

Instructor:	Frank Secretain
Course:	CIVL 11
Assessment:	Test 2
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	4 questions worth 30 marks + 1 bonus question worth 2 marks
Percentage of final grade:	25% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

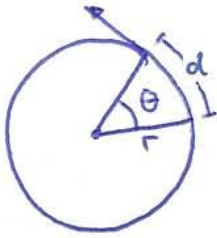
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a}$$

$$\vec{F}_g = \frac{Gm_1 m_2}{r^2} = mg$$

$$\vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

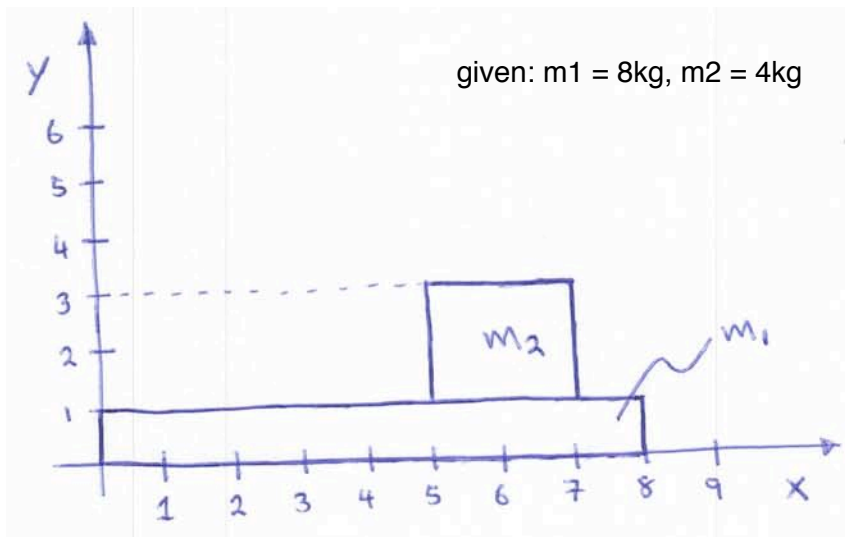
$$E_f - E_o = Q - W$$

$$E_{PE} = mg(y - y_o)$$

$$E_{KE} = \frac{1}{2} m v^2$$

$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

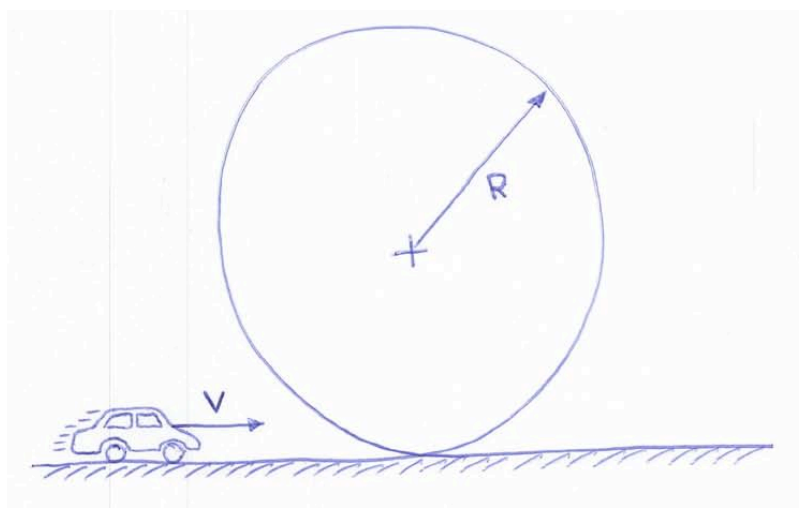
(4 marks) Determine the centre of mass for the two mass system and draw it on the diagram.



(6 marks) Determine the minimum constant velocity needed by the car to travel around the loop without falling off.

