

Instructor:	Frank Secretain
Course:	Civil 11
Assessment:	Final Exam
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 60 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

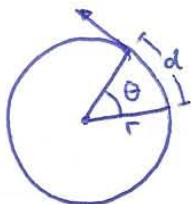
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a}$$

$$\vec{F}_g = \frac{Gm_1 m_2}{r^2} = mg$$

$$\vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p$$

$$\vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

$$E_f - E_o = Q - W$$

$$E_{PE} = mg(y - y_o)$$

$$E_{KE} = \frac{1}{2} m v^2$$

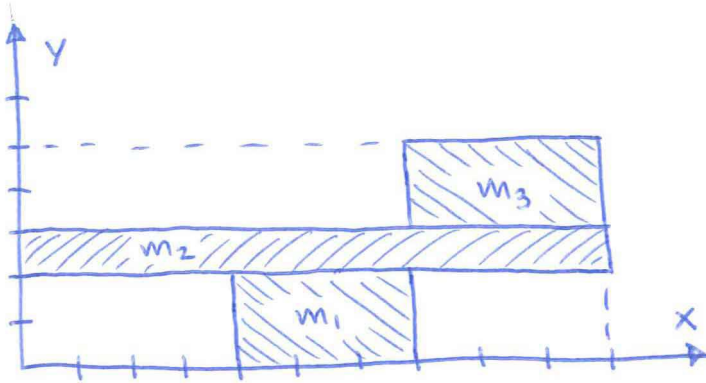
$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

Moment of Inertia Table

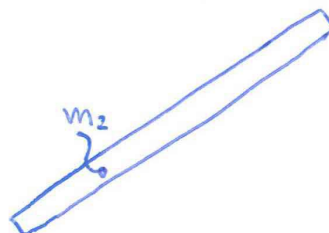
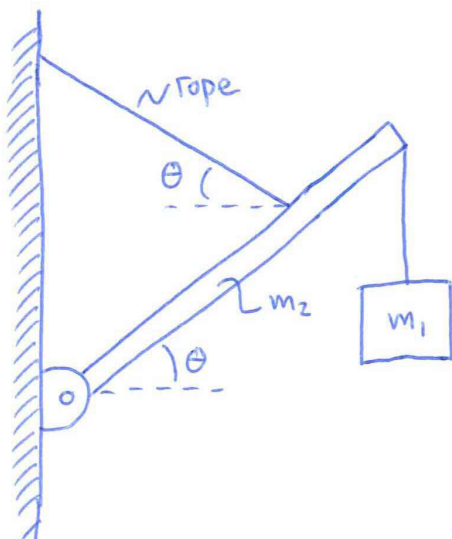
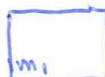
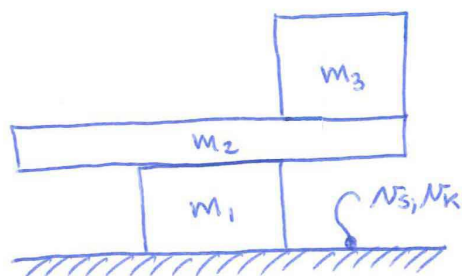
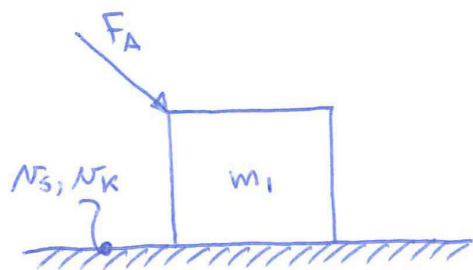
Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + l^2)$ $I_{yy} = \frac{1}{12} m(b^2 + l^2)$ $I_{zz} = \frac{1}{12} m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12} mb^2 + \frac{1}{3} ml^2$
Circular Cylinder		—	$I_{xx} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Sphere		—	$I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Uniform Slender Rod		—	$I_{yy} = \frac{1}{12} ml^2$ $I_{y_1y_1} = \frac{1}{3} ml^2$

