

Instructor:	Frank Secretain
Course:	CIVL 11
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 25 marks
Percentage of final grade:	25% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

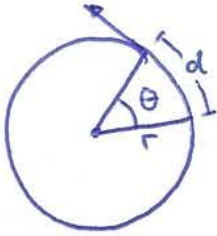
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \vec{F}_g = \frac{Gm_1m_2}{r^2} = mg \quad \vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{r} \times \vec{F} = \vec{F} \perp \vec{r}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

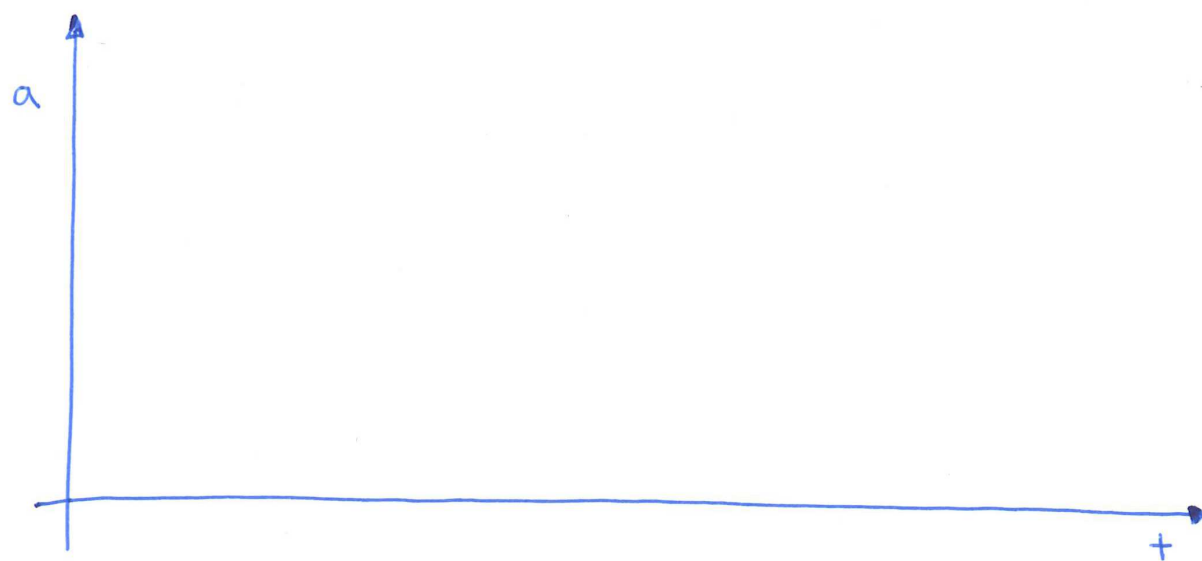
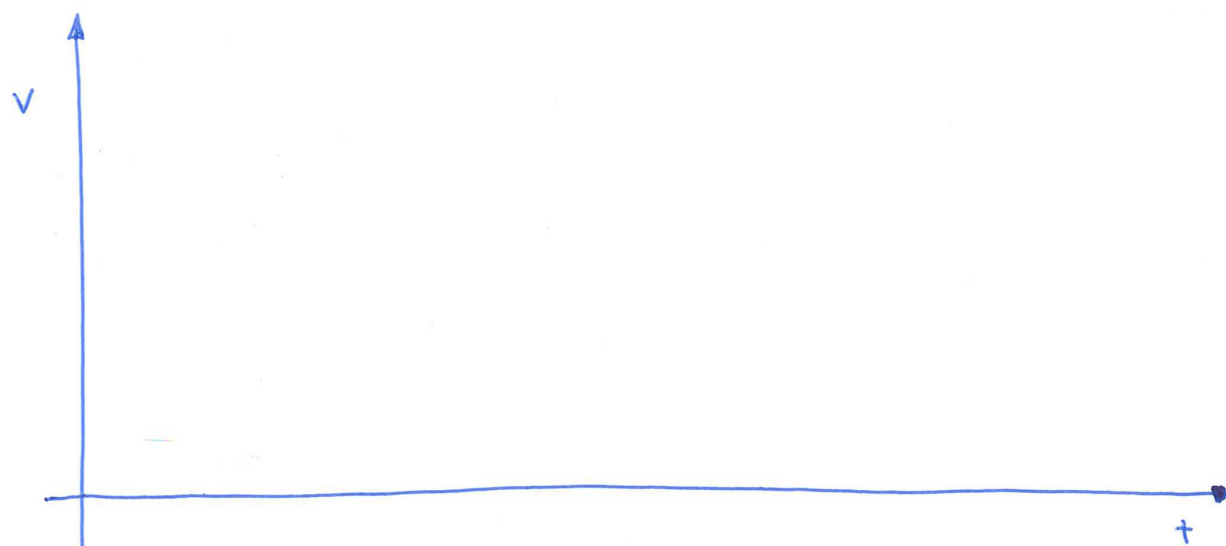
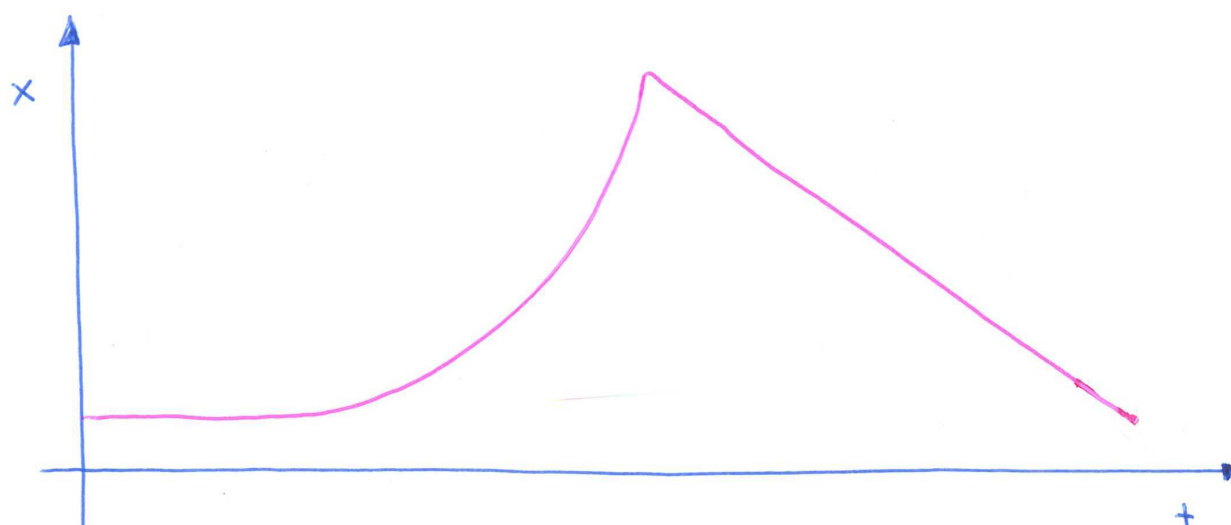
$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

