

Instructor:	Frank Secretain
Course:	Civil 11
Assessment:	Test 1
Time allowed:	110 minutes
Devices allowed:	Pencil, pen, eraser, calculator
Notes from instructor:	Be neat. Show your work where needed. Box final answers. This sheet can be ripped off and used as formula sheet. Write your name at the top of the second sheet (question 1).
Marks allocated:	4 questions worth 40 marks + bonus worth 2 marks (on back)
Percentage of final grade:	20% of final grade

Formula Sheet

Line equation

$$y = ax + b$$

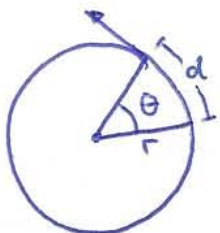
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_r = -\omega^2 r = -\frac{v^2}{r}$$

Kinematics

$$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$\vec{v} = \vec{v}_o + \vec{a} t$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$$

$$\omega = \omega_o + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt} = m\vec{a} \quad \vec{F}_g = \frac{Gm_1 m_2}{r^2} = mg \quad \vec{F}_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum \vec{M}_p = \frac{d(I_p \vec{\omega}_p)}{dt} = I_p \vec{\alpha}_p \quad \vec{M} = \vec{F} \times \vec{x} = \vec{F} \perp \vec{x}$$

Conservation of momentum (integral of Newton's 2nd Law with time)

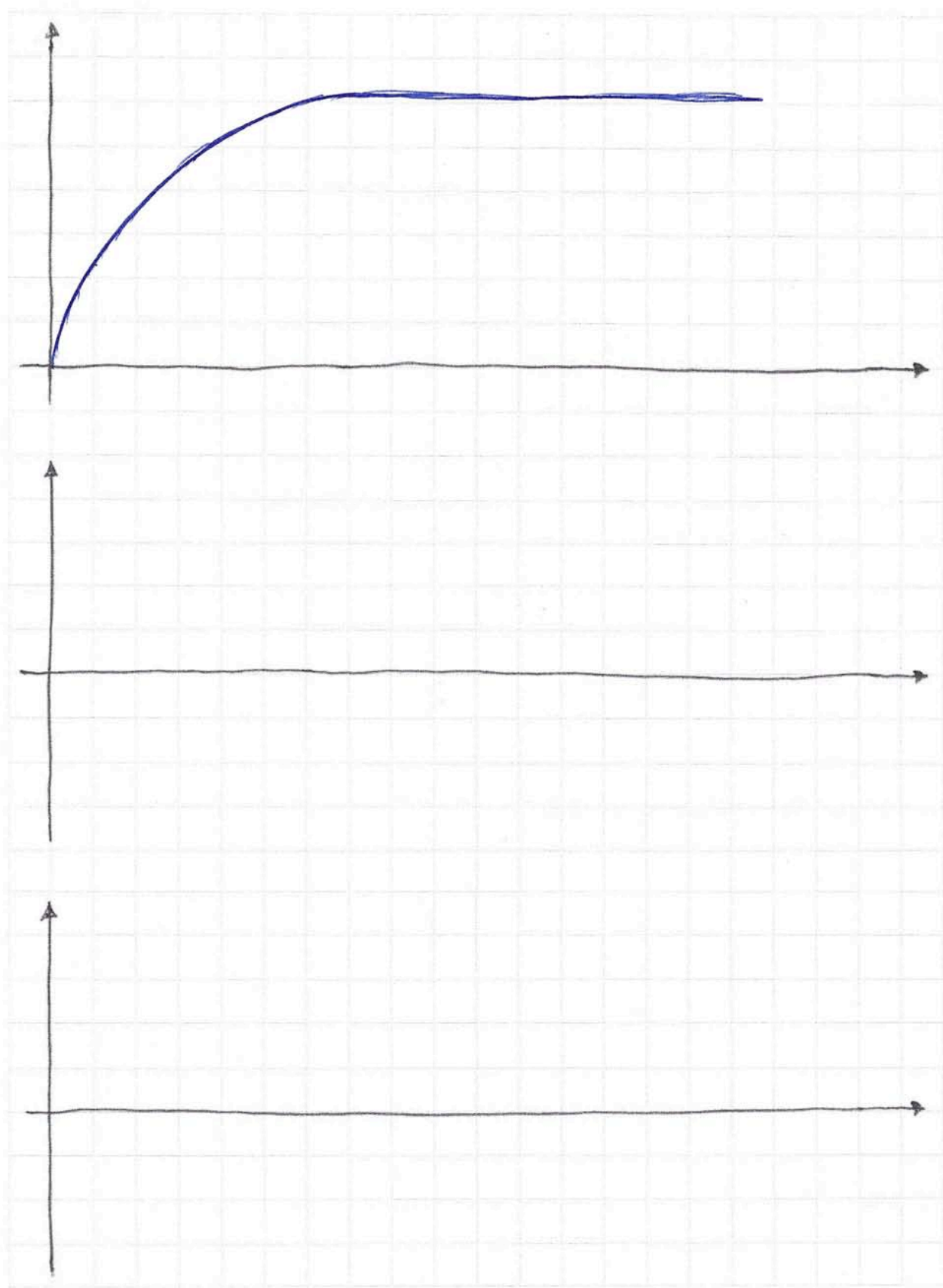
$$\vec{p} = m\vec{v} \quad \vec{p} - \vec{p}_o = \int \vec{F} dt = \vec{F}_{ave.}(t - t_o)$$