(5 marks) Determine the centre of mass for the three mass system and draw it on the diagram, given:

$$m_1 = 6\text{kg}, m_2 = 10\text{kg}, m_3 = 6\text{kg}$$

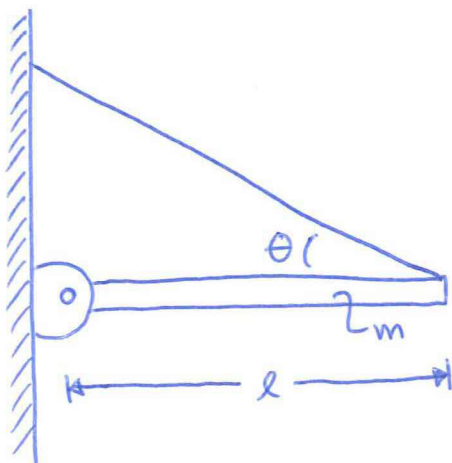


(15 marks) Draw the free body diagram (FBD) for each of the 3 problems on the left hand side. Draw the force arrows on the provided blocks on the right hand side of the page. You do not need to solve Newton's laws of motion for these, just and accurate free body diagram of the forces.

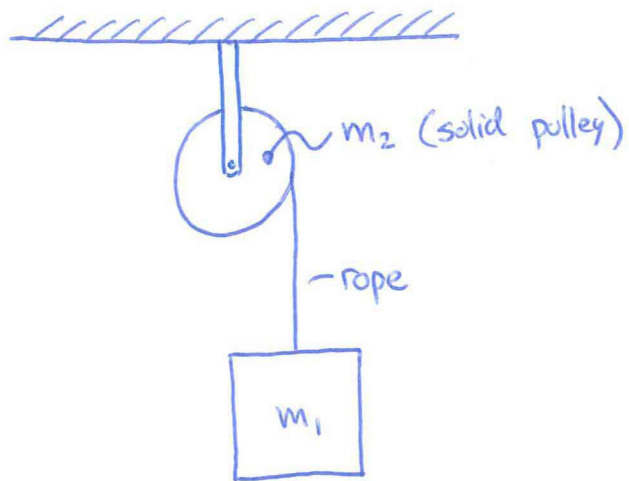


(15 marks) Solve for the magnitude and direction of the force in the pin, given:

$m=100\text{kg}$, $\theta=30^\circ$ and $L=10\text{m}$



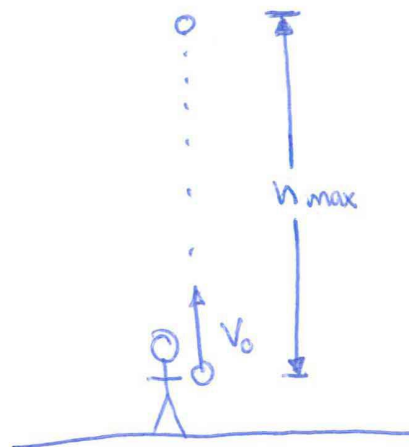
(15 marks) Determine the tension in the rope and the acceleration of the system, given.



$m_1=100\text{kg}$, $m_2=100\text{kg}$ (evenly distributed)
 $R_{\text{Pulley}}=1\text{m}$

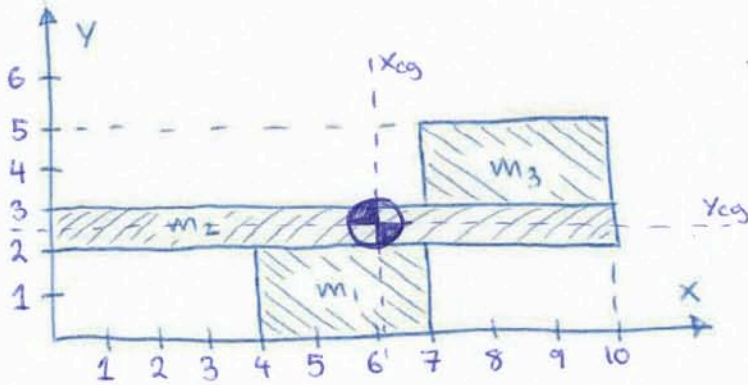
(10 marks) Determine the theoretical maximum height the ball could go using:

