

Instructor:	Frank Secretain
Course:	CIVL 11
Assessment:	Final Test
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers.
Marks allocated:	5 questions worth 30 marks + 1 bonus question worth 2 marks
Percentage of final grade:	20% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

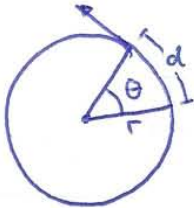
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \vec{F}_g = \frac{Gm_1 m_2}{r^2} = mg \quad \vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{r} \times \vec{F} = \vec{F} \perp \vec{r}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

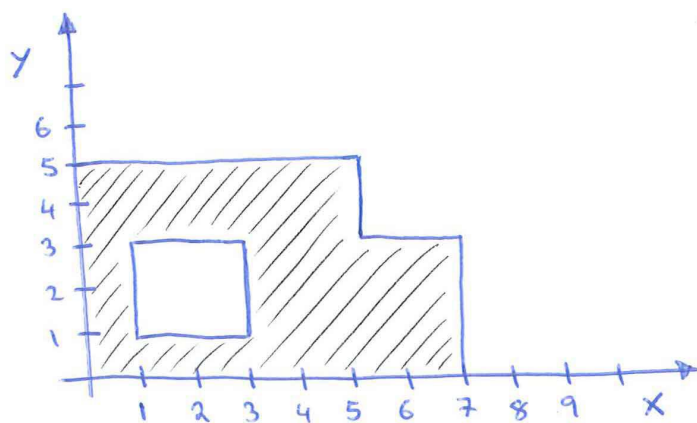
$$E_f - E_o = Q - W \quad E_{PE} = mg(y - y_o) \quad E_{KE} = \frac{1}{2}mv^2$$

$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

Moment of Inertia Table

Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + l^2)$ $I_{yy} = \frac{1}{12} m(b^2 + l^2)$ $I_{zz} = \frac{1}{12} m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12} mb^2 + \frac{1}{3} ml^2$
Circular Cylinder		—	$I_{xx} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Sphere		—	$I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Uniform Slender Rod		—	$I_{yy} = \frac{1}{12} ml^2$ $I_{y_1y_1} = \frac{1}{3} ml^2$

(4 marks) Determine the centre of mass of the shaded area assuming uniform composition (density of the object is constant) and draw it on the diagram.



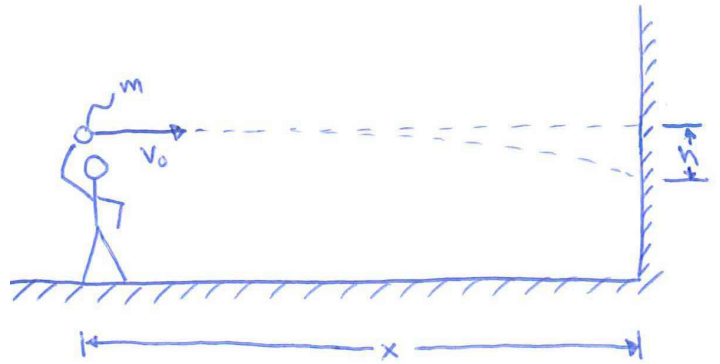
(4 marks) A ball is thrown horizontally towards a wall with an initial velocity of 10 m/s. The ball hits the wall 1.5 m below the horizontal plane in which it was thrown. Using the 1st law of thermodynamics.

determine:

the velocity of the ball just before it hits the wall.

given:

$m = 0.2 \text{ kg}$, $h = 1.5 \text{ m}$, $v_o = 10 \text{ m/s}$



(4 marks) A ball is thrown horizontally towards a wall with an initial velocity of 10 m/s. The ball hits the wall 1.5 m below the horizontal plane in which it was thrown. Using kinematics.

determine:

the distance the wall is away from the thrower (x).

