

Instructor:	Frank Secretain
Course:	CIVL 11
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 25 marks + 1 bonus question worth 2 marks
Percentage of final grade:	25% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

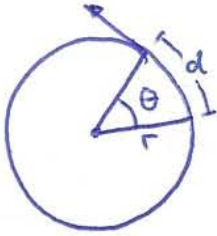
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \vec{F}_g = \frac{Gm_1m_2}{r^2} = mg \quad \vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

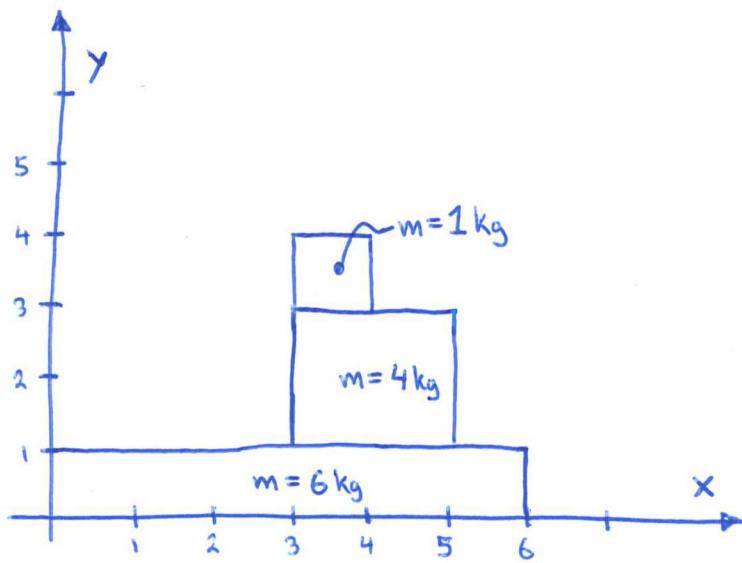
$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

$$E_f - E_o = Q - W \quad E_{PE} = mg(y - y_o) \quad E_{KE} = \frac{1}{2}mv^2$$

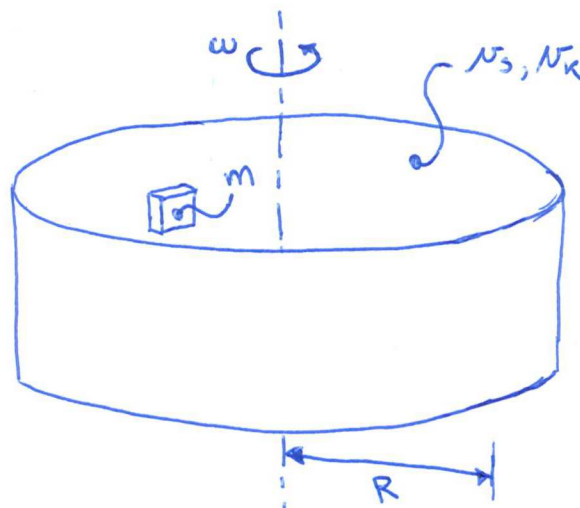
$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

(4 marks) Determine the centre of mass for the three mass system and draw it on the diagram.