- a) kinematics
- b) Laws of thermodynamics



(5 marks) Determine the centre of mass for the three mass system and draw it on the diagram, given:

$$m_1 = 6\text{kg}, m_2 = 10\text{kg}, m_3 = 6\text{kg}$$



Centre of mass:

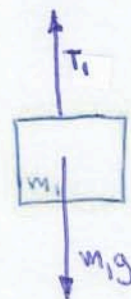
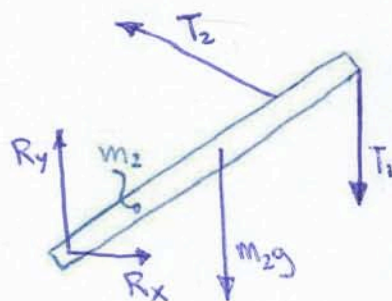
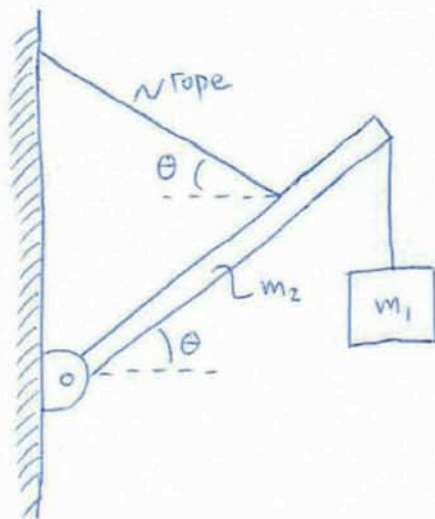
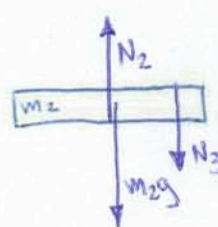
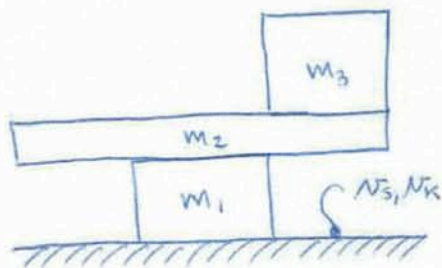
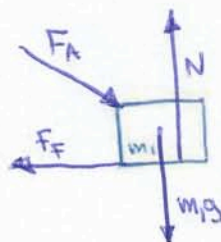
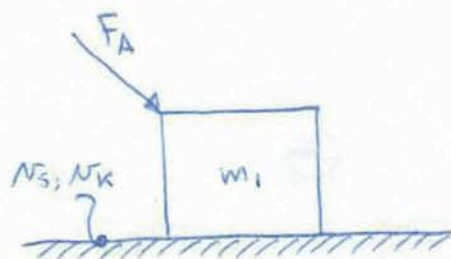
$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$\begin{aligned} x_{cg} &= \frac{\sum m_i x_i}{\sum m_i} \\ &= \frac{(6)(5.5) + (10)(5) + (6)(8.5)}{6 + 10 + 6} \\ &= 6.091 \end{aligned}$$

$$\begin{aligned} y_{cg} &= \frac{\sum m_i y_i}{\sum m_i} \\ &= \frac{(6)(1) + (10)(2.5) + (6)(4)}{6 + 10 + 6} \\ &= 2.5 \end{aligned}$$