1st Law of Thermodynamics

$$E_f - E_o = Q - W \quad E_{PE} = mg(y - y_o) \quad E_{KE} = \frac{1}{2} m v^2$$

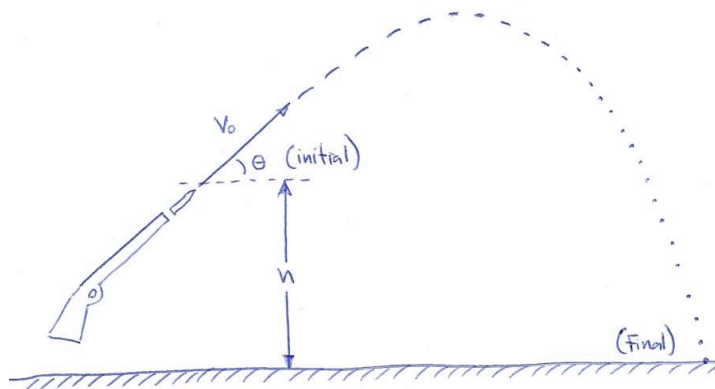
$$W = \vec{F} \cdot (\vec{x} - \vec{x}_o)$$

(5 marks) Complete the distance-velocity-acceleration graph.



(10 marks) Determine the time of flight for the given initial conditions:

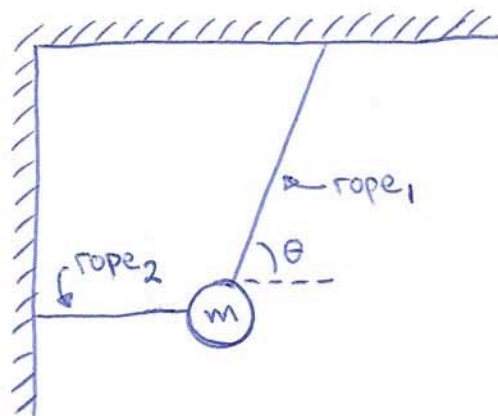
- a. $v_o = 330 \text{ m/s}$, $h = 2 \text{ m}$, $\theta = 90 \text{ degrees}$
- b. $v_o = 330 \text{ m/s}$, $h = 2 \text{ m}$, $\theta = 30 \text{ degrees}$



(10 marks) Determine the tension in each rope given initial conditions:

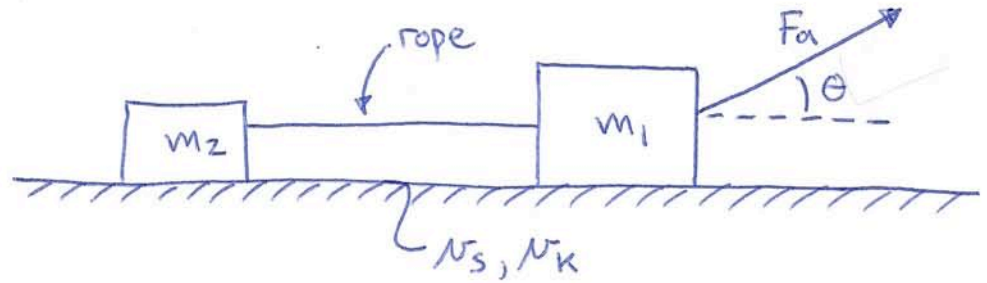
$$m = 100 \text{ kg}$$

$$\theta = 75^\circ$$



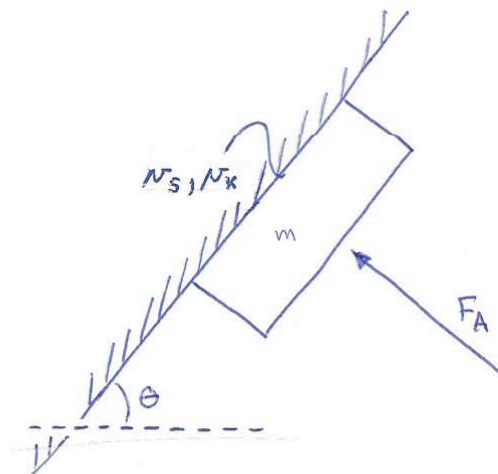
(15 marks) Determine the tension in the rope and the acceleration given initial conditions:

$F_a = 500 \text{ N}$
 $\theta = 35 \text{ degrees}$
 $m_1 = 100 \text{ kg}$
 $m_2 = 50 \text{ kg}$
 $\mu_s = 0.1$
 $\mu_k = 0.05$

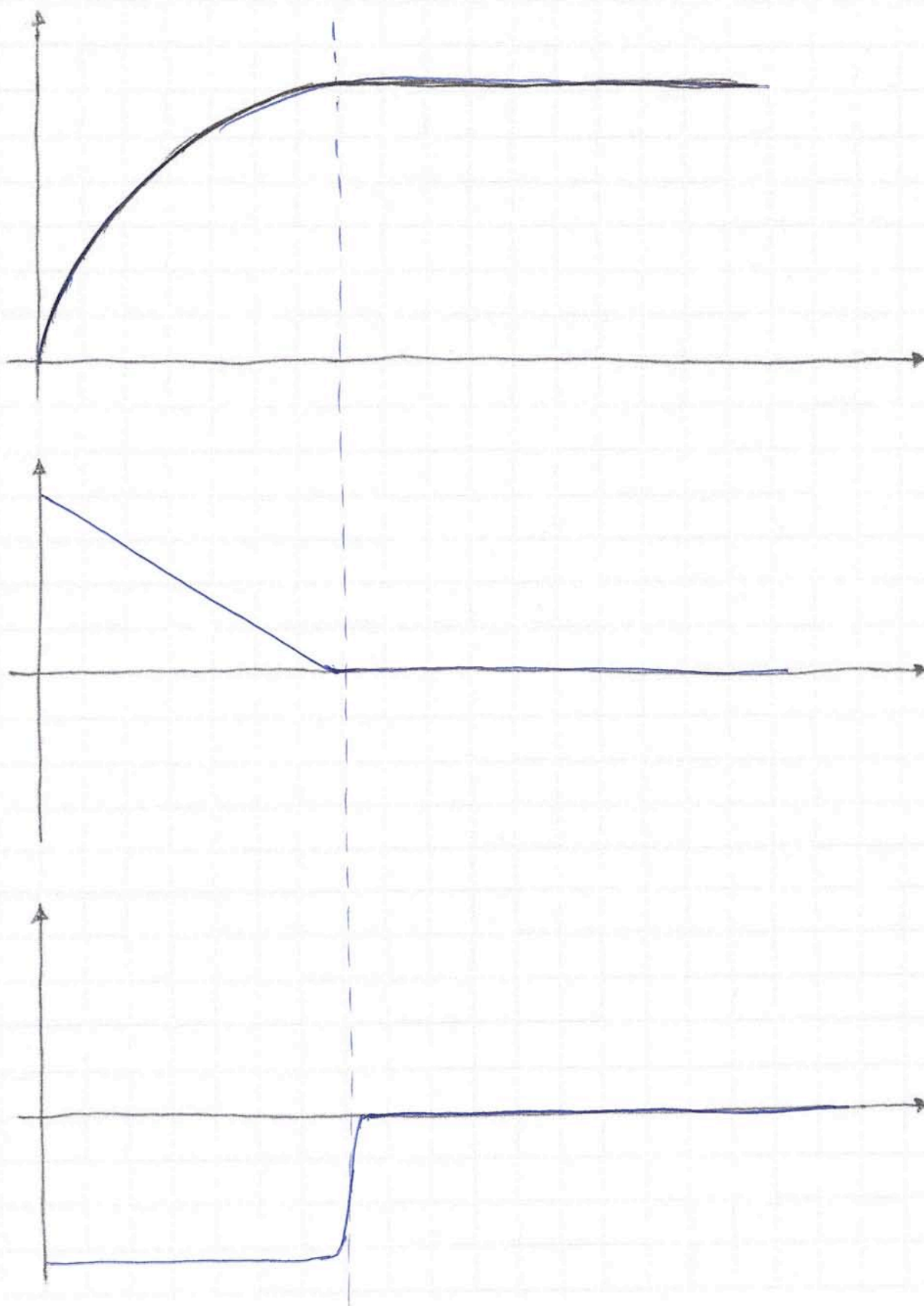


BONUS (2 marks)

Determine the minimum force need to keep the mass from moving.
Does the mass fall or does it slip first?

