sum F_y = \cancel{w} \cancel{dy} = R_y - V \quad (5)$$

$$\sum M = \cancel{I} \cancel{dx} = xV - M \quad (6)$$

Solve (4) for A:

$$A = 0$$

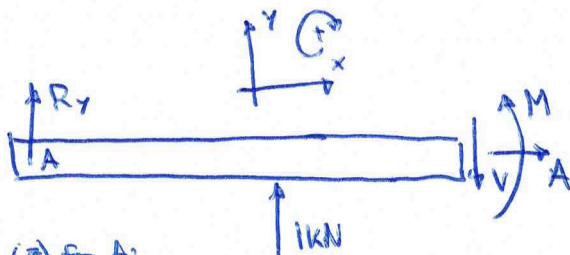
Solve (5) for V:

$$V = R_y = -1000 \text{ N}$$

Solve (6) for M:

$$M = xV = -1000x$$

Section cut ($1 < x < 2$)



$$\sum F_x = \cancel{w} \cancel{dx} = A \quad (7)$$

$$\sum F_y = \cancel{w} \cancel{dy} = R_y + 1000 - V \quad (8)$$

$$\sum M = \cancel{I} \cancel{dx} = -(1)(1000) + xV - M \quad (9)$$

Solve (7) for A:

$$A = 0$$

Solve (8) for V:

$$V = R_y + 1000$$

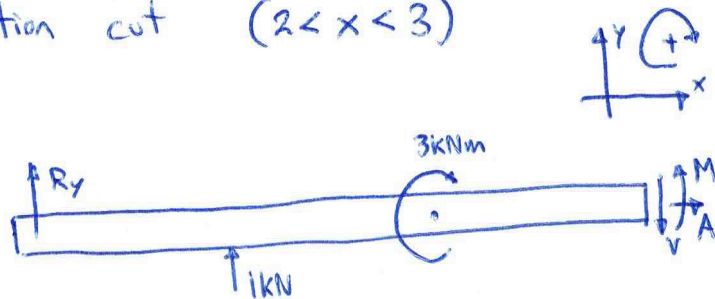
$$V = [-1000] + 1000 = 0$$

Solve (9) for M:

$$M = xV - 1000$$

$$M = x[0] - 1000$$

Section cut ($2 < x < 3$)



$$\sum F_x = \cancel{w_x} = A \quad (10)$$

$$\sum F_y = \cancel{w_y} = R_y + 1000 - V \quad (11)$$

$$\sum M = \cancel{I_x} = -(1)(1000) + 3000 + xV - M \quad (12)$$

Solve (10) for A:

$$\boxed{A = 0}$$

Solve (11) for V:

$$V = R_y + 1000$$

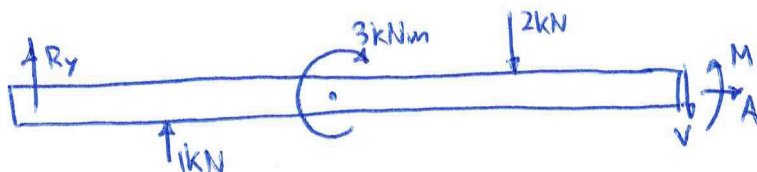
$$\boxed{V = [-1000] + 1000 = 0}$$

Solve (12) for M:

$$M = -1000 + 3000 + xV$$

$$\boxed{M = 2000 + x[0] = 2000}$$

Section cut ($3 < x < 4$)



$$\sum F_x = \cancel{w_x} = A \quad (13)$$

$$\sum F_y = \cancel{w_y} = R_y + 1000 - 2000 - V \quad (14)$$

$$\sum M = \cancel{I_x} = -(1)(1000) + 3000 + (3)(2000) + xV - M \quad (15)$$

Solve (13) for A:

$$\boxed{A = 0}$$

Solve (14) for V:

$$V = R_y - 1000$$

$$\boxed{V = [-1000] - 1000 = -2000 \text{ N}}$$

Solve (15) for M:

$$M = 8000 + xV$$

$$\boxed{M = -2000x + 8000}$$