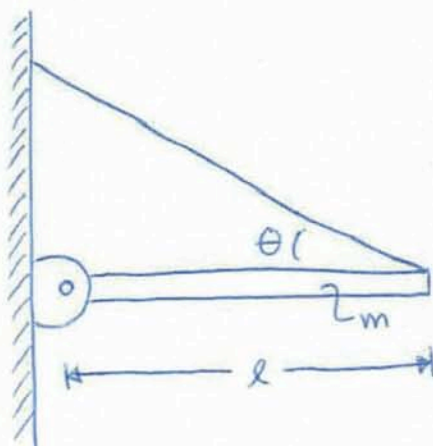
$$\begin{aligned} x_{cg} &= 6.1 \\ y_{cg} &= 2.5 \end{aligned}$$

(15 marks) Draw the free body diagram (FBD) for each of the 3 problems on the left hand side. Draw the force arrows on the provided blocks on the right hand side of the page. You do not need to solve Newton's laws of motion for these, just and accurate free body diagram of the forces.

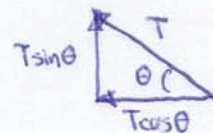
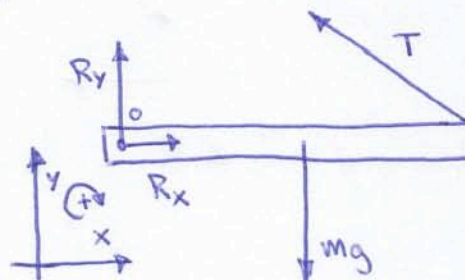


(15 marks) Solve for the magnitude and direction of the force in the pin, given:

$$m=100\text{kg}, \theta=30^\circ \text{ and } L=10\text{m}$$



FBD:



Newton's 2nd Law:

$$\sum F_x = m a_x = R_x - T \cos \theta \quad (1)$$

$$\sum F_y = m a_y = R_y + T \sin \theta - mg \quad (2)$$

$$\sum M_o = I_o \alpha_o = (mg)\left(\frac{L}{2}\right) - (T \sin \theta)(L) \quad (3)$$

Solve eqn. (3) for T:

$$0 = \frac{mgL}{2} - T \sin \theta L$$

$$\boxed{T = \frac{mg}{2 \sin \theta}} \quad (3a)$$

sub (3a) into (1) & (2) and solve for R_x & R_y :

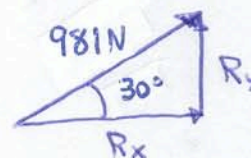
$$R_x = \left[\frac{mg}{2 \sin \theta} \right] \cos \theta = \frac{mg \cos \theta}{2 \sin \theta} = \frac{(100)(9.81) \cos(30)}{2 \sin(30)} = 850 \text{ N}$$

$$R_y = mg - \left[\frac{mg}{2 \sin \theta} \right] \sin \theta = \frac{mg}{2} = \frac{(100)(9.81)}{2} = 491 \text{ N}$$

Magnitude and direction:

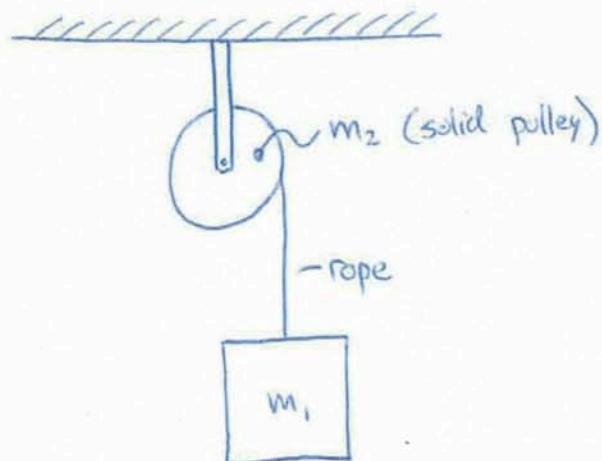
$$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{850^2 + 491^2} = 981 \text{ N}$$

$$\angle R = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{491}{850}\right) = 30^\circ$$

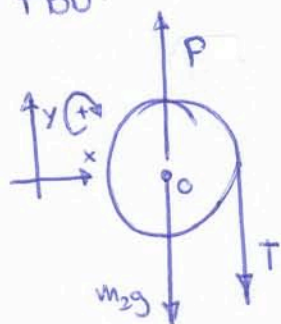


(15 marks) Determine the tension in the rope and the acceleration of the system, given.