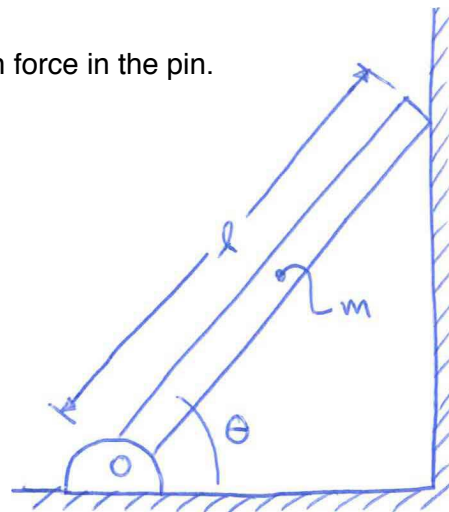
given:

$m = 0.2 \text{ kg}$, $h = 1.5 \text{ m}$, $v_o = 10 \text{ m/s}$



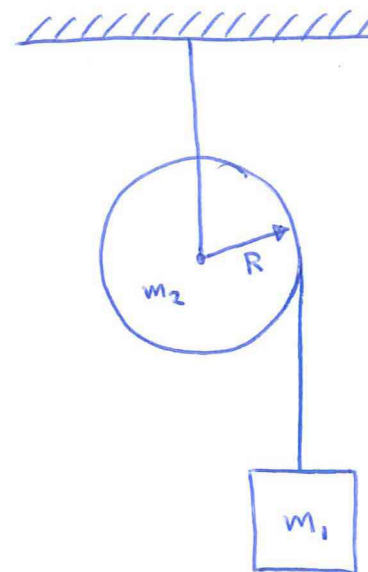
(10 marks) Determine the magnitude and direction of the reaction force in the pin.

given: $L = 8 \text{ m}$, $\theta = 60^\circ$
 $m = 50 \text{ kg}$



(8 marks) Determine the tension in the rope and the acceleration of the system assuming the pulley is a solid disk.

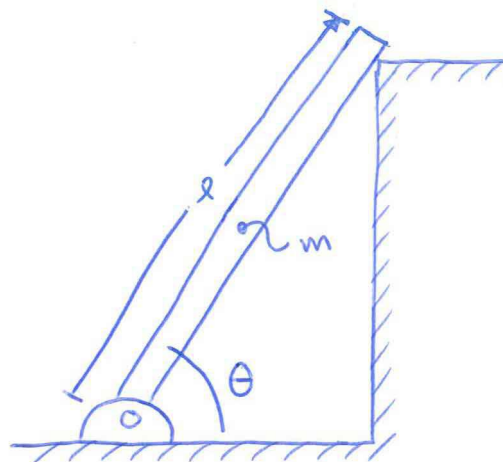
given: $m_1 = 100 \text{ kg}$, $m_2 = 100 \text{ kg}$, $R = 1 \text{ m}$



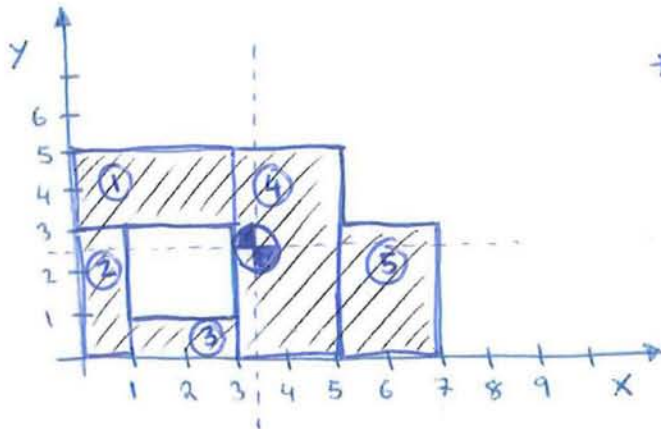
BONUS

(2 marks) Determine the magnitude and direction of the reaction force in the pin assuming the normal force at the corner is perpendicular to the beam.

given: $L = 8 \text{ m}$, $\theta = 60^\circ$
 $m = 50 \text{ kg}$



(4 marks) Determine the centre of mass of the shaded area assuming uniform composition (density of the object is constant) and draw it on the diagram.