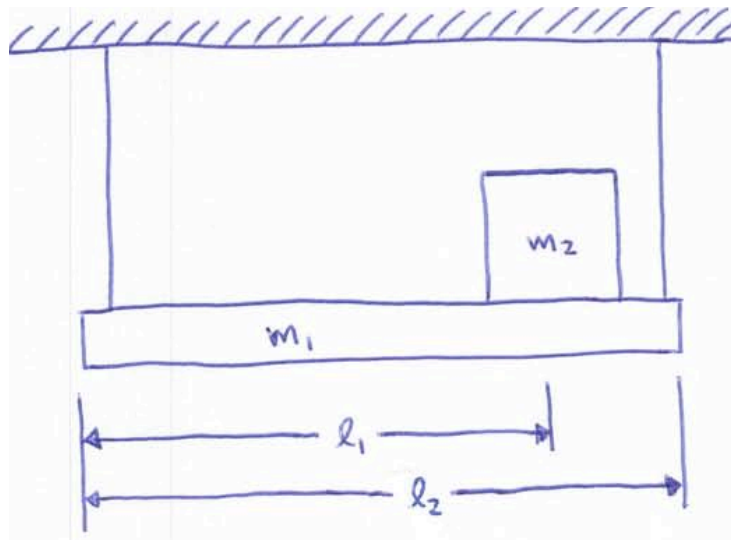
Hint: the point at which the car would fall off would be at the top of the loop, this is the point which the minimum velocity should be solved for.

Given: $m = 1000 \text{ kg}$
 $R = 10 \text{ m}$



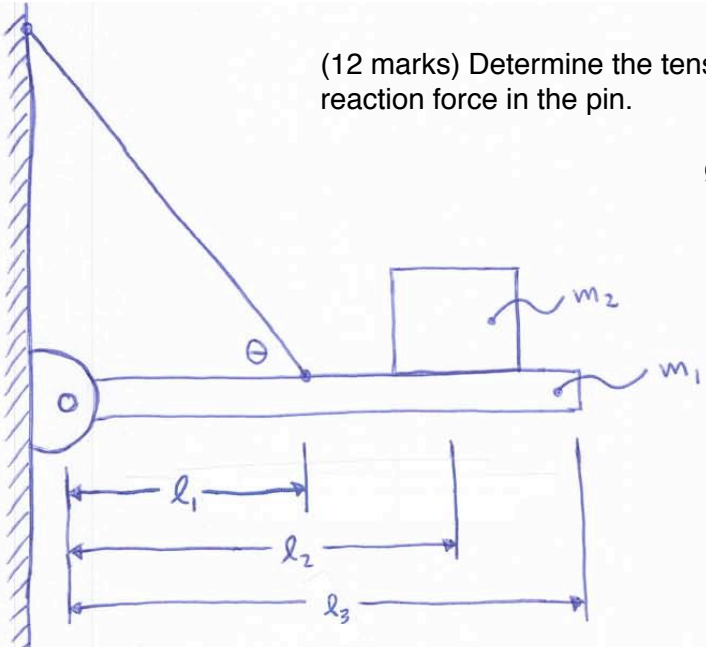
(8 marks) Determine the tension in each rope.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $L_1 = 8 \text{ m}$
 $L_2 = 10 \text{ m}$



(12 marks) Determine the tension in each rope and the magnitude of the reaction force in the pin.

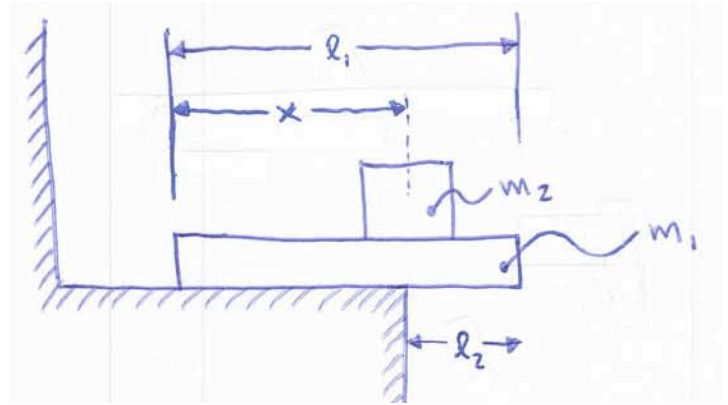
given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $\theta = 50^\circ$
 $L_1 = 4 \text{ m}$
 $L_2 = 8 \text{ m}$
 $L_3 = 10 \text{ m}$