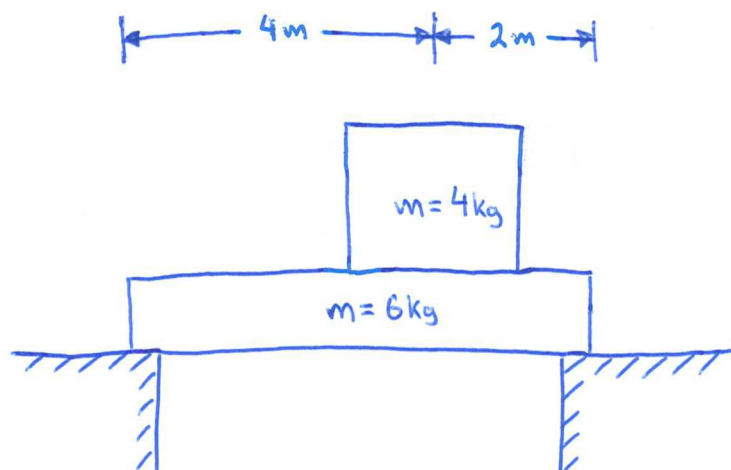


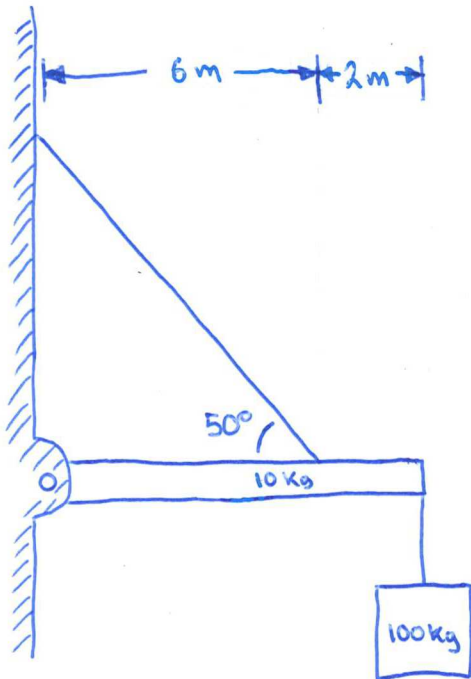
(6 marks) Determine the minimum constant angular velocity [in rpm, repetitions per minute] of the hoop so the 1 kg block inside does not slide down.

Given: $m = 1 \text{ kg}$
 $R = 2.5 \text{ m}$
 $\mu_s = 0.2$
 $\mu_k = 0.1$



(6 marks) Determine both normal forces on either side of the 6 kg block.

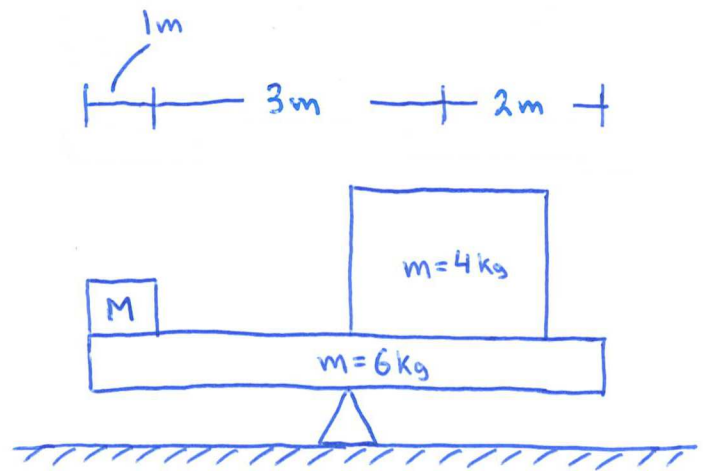




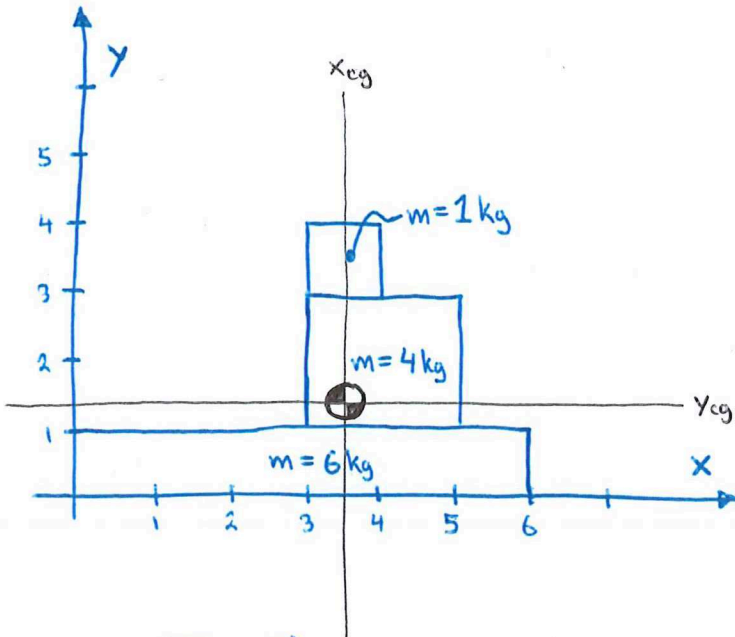
(9 marks) Determine the tension in each rope and the magnitude of the reaction force in the pin.