* assume  1 has mass m

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6)(1.5) + (3)(0.5) + (2)(2) + (10)(4) + (6)(6)}{6 + 3 + 2 + 10 + 6}$$

$$= 3.35$$

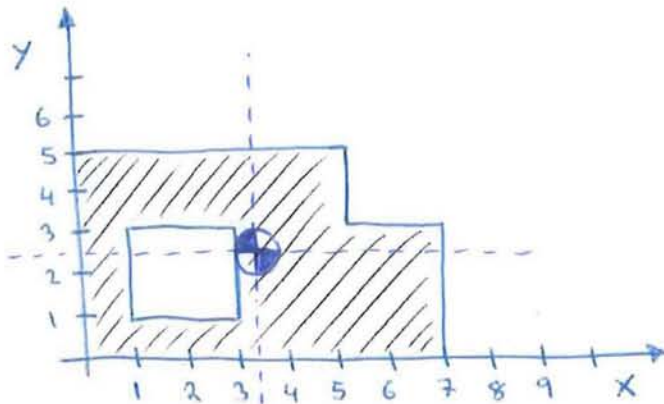
$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6)(4) + (3)(1.5) + (2)(0.5) + (10)(2.5) + (6)(1.5)}{6 + 3 + 2 + 10 + 6}$$

$$= 2.35$$

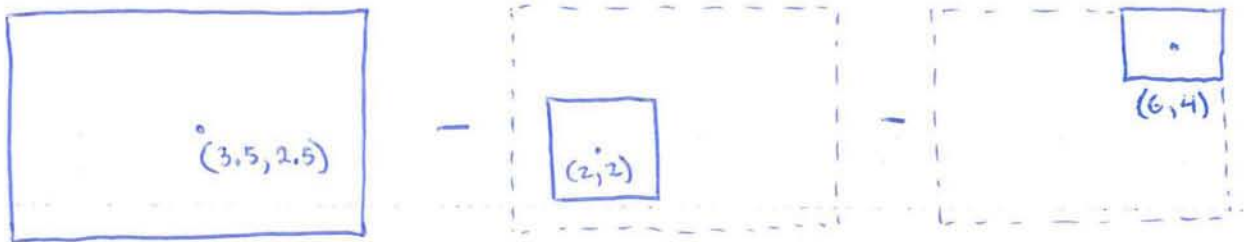
$$x_{cg} = 3.35$$

$$y_{cg} = 2.35$$

(4 marks) Determine the centre of mass of the shaded area assuming uniform composition (density of the object is constant) and draw it on the diagram.



* assume  has mass m



$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(35)(3.5) - (4)(2) - (4)(6)}{35 - 4 - 4} = 3.35$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(35)(2.5) - (4)(2) - (4)(4)}{35 - 4 - 4} = 2.35$$

$$\boxed{\begin{aligned} x_{cg} &= 3.35 \\ y_{cg} &= 2.35 \end{aligned}}$$

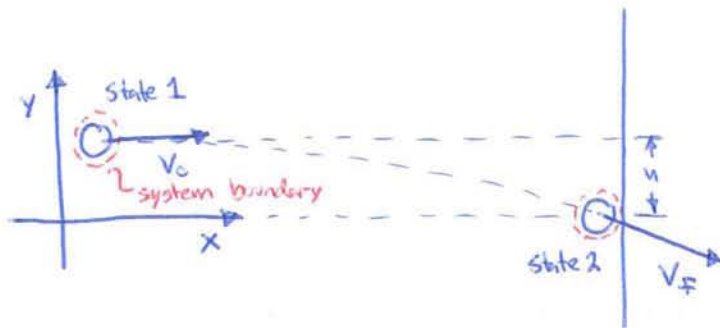
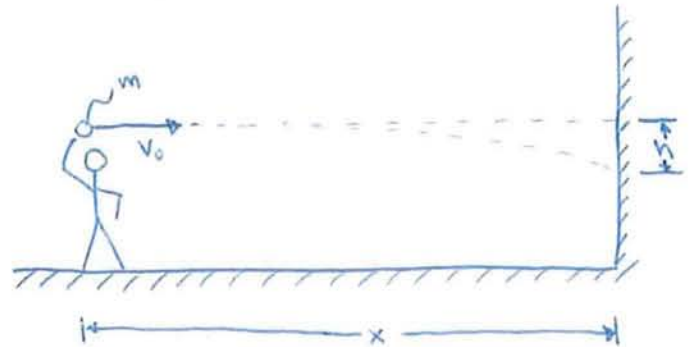
(4 marks) A ball is thrown horizontally towards a wall with an initial velocity of 10 m/s. The ball hits the wall 1.5 m below the horizontal plane in which it was thrown. Using the 1st law of thermodynamics.