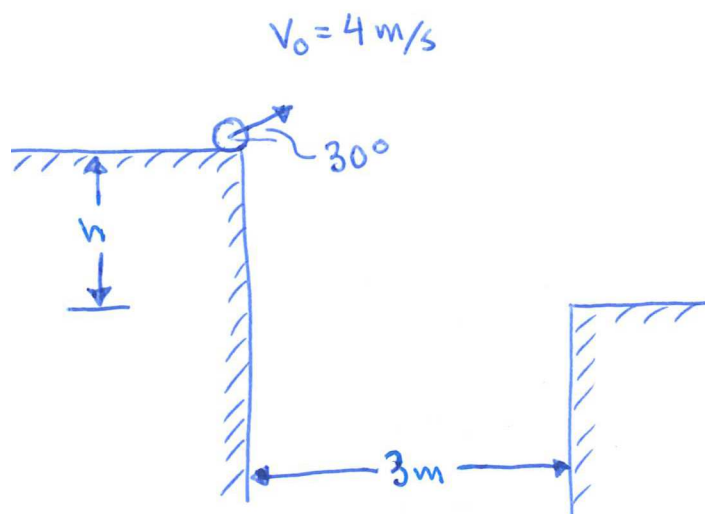
$$E_f - E_o = Q - W \quad E_{PE} = mg(y - y_o) \quad E_{KE} = \frac{1}{2}mv^2$$