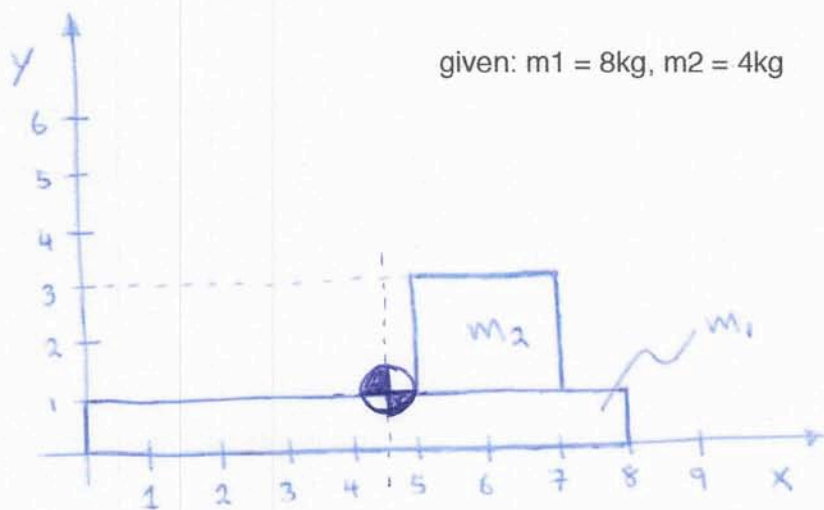
(2 marks) Bonus

Determine the maximum distance x that m_2 could be without the system (m_1 and m_2) tipping over.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $L_1 = 8 \text{ m}$
 $L_2 = 3 \text{ m}$



(4 marks) Determine the centre of mass for the two mass system and draw it on the diagram.



centre of mass: $\vec{x}_{cg} = \frac{\sum \vec{x}_i m_i}{\sum m_i}$

$$x_{cg} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(4)(8) + (6)(4)}{8 + 4} = \frac{56}{12} = 4.67$$

$$x_{cg} = 4.67$$

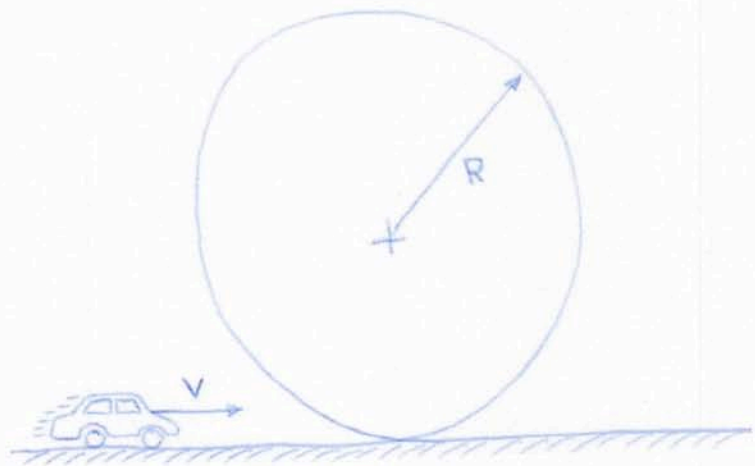
$$y_{cg} = \frac{\sum y_i m_i}{\sum m_i} = \frac{(0.5)(8) + (2)(4)}{8 + 4} = \frac{12}{12} = 1$$

$$y_{cg} = 1.0$$

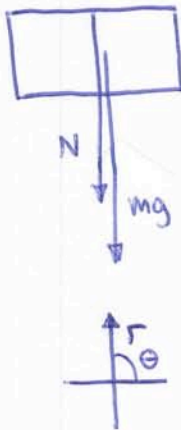
(6 marks) Determine the minimum constant velocity needed by the car to travel around the loop without falling off.