determine:

the velocity of the ball just before it hits the wall.

given:

$m = 0.2 \text{ kg}$, $h = 1.5 \text{ m}$, $v_o = 10 \text{ m/s}$



$$E_1 = \frac{1}{2} m v_o^2 + mgh$$

$$E_2 = \frac{1}{2} m v_f^2$$

1st Law of Thermodynamics

$$E_f - E_o = \cancel{Q} - \cancel{W}$$

$$\left[\frac{1}{2} m v_f^2 \right] - \left[\frac{1}{2} m v_o^2 + mgh \right] = 0$$

$$\cancel{\frac{1}{2} m} v_f^2 = \cancel{\frac{1}{2} m} v_o^2 + \cancel{m} gh$$

$$v_f = \pm \sqrt{v_o^2 + 2gh}$$

$$v_f = \pm \sqrt{(10)^2 + (2)(9.81)(1.5)}$$

$$= \pm 11.4$$

$$v_f = 11.4 \text{ m/s}$$

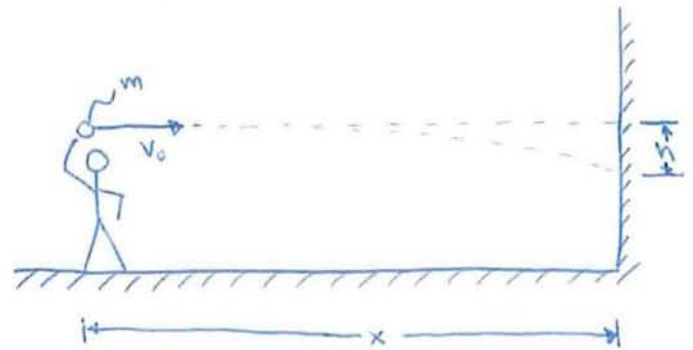
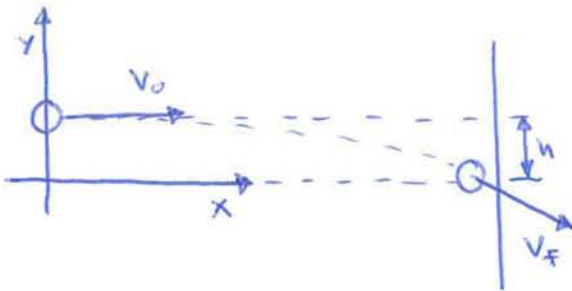
(4 marks) A ball is thrown horizontally towards a wall with an initial velocity of 10 m/s. The ball hits the wall 1.5 m below the horizontal plane in which it was thrown. Using kinematics.

determine:

the distance the wall is away from the thrower (x).

given:

$m = 0.2 \text{ kg}$, $h = 1.5 \text{ m}$, $v_o = 10 \text{ m/s}$



kinematic equations

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$v_{fy}^2 = v_{oy}^2 + 2a_y(x_f - x_o)$$

$$v_{fy} = \pm \sqrt{-2a_y y_o}$$

$$v_{fy} = \pm \sqrt{-2(-9.81)(1.5)}$$

$$v_{fy} = -5.42 \text{ m/s}$$

$$\vec{v} = \vec{v}_o + \vec{a}t$$

$$v_{fy} = v_{oy} + a_y t$$

$$t = \frac{v_{fy}}{a_y}$$

$$t = \frac{-5.42}{-9.81} = 0.55 \text{ s}$$

<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_o = 0$	$y_o = 1.5$	$t =$
$x_f =$	$y_f = 0$	
$v_{ox} = 10$	$v_{oy} = 0$	
$v_{fx} = 10$	$v_{fy} =$	
$a_x = 0$	$a_y = -9.81$	