$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

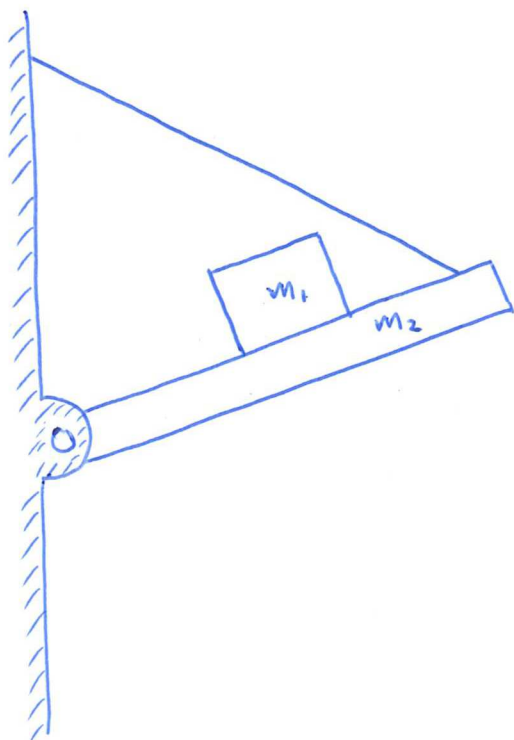
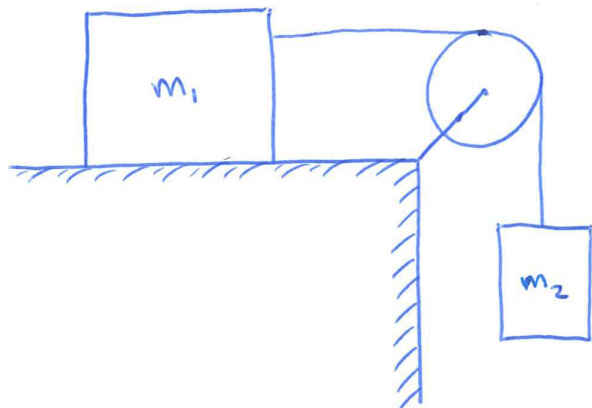
(4 marks) Complete the distance-velocity-acceleration graph.



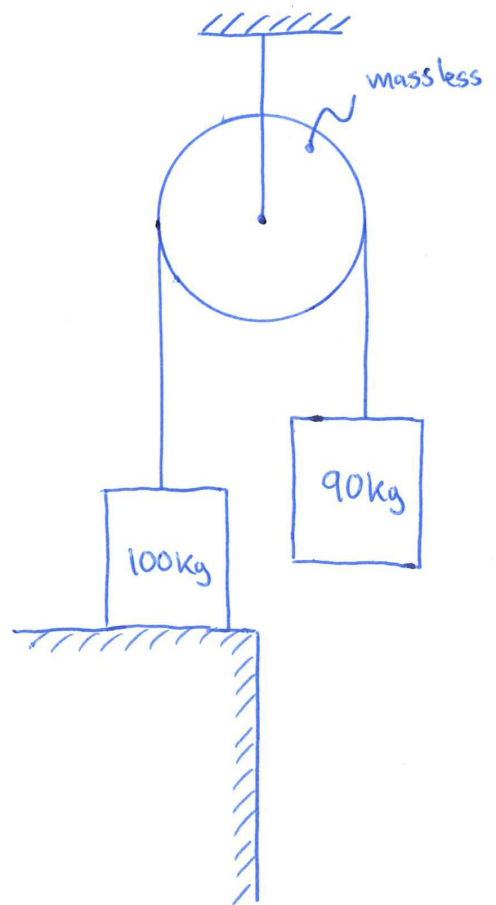
(8 marks) Determine the minimum height (h) that would result with the ball landing on the ledge which is 3 meters away given initial conditions: $v_o = 4 \text{ m/s}$, $m = 1 \text{ kg}$, $\theta = 30^\circ$



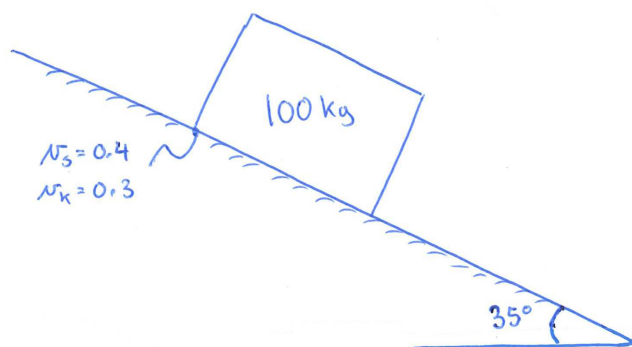
(4 marks) Draw the free body diagram for each mass.