Hint: the point at which the car would fall off would be at the top of the loop, this is the point which the minimum velocity should be solved for.

Given: $m = 1000 \text{ kg}$
 $R = 10 \text{ m}$



FBD at top of loop



Newton's 2nd Law

$$\sum F_r = ma_r = -N - mg \quad (1)$$

$$\sum F_\theta = ma_\theta = 0 \quad (2)$$

uniform circular motion

$$a_r = -\omega^2 R = -\frac{v^2}{R} \quad (3)$$

sub (3) into (1)

$$m\left(-\frac{v^2}{R}\right) = -N - mg$$

set $N=0$ for minimum velocity.

$$-\frac{m v_{\min}^2}{R} = -mg$$

$$v_{\min} = \sqrt{gR}$$

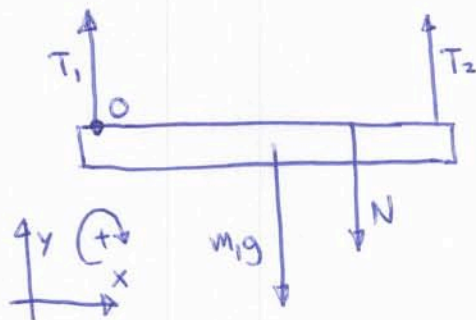
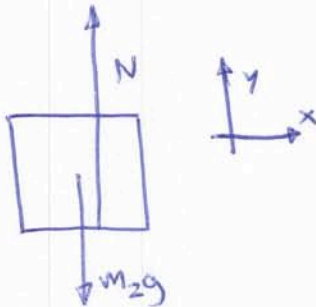
$$= \sqrt{(9.81)(10)} = 9.90$$

$$v_{\min} = 9.9 \text{ m/s}$$

(8 marks) Determine the tension in each rope.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $L_1 = 8 \text{ m}$
 $L_2 = 10 \text{ m}$

FBD:



Newton's 2nd Law:

$$\sum F_x = m_2 \ddot{x} = 0$$

$$\sum F_y = m_2 \ddot{y} = N - m_2 g \Rightarrow N = m_2 g \quad (1)$$

$$\sum F_x = m_1 \ddot{x} = 0$$

$$\sum F_y = m_1 \ddot{y} = T_1 + T_2 - m_1 g - N \quad (2)$$

$$\sum M_o = I_o \ddot{\alpha} = \left(\frac{L_2}{2}\right)(m_1 g) + (N)(L_1) - (T_2)(L_2) \quad (3)$$

sub (1) into (3) and solve for T_2 :

$$0 = \frac{m_1 g L_2}{2} + [m_2 g] L_1 - T_2 L_2$$

$$T_2 = \frac{m_1 g}{2} + m_2 g \left(\frac{L_1}{L_2}\right) \quad (3a)$$

$$T_2 = \frac{(10)(9.81)}{2} + (100)(9.81)\left(\frac{8}{10}\right) = 834$$

sub (1) and (3a) into (2) and solve for T_1

$$0 = T_1 + \left[\frac{m_1 g}{2} + \frac{m_2 g L_1}{L_2}\right] - m_1 g - m_2 g$$