(2 marks) Bonus

Determine the mass M so the system is balanced on the triangle.



(4 marks) Determine the centre of mass for the three mass system and draw it on the diagram.



$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

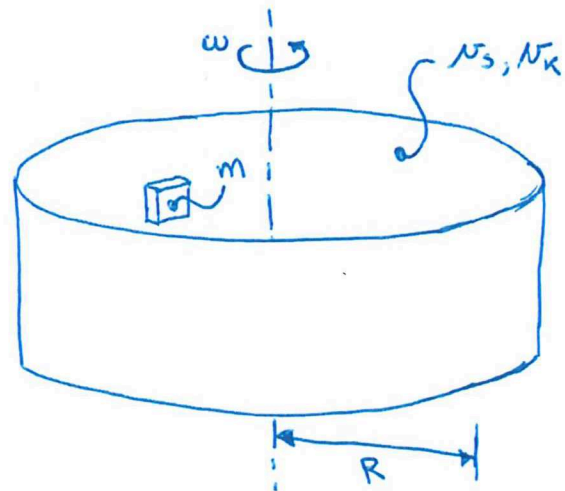
$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6)(3) + (4)(4) + (1)(3.5)}{(6) + (4) + (1)} = 3.41$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6)(0.5) + (4)(2) + (1)(3.5)}{(6) + (4) + (1)} = 1.32$$

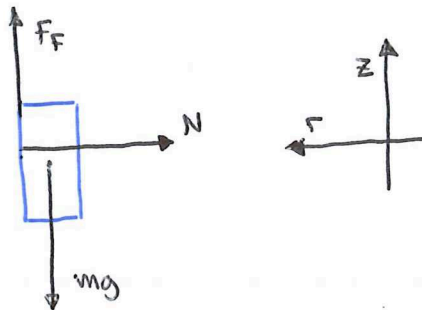
$x_{cg} = 3.41$ $y_{cg} = 1.32$

(6 marks) Determine the minimum constant angular velocity [in rpm, repetitions per minute] of the hoop so the 1 kg block inside does not slide down.

Given: $m = 1 \text{ kg}$
 $R = 2.5 \text{ m}$
 $\mu_s = 0.2$
 $\mu_k = 0.1$



FBD:



Newton's 2nd Law:

$$\sum F_z = m a_z = 0 = F_F - mg \quad (1)$$

$$\sum F_r = m a_r = -N \quad (2)$$

Maximum Friction:

$$F_F = \mu_s N \quad (3)$$

Constant angular velocity:

$$a_r = -\omega^2 R \quad (4)$$

sub (4) into (2)

$$m[-\omega^2 R] = -N$$

$$N = m\omega^2 R \quad (2a)$$

sub (3) into (1)

$$0 = [\mu_s N] - mg \quad (1a)$$

sub (2a) into (1a)

$$0 = \mu_s [m\omega^2 R] - mg$$

$$\mu_s \omega^2 R = g$$

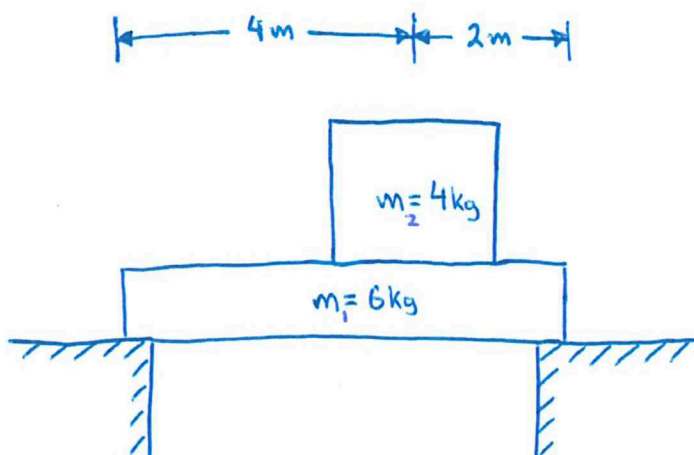
$$\omega = \sqrt{\frac{g}{\mu_s R}} = \sqrt{\frac{9.81}{(0.2)(2.5)}}$$

$$= 4.43 \frac{1}{\text{sec}} \left(\frac{1 \text{ cycle}}{2\pi \text{ rads}} \right) \left(\frac{60 \text{ sec}}{1 \text{ min}} \right)$$