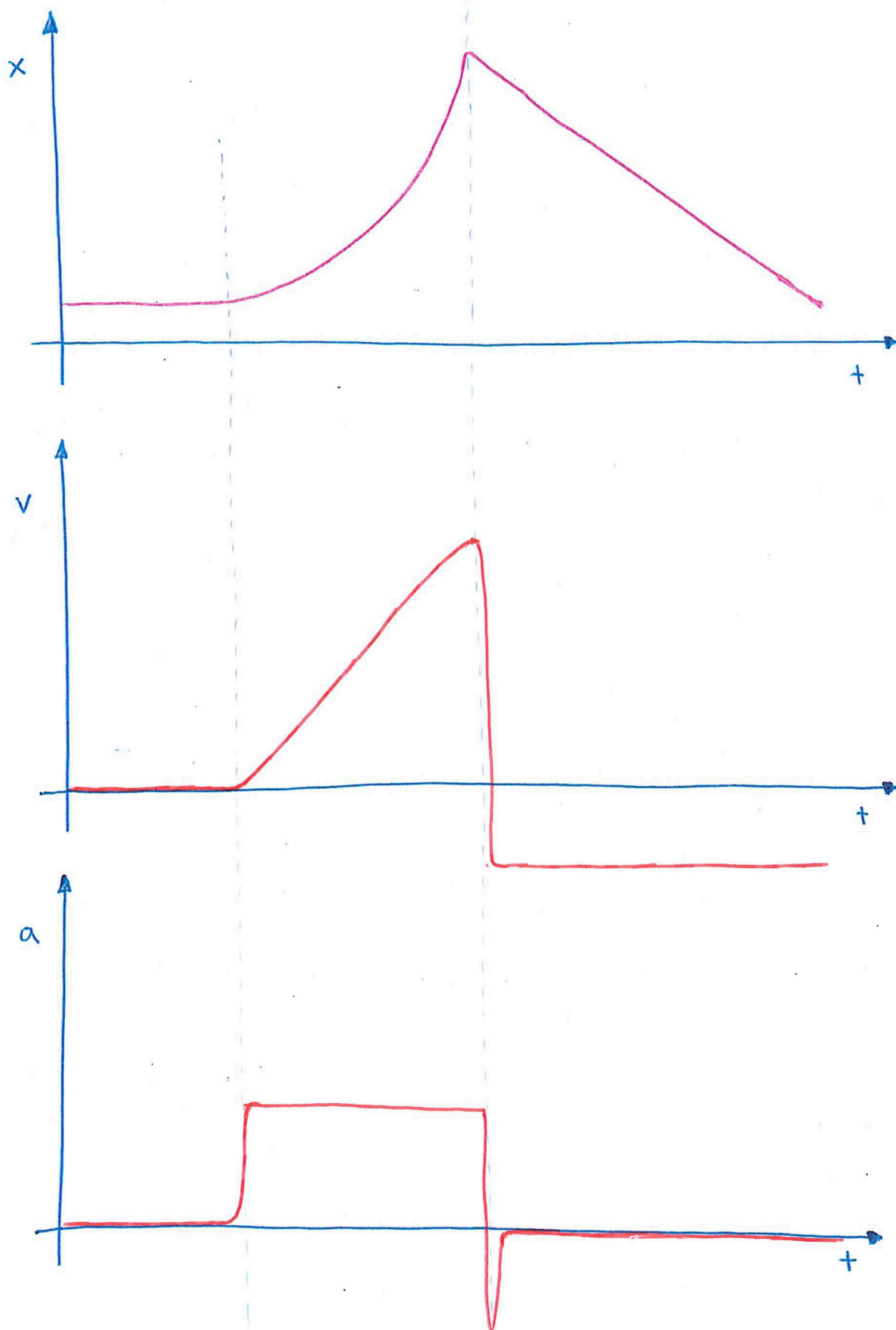
(2 marks) Determine the normal force of 100 kg mass.