$$T_1 = \frac{m_1 g}{2} + m_2 g \left(1 - \frac{L_1}{L_2}\right)$$

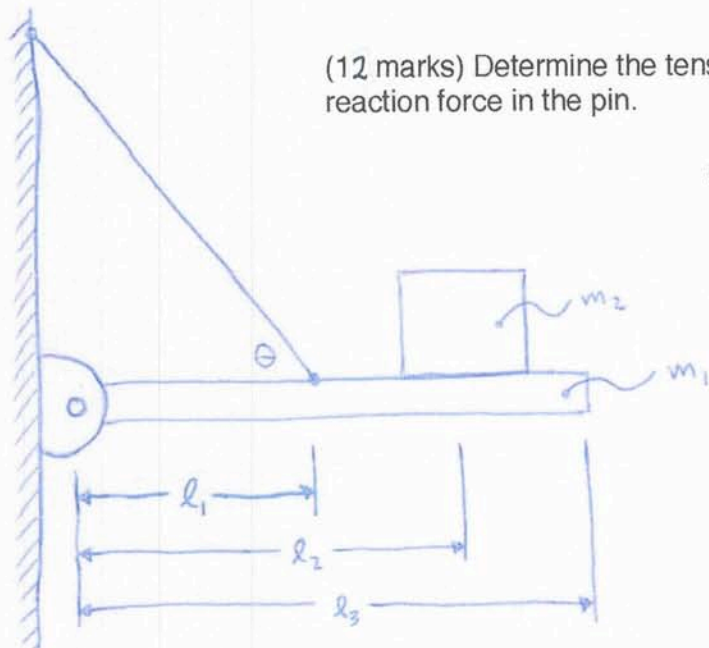
$$T_1 = \frac{(10)(9.81)}{2} + (100)(9.81)\left(1 - \frac{8}{10}\right) = 245$$

$$T_1 = 245 \text{ N}$$

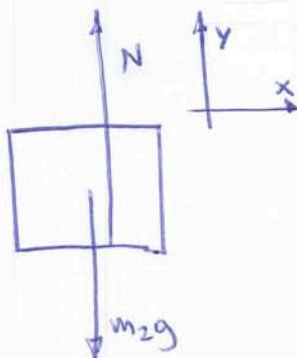
$$T_2 = 834 \text{ N}$$

(12 marks) Determine the tension in each rope and the magnitude of the reaction force in the pin.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $\theta = 50^\circ$
 $L_1 = 4 \text{ m}$
 $L_2 = 8 \text{ m}$
 $L_3 = 10 \text{ m}$



FBD:



Newton's 2nd Law:

$$\sum F_x = m_2 a_x = 0$$

$$\sum F_y = m_2 a_y = N - m_2 g \Rightarrow N = m_2 g \quad (1)$$

$$\sum F_x = m_1 a_x = R_x - T \cos \theta \quad (2)$$

$$\sum F_y = m_1 a_y = R_y + T \sin \theta - m_1 g - N \quad (3)$$

$$\sum M_o = I_o \alpha_o = (L_1)(T \sin \theta) - \left(\frac{L_3}{2}\right)(m_1 g) - (L_2)(N) \quad (4)$$

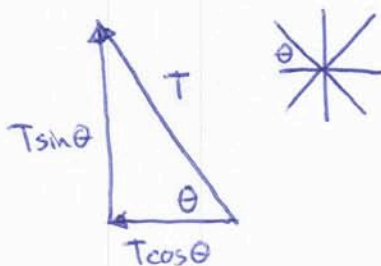
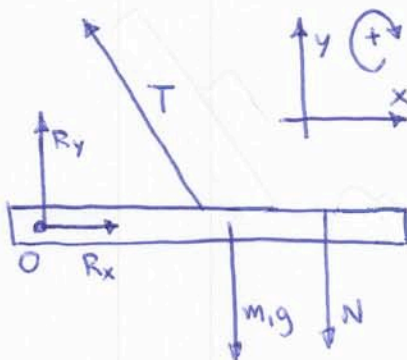
sub (1) into (4) and solve for T:

$$0 = L_1 T \sin \theta - \frac{L_3}{2} m_1 g - L_2 m_2 g$$

$$T = \frac{\frac{L_3}{2} m_1 g + L_2 m_2 g}{L_1 \sin \theta} \quad (4a)$$

$$= \frac{\left(\frac{10}{2}\right)(10)(9.81) + (8)(100)(9.81)}{(4) \sin(50)}$$

$$= 2721 \text{ N}$$



sub (4a) into (2) and solve for R_x :

$$0 = R_x - \left[\frac{l_3 m_1 g + l_2 m_2 g}{l_1 \sin \theta} \right] \cos \theta$$

$$R_x = \frac{l_3 m_1 g + l_2 m_2 g}{2 l_1 \sin \theta} \cos \theta$$

$$R_x = \frac{(10)(10)(9.81) + (2)(8)(100)(9.81)}{(2)(4) \sin(50)} \cos(50)$$

$$R_x = 1749$$

sub (4a) and (1) into (3) and solve for R_y :

$$0 = R_y + \left[\frac{l_3 m_1 g + l_2 m_2 g}{l_1 \sin \theta} \right] \sin \theta - m_1 g - m_2 g$$

$$R_y = m_1 g + m_2 g - \left(\frac{l_3}{2 l_1} \right) m_1 g - \left(\frac{l_2}{l_1} \right) m_2 g$$

$$R_y = \left(1 - \frac{l_3}{2 l_1} \right) m_1 g + \left(1 - \frac{l_2}{l_1} \right) m_2 g$$

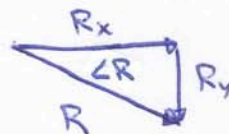
$$R_y = \left(1 - \frac{10}{(2)(4)} \right) (10)(9.81) + \left(1 - \frac{8}{4} \right) (100)(9.81)$$

$$R_y = -1006$$

magnitude of reaction force:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1749)^2 + (-1006)^2} = 2017 \text{ N}$$

$$\angle R = \tan^{-1} \left(\frac{R_y}{R_x} \right) = \tan^{-1} \left(\frac{1006}{1749} \right) = 29.9^\circ$$



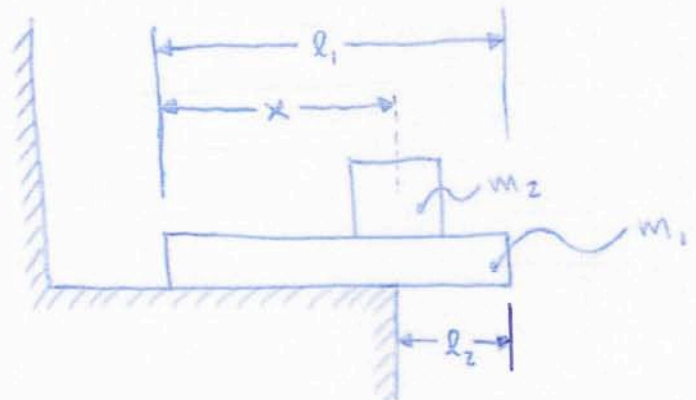
$$T = 2721 \text{ N}$$

$$R = 2071 \text{ N}$$

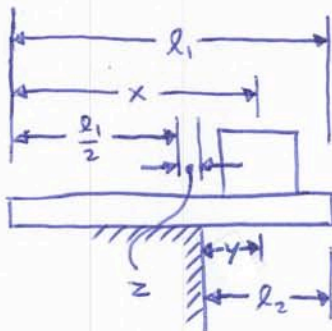
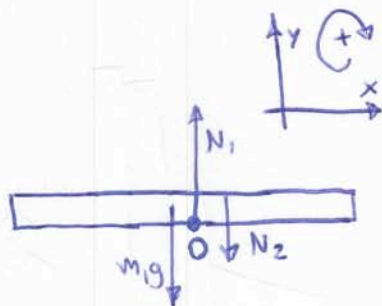
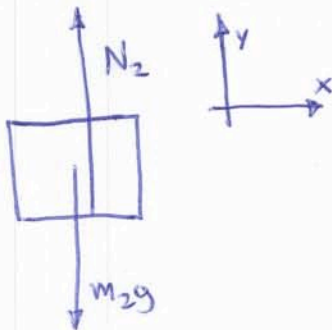
(2 marks) Bonus