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$x_f = x_o + v_{ox} t + \frac{1}{2} a_x t^2$$

$$x_f = (10)(0.55) = 5.5 \text{ m}$$

$$\text{distance } x = 5.5 \text{ m}$$

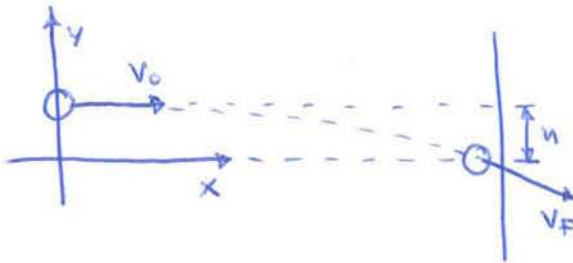
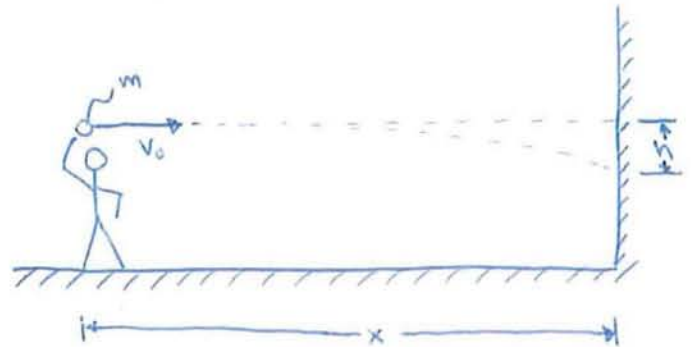
(4 marks) A ball is thrown horizontally towards a wall with an initial velocity of 10 m/s. The ball hits the wall 1.5 m below the horizontal plane in which it was thrown. Using kinematics.

determine:

the distance the wall is away from the thrower (x).

given:

$m = 0.2 \text{ kg}$, $h = 1.5 \text{ m}$, $v_o = 10 \text{ m/s}$



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_o = 0$	$y_o = 1.5$	$t =$
$x_f =$	$y_f = 0$	
$v_{ox} = 10$	$v_{oy} = 0$	
$v_{fx} = 10$	$v_{fy} =$	
$a_x = 0$	$a_y = -9.81$	

kinematic equations

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$y_f = y_o + v_{oy} t + \frac{1}{2} a_y t^2$$

$$\frac{1}{2} a_y t^2 = -y_o$$

$$t = \pm \sqrt{-\frac{2y_o}{a_y}}$$

$$t = \pm \sqrt{-\frac{(2)(1.5)}{-9.81}}$$

$$t = 0.55 \text{ sec}$$

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

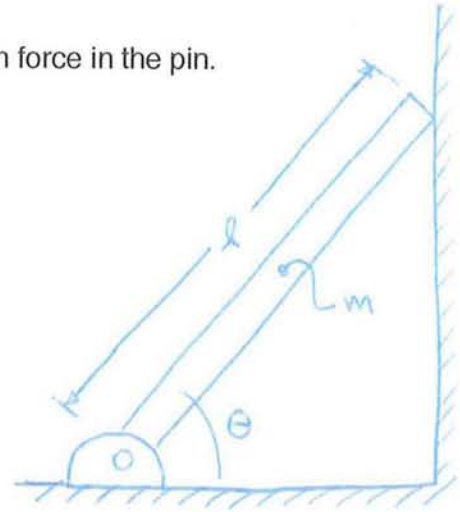
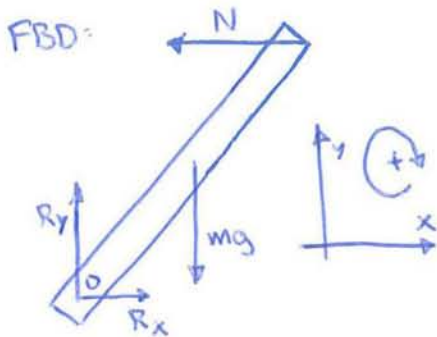
$$x_f = x_o + v_{ox} t + \frac{1}{2} a_x t^2$$

$$x_f = (10)(0.55) = 5.5 \text{ m}$$

$$\text{distance } x = 5.5 \text{ m}$$

(10 marks) Determine the magnitude and direction of the reaction force in the pin.