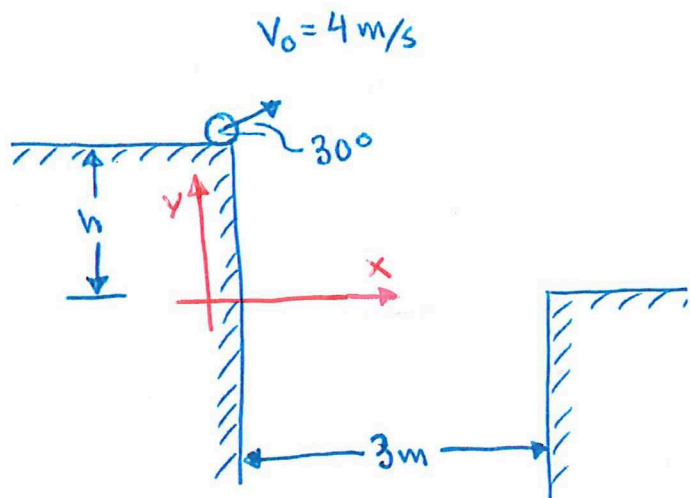
(7 marks) Determine if the moves and the resultant acceleration (if applicable).



(4 marks) Complete the distance-velocity-acceleration graph.



(8 marks) Determine the minimum height (h) that would result with the ball landing on the ledge which is 3 meters away given initial conditions: $v_0 = 4 \text{ m/s}$, $m = 1 \text{ kg}$, $\theta = 30^\circ$



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_0 = 0$	$y_0 =$	$t_i =$
$x_i = 3$	$y_i = 0$	
$v_{0x} = 3.464$	$v_{0y} = 2$	
$v_{ix} = 3.464$	$v_{iy} =$	
$a_x = 0$	$a_y = -9.81$	

$v_0 \sin \theta = 4 \sin(30^\circ)$
 $= 2$
 $= v_0 \cos \theta$
 $= 4 \cos(30^\circ)$
 $= 3.464$

solve for t in kinematics_x (1):

$$x_i = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad (a_x = 0)$$

$$\begin{aligned}
 t_i &= (x_i - x_0) / v_{0x} \\
 &= (3 - 0) / 3.464 \\
 &= 0.866 \text{ s}
 \end{aligned}$$