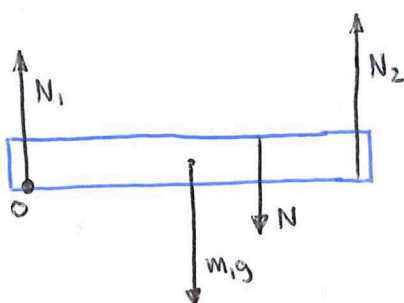
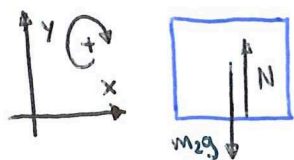
$$\omega = 42.3 \text{ rpm}$$

$$V = 11.1 \text{ m/s}$$

(6 marks) Determine both normal forces on either side of the 6 kg block.



FBD:



Newton's Laws (m_2):

$$\sum F_x = m_2 a_{x2} = 0 \quad (a)$$

$$\sum F_y = m_2 a_{y2} = N - m_2g \quad (b)$$

solve (b) for N :

$$N = m_2g \quad (b)$$

Newton's Laws (m_1):

$$\sum F_x = m_1 a_{x1} = 0 \quad (1)$$

$$\sum F_y = m_1 a_{y1} = N_1 + N_2 - N - m_1g \quad (2)$$

$$\sum M_o = I_o \alpha_o = (m_1g)(3) + (N)(4) - (N_2)(6) \quad (3)$$

sub (b) into (3):

$$0 = 3m_1g + 4[m_2g] - 6N_2$$

$$N_2 = \frac{1}{6}(3m_1g + 4m_2g)$$

$$= \frac{1}{6}((3)(6)(9.81) + (4)(4)(9.81))$$

$$N_2 = 55.6 \text{ N}$$

(3a)

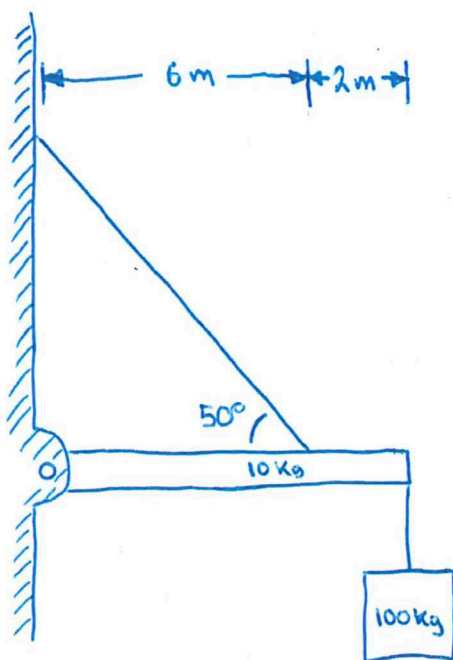
sub (3a) & (b) into

$$0 = N_1 + [55.6] - [m_2g] - m_1g$$

$$N_1 = m_2g + m_1g - 55.6$$

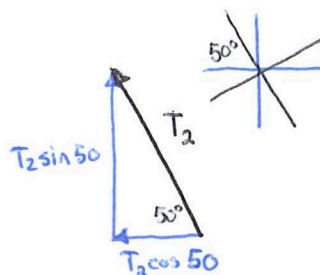
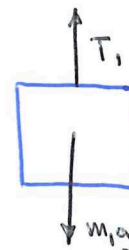
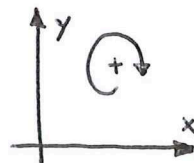
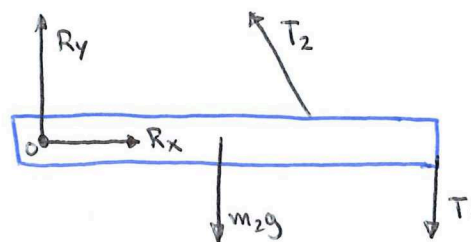
$$= (4)(9.81) + (6)(9.81) - 55.6$$

$$N_1 = 42.5 \text{ N}$$



(9 marks) Determine the tension in each rope and the magnitude of the reaction force in the pin.