Determine the maximum distance x that m_2 could be without the system (m_1 and m_2) tipping over.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $L_1 = 8 \text{ m}$
 $L_2 = 3 \text{ m}$



FBD:



Newton's 2nd Law:

$$\sum F_x = \cancel{m_2 a_x} = 0$$

$$\sum F_y = \cancel{m_2 a_y} = N_2 - m_2 g \Rightarrow N_2 = m_2 g \quad (1)$$

$$\sum F_x = \cancel{m_1 a_x} = 0$$

$$\sum F_y = \cancel{m_1 a_y} = N_1 - N_2 - m_1 g \quad (2)$$

$$\sum M_o = \cancel{I_o \alpha_o} = (x + L_2 - L_1)(N_2) - \left(\frac{L_1}{2} - L_2\right)(m_1 g) \quad (3)$$

sub (1) into (3) and solve for x :

$$0 = (x + L_2 - L_1)(m_2 g) - \left(\frac{L_1}{2} - L_2\right)(m_1 g)$$

$$0 = x m_2 g + (L_2 - L_1)(m_2 g) - \left(\frac{L_1}{2} - L_2\right)(m_1 g)$$

$$x = \frac{\left(\frac{L_1}{2} - L_2\right) m_1 g + (L_1 - L_2)(m_2 g)}{m_2 g}$$

$$x = 5.1 \text{ m}$$

$$L_1 = x - y + L_2$$

$$\frac{L_1}{2} = z + L_2$$

so

$$y = x + L_2 - L_1$$

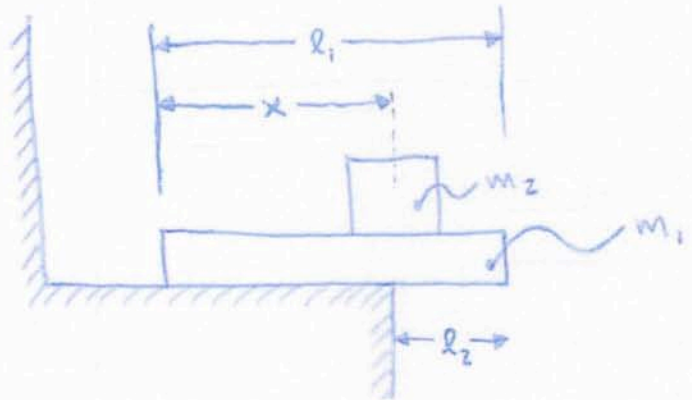
so

$$z = \frac{L_1}{2} - L_2$$

(2 marks) Bonus

Determine the maximum distance x that m_2 could be without the system (m_1 and m_2) tipping over.

given: $m_1 = 10 \text{ kg}$
 $m_2 = 100 \text{ kg}$
 $L_1 = 8 \text{ m}$
 $L_2 = 3 \text{ m}$



Centre of mass: $\vec{x}_{cg} = \frac{\sum x_i m_i}{\sum m_i}$

$$x_{cg} = \frac{(x)(m_2) + (\frac{L_1}{2})(m_1)}{m_1 + m_2}$$

$$L_1 - L_2 = \frac{x m_2 + \frac{L_1}{2} m_1}{m_1 + m_2}$$

$$x m_2 = (m_1 + m_2)(L_1 - L_2) - \frac{L_1 m_1}{2}$$

$$x = \frac{(m_1 + m_2)(L_1 - L_2) - \frac{L_1 m_1}{2}}{m_2}$$

$$= \frac{(10 + 100)(8 - 3) - \frac{(8)(10)}{2}}{100}$$

$$= 5.1$$

$$x = 5.1 \text{ m}$$