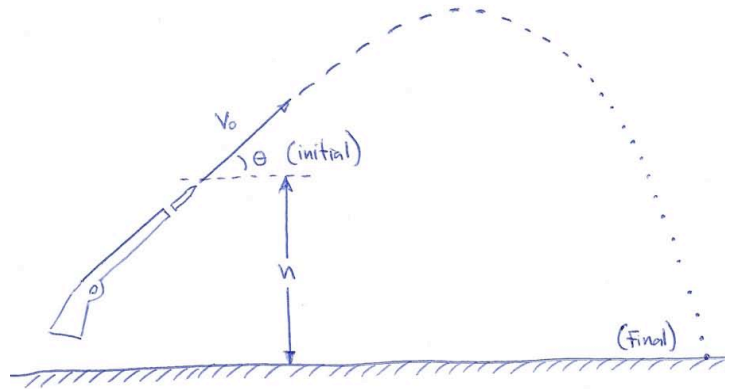


(5 marks) Complete the distance-velocity-acceleration graph.

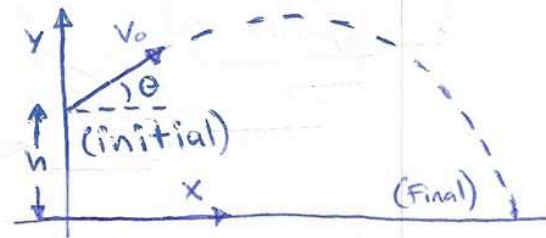


(10 marks) Determine the time of flight for the given initial conditions:

- a. $v_o = 330 \text{ m/s}$, $h = 2 \text{ m}$, $\theta = 90 \text{ degrees}$
 b. $v_o = 330 \text{ m/s}$, $h = 2 \text{ m}$, $\theta = 30 \text{ degrees}$



<u>x-dir</u>	<u>y-dir</u>	<u>time</u>
$x_o = 0$	$y_o = h$	$t =$
$x_f =$	$y_f = 0$	
$v_{ox} = v_o \cos \theta$	$v_{oy} = v_o \sin \theta$	
$v_{fx} =$	$v_{fy} =$	
$a_x = 0$	$a_y = -9.81 \text{ m/s}^2$	



kinematic equation:

$$\vec{v}^2 = \vec{v}_o^2 + 2\vec{a}(\vec{x} - \vec{x}_o)$$

$$v_{fy}^2 = v_{oy}^2 + 2a_y(y_f - y_o)$$

$$v_{fy} = \pm \sqrt{v_{oy}^2 - 2a_y h}$$

$$\vec{v} = \vec{v}_o + a t$$

$$v_{fy} = v_{oy} + a_y t$$

$$t = \frac{v_{fy} - v_{oy}}{a_y}$$

a)

$$v_{fy} = -\sqrt{(330 \sin 90)^2 - 2(-9.81)(2)}$$

$$= -330.1 \text{ m/s}$$

$$t = \frac{-330.1 - 330 \sin 90}{-9.81}$$

$$t = 67 \text{ sec}$$

b)

$$v_{fy} = -\sqrt{(330 \sin 30)^2 - 2(-9.81)(2)}$$

$$= -165.1 \text{ m/s}$$

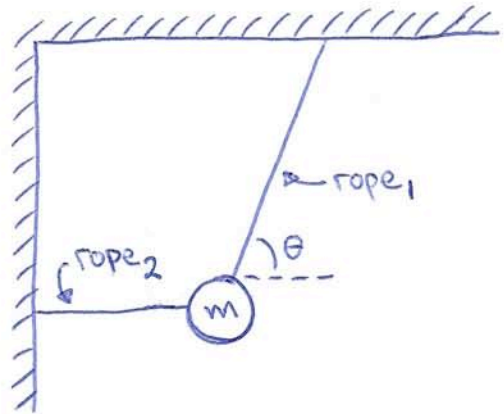
$$t = \frac{-165.1 - 330 \sin 30}{-9.81}$$

$$t = 34 \text{ sec}$$

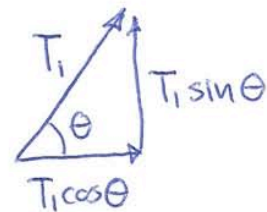