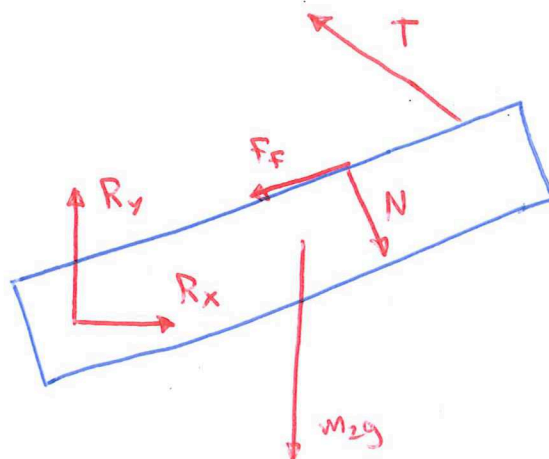
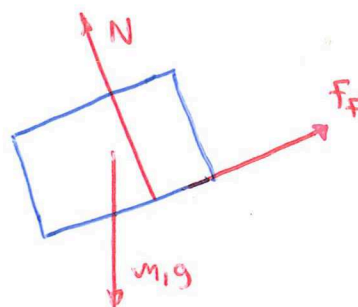
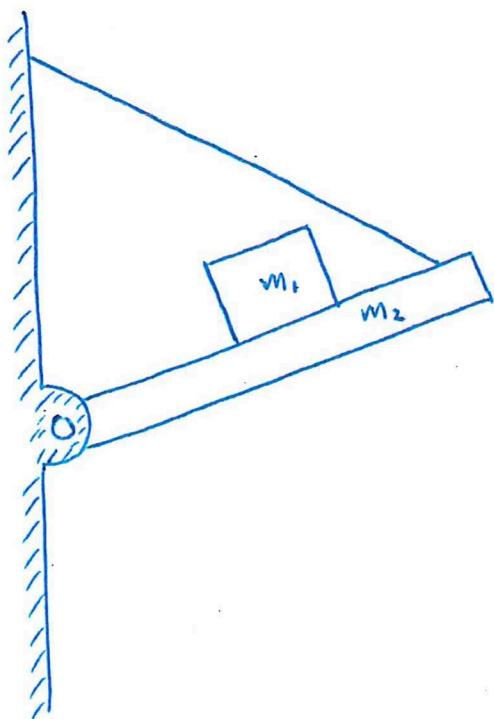
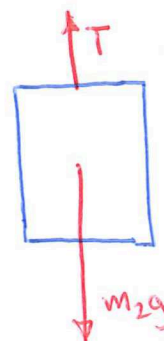
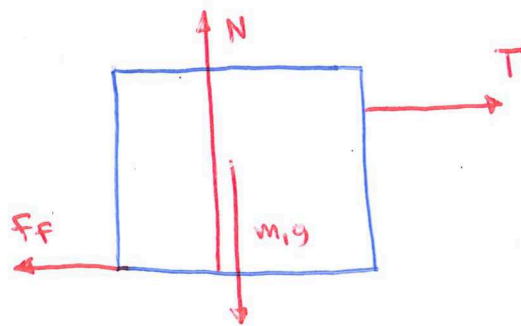
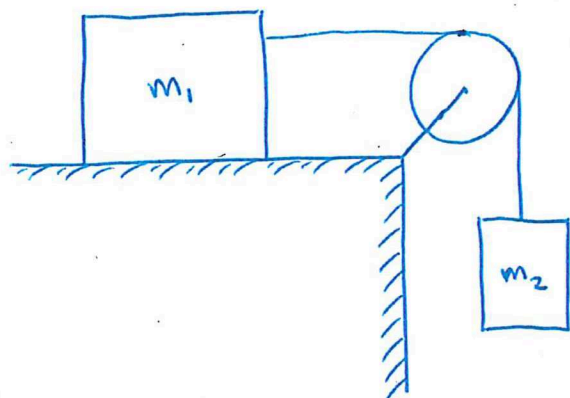
solve for y_0 in kinematics_y (1):

$$y_i = y_0 + v_{0y}t_i + \frac{1}{2}a_y t_i^2$$

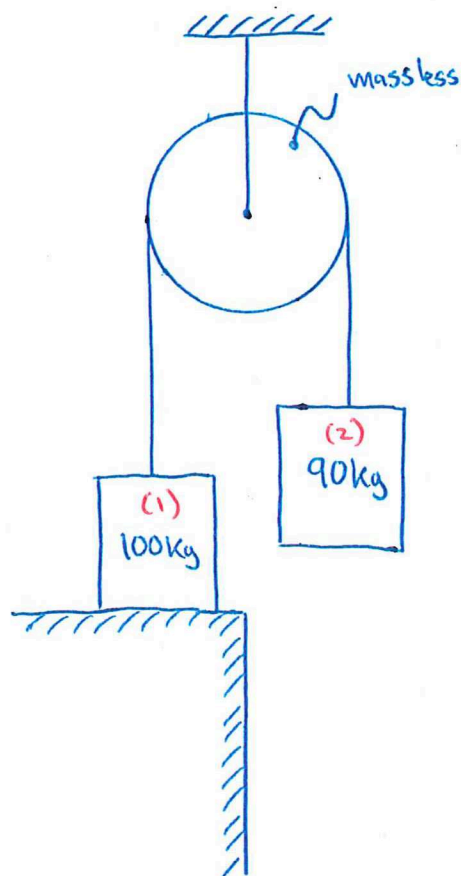
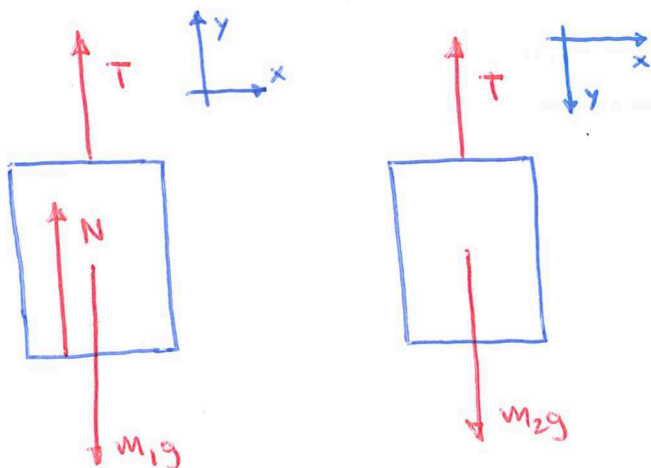
$$\begin{aligned}
 y_0 &= y_i - v_{0y}t_i - \frac{1}{2}a_y t_i^2 \\
 &= 0 - (2)(0.866) - \frac{1}{2}(-9.81)(0.866)^2 \\
 &= 1.95
 \end{aligned}$$