given: $L = 8 \text{ m}$, $\theta = 60^\circ$
 $m = 50 \text{ kg}$

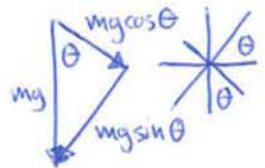
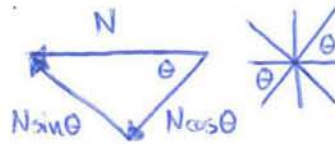


Newton's 2nd Law:

$$\sum F_x = m a_x = R_x - N \quad (1)$$

$$\sum F_y = m a_y = R_y - mg \quad (2)$$

$$\sum M_o = I_o \alpha_o = (mg \cos \theta) \left(\frac{L}{2} \right) - (N \sin \theta)(L) \quad (3)$$



solve for N in (3)

$$0 = \frac{1}{2} mg L \cos \theta - N L \sin \theta$$

$$N = \frac{mg \cos \theta}{2 \sin \theta} = \frac{(50)(9.81) \cos(60)}{2 \sin(60)} = 142 \quad (3a)$$

sub (3a) into (1) and solve for R_x :

$$R_x = N = 142$$

solve for R_y in (2)

$$R_y = mg = (50)(9.81) = 490$$

Magnitude and direction of Reaction Force

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(142)^2 + (490)^2}$$

$$R = 511 \text{ N}$$

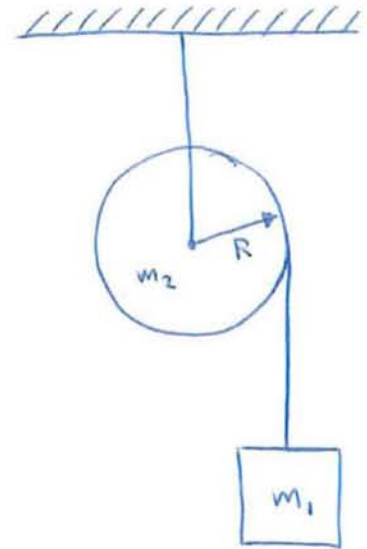


$$\phi = \tan^{-1} \left(\frac{R_y}{R_x} \right) = \tan^{-1} \left(\frac{490}{142} \right)$$

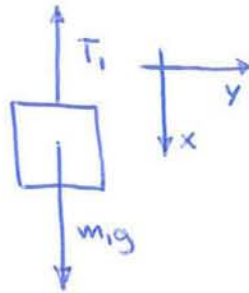
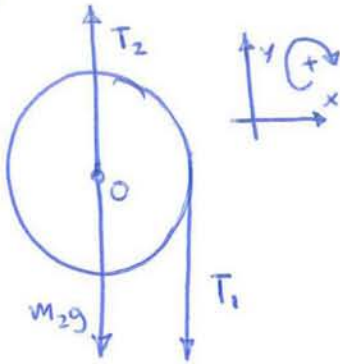
$$\phi = 74^\circ$$

(8 marks) Determine the tension in the rope and the acceleration of the system assuming the pulley is a solid disk.

given: $m_1 = 100 \text{ kg}$, $m_2 = 100 \text{ kg}$, $R = 1 \text{ m}$



FBD:



Newton's 2nd Law

$$\sum F_x = m_2 a_{x2} = 0$$

$$\sum F_y = m_2 a_{y2} = T_2 - m_2 g - T_1 \quad (1)$$

$$\sum M_O = I_O \alpha_O = R T_1 \quad (2)$$

$$\sum F_x = m_1 a_{x1} = m_1 g - T_1 \quad (3)$$

$$\sum F_y = m_1 a_{y1} = 0$$

massless, stretchless rope

$$a_{x1} = \alpha_O R = a_x \quad (4)$$

solid disk

$$I_O = \frac{1}{2} m_2 R^2 \quad (5)$$

solve for T_1 in (3)

$$T_1 = m_1 g - m_1 a_x \quad (3a)$$

sub (3a), (4) and (5) into (2)

$$\left[\frac{1}{2} m_2 \right] \left[\frac{a_x}{R} \right] = R [m_1 g - m_1 a_x]$$