FBD:



Newton's Law's m_1 :

$$\sum F_x = m_1 a_{x1} = 0 \quad (1)$$

$$\sum F_y = m_1 a_{y1} = T_1 - m_1 g \quad (2)$$

solve (2) for T_1 :

$$T_1 = m_1 g = (100)(9.81)$$

$$\boxed{T_1 = 981 \text{ N}} \quad (2a)$$

Newton's Law m_2 :

$$\sum F_x = m_2 a_{x2} = R_x - T_2 \cos 50 \quad (1)$$

$$\sum F_y = m_2 a_{y2} = R_y + T_2 \sin 50 - m_2 g - T_1 \quad (2)$$

$$\sum M_o = I_o \alpha_o = (m_2 g)(4) + (T_1)(8) - (T_2 \sin 50)(6) \quad (3)$$

sub (2a) into (3):

$$0 = 4m_2 g + 8[m_1 g] - 6T_2 \sin 50$$

$$T_2 = \frac{4m_2 g + 8m_1 g}{6 \sin 50} = \frac{(4)(10)(9.81) + (8)(100)(9.81)}{6 \sin 50}$$

$$\boxed{T_2 = 1793 \text{ N}}$$

(3a)

sub (3a) into (1)

$$0 = R_x - [1793] \cos 50$$

$$R_x = 1793 \cos 50$$

$$\boxed{R_x = 1152 \text{ N}}$$

sub (2a) & (3a) into (2)

$$0 = R_y + [1793] \sin 50 - m_2 g - [981]$$

$$\begin{aligned} R_y &= m_2 g + 981 - 1793 \sin 50 \\ &= (10)(9.81) + 981 - 1793 \sin 50 \end{aligned}$$

$$\boxed{R_y = -294 \text{ N}}$$

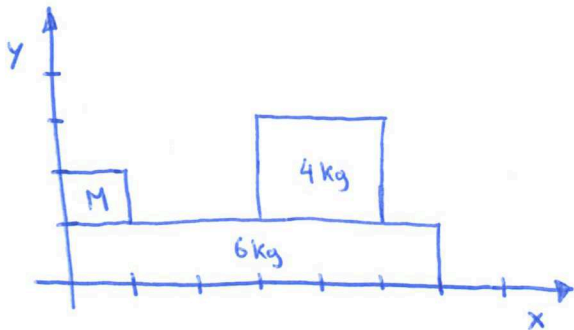
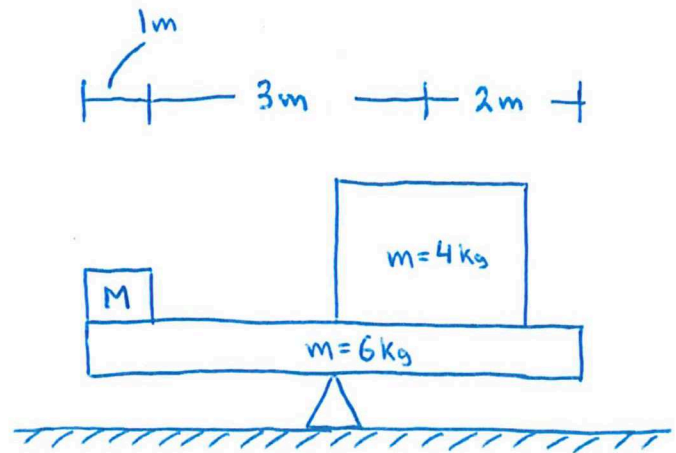
solve for reaction force

$$\begin{aligned} |\vec{R}| &= \sqrt{R_x^2 + R_y^2} \\ &= \sqrt{(1152)^2 + (-294)^2} \end{aligned}$$

$$\boxed{|\vec{R}| = 1189 \text{ N}}$$

(2 marks) Bonus

Determine the mass M so the system is balanced on the triangle.



$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(M)(0.5) + (6)(3) + (4)(4)}{M + 6 + 4} \stackrel{\text{set to}}{=} 3 \quad (\text{balanced on triangle})$$

$$0.5M + 18 + 16 = 3(M + 10)$$

$$0.5M + 34 = 3M + 30$$

$$-2.5M = -4$$

$$M = 1.6\text{ kg}$$