$m_1 = 100\text{kg}$, $m_2 = 100\text{kg}$ (evenly distributed)
 $R_{\text{Pulley}} = 1\text{m}$



FBD:

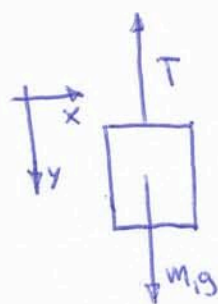


Newton's 2nd Law:

$$\sum F_x = m_2 a_{x2} = 0$$

$$\sum F_y = m_2 a_{y2} = P - m_2g - T \quad (1)$$

$$\sum M_O = I_O \alpha_O = RT \quad (2)$$



$$\sum F_x = m_1 a_{x1} = 0$$

$$\sum F_y = m_1 a_{y1} = m_1g - T \quad (3)$$

$$\sum M_O = I_O \alpha_O = 0$$

Assume rolling without slipping so $a_y = \alpha_O R$:

Solve $I_O = \frac{1}{2} m_2 R^2$ and eq. (3) into (2):

$$\left[\frac{1}{2} m_2 R^2 \right] \left[\frac{a_y}{R} \right] = R [m_1g - m_1a_y] \Rightarrow$$

$$a_y = \frac{m_1g}{m_1 + \frac{m_2}{2}} = \frac{(100)(9.81)}{(100) + \frac{(100)}{2}} = 6.5 \text{ m/s}^2$$

solve for T in eq. (3)

$$T = m_1g - m_1a_y = (100)(9.81) - (100)(6.5) = 327 \text{ N}$$

(10 marks) Determine the theoretical maximum height the ball could go using:

- a) kinematics
- b) Laws of thermodynamics

a) kinematics

y-dir

$$y_0 = 0$$

$$y_F = h$$

$$v_{0y} = v_0$$

$$v_{Fy} = 0$$

$$a_y = -g, \text{ where } g = 9.81 \text{ m/s}^2$$

use Kinematics equ. (2):

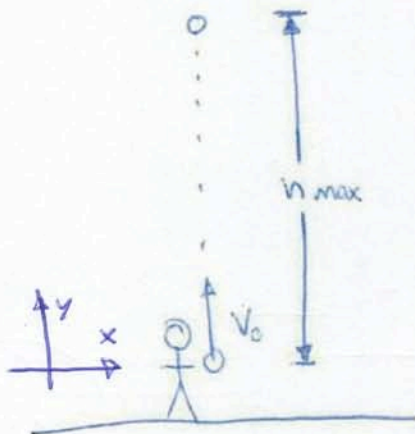
$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{x} - \vec{x}_0)$$

$$v_{Fy}^2 = v_{0y}^2 + 2a_y(y_F - y_0)$$

$$0 = v_{0y}^2 + 2a_y y_F$$

$$y_F = -\frac{v_{0y}^2}{2a_y} = \frac{v_0^2}{2g}$$

$$h = \frac{v_0^2}{2g}$$



b) Laws of Thermodynamics:

<u>initial</u>	<u>Final</u>
$E_{KE} = \frac{1}{2}mv_0^2$	$E_{KE} = \frac{1}{2}mv_F^2 = 0$
$E_{PE} = mgh_i = 0$	$E_{PE} = mgh$

1st Law of Thermodynamics:

$$E_F - E_0 = \cancel{Q} - \cancel{W}$$

$$[mgh] - [\frac{1}{2}mv_0^2] = 0$$

$$gh = \frac{v_0^2}{2}$$

$$h = \frac{v_0^2}{2g}$$