$h = 1.95 \text{ m}$

(4 marks) Draw the free body diagram for each mass.



(2 marks) Determine the normal force of 100 kg mass.



mass 1:

$$\sum F_y = m_1 a_y = T + N - m_1 g \quad (1)$$

mass 2:

$$\sum F_y = m_2 a_y = m_2 g - T \quad (2)$$

assume $a_{y1} = a_{y2} = a_y = 0$

solve (2) for T:

$$\begin{aligned} [0] &= m_2 g - T \\ T &= m_2 g = (90)(9.81) \\ &= 882.9 \text{ N} \end{aligned} \quad (2a)$$

solve (1) for N w sub (2a)

$$[0] = [882.9] + N - m_1 g$$

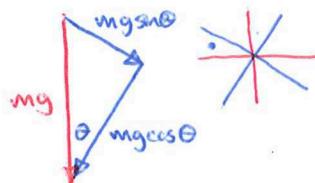
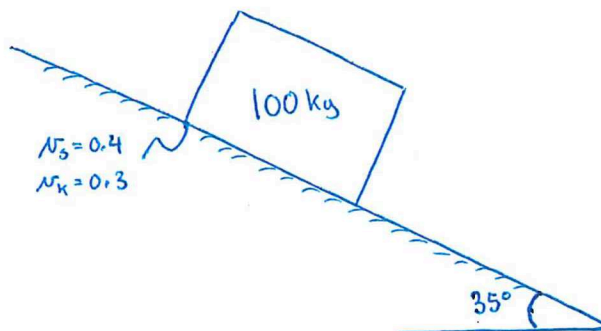
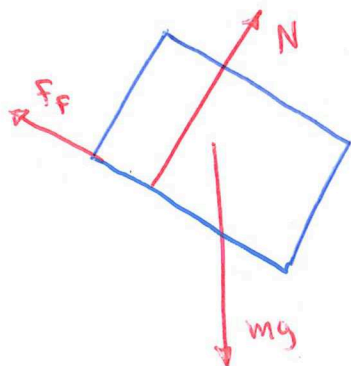
$$\begin{aligned} N &= m_1 g - 882.9 \\ &= (100)(9.81) - 882.9 \end{aligned}$$

$$N = 98.1 \text{ N}$$

$$N = 98.1 \text{ N}$$

(7 marks) Determine if the moves and the resultant acceleration (if applicable).

FBD:



$$\sum F_x = m a_x = m g \sin \theta - F_f \quad (1)$$

$$\sum F_y = m a_y = N - m g \cos \theta \quad (2)$$

assume $a_y = 0$, solve for N in (2):

$$[0] = N - m g \cos \theta$$

$$N = m g \cos \theta = (100)(9.81) \cos (35)$$

$$= 803.6$$

assume moving so $F_f = \mu N$

sub into (1) and solve for a_x :

$$m a_x = m g \sin \theta - [\mu N]$$

$$a_x = g \sin \theta - \frac{\mu}{m} N$$

$$= 9.81 \sin 35 - \frac{(0.4)}{100} (803.6)$$

$$= 2.41 \text{ m/s}^2 \quad (\text{moving!})$$

$$a_x = 9.81 \sin 35 - \frac{(0.3)(803.6)}{100}$$

$$a_x = 3.216 \text{ m/s}^2$$

$$a_x = 3.2 \text{ m/s}^2$$