$$\frac{1}{2} m_2 a_x = m_1 g - m_1 a_x$$

$$\frac{1}{2} m_2 a_x + m_1 a_x = m_1 g$$

$$\left(\frac{1}{2} m_2 + m_1 \right) a_x = m_1 g$$

$$a_x = g \frac{m_1}{m_1 + \frac{m_2}{2}}$$

$$a_x = (9.81) \frac{100}{100 + \frac{100}{2}} = 6.54 \text{ m/s}^2$$

$$T = m_1 g - m_1 a_x$$

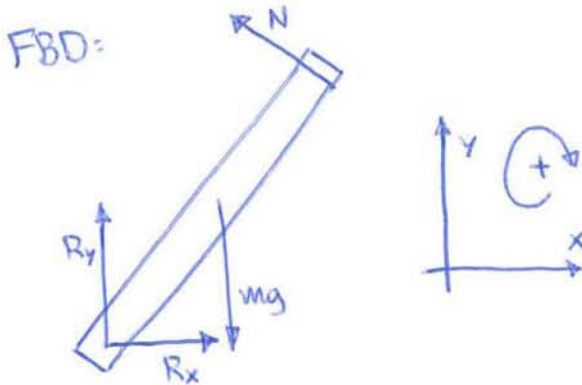
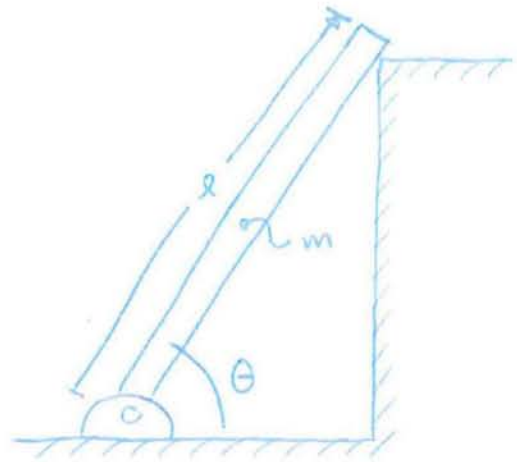
$$T = (100)(9.81) - (100)(6.54)$$

$$T = 327 \text{ N}$$

BONUS

(2 marks) Determine the magnitude and direction of the reaction force in the pin assuming the normal force at the corner is perpendicular to the beam.

given: $L = 8 \text{ m}$, $\theta = 60^\circ$
 $m = 50 \text{ kg}$

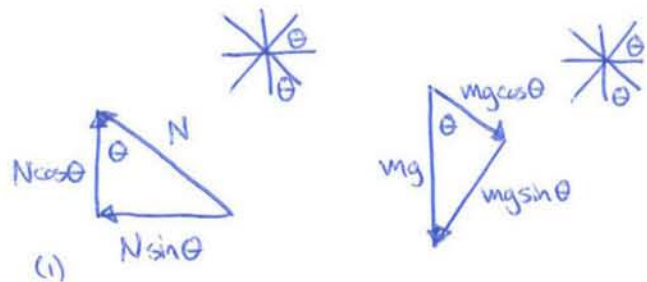


Newton's 2nd Law

$$\sum F_x = m a_x = R_x - N \sin \theta$$

$$\sum F_y = m a_y = R_y + N \cos \theta - mg$$

$$\sum M_o = I_o \alpha_o = (mg \cos \theta) \left(\frac{L}{2} \right) - (N)(L)$$



solve (3) for N :

$$N = \frac{1}{2} mg \cos \theta = \frac{1}{2} (50)(9.81) \cos(60) = 123 \quad (3a)$$

sub (3a) into (1) and solve for R_x :

$$R_x = N \sin \theta = (123) \sin(60) = 106$$

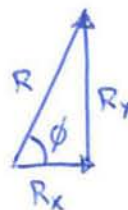
sub (3a) into (2) and solve for R_y :

$$R_y = mg - N \cos \theta = (50)(9.81) - (123) \cos(60) = 429$$

Magnitude and direction of Reaction Force:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(106)^2 + (429)^2}$$

$$R = 442 \text{ N}$$



$$\phi = \tan^{-1} \left(\frac{R_y}{R_x} \right) = \tan^{-1} \left(\frac{429}{106} \right)$$

$$\phi = 76^\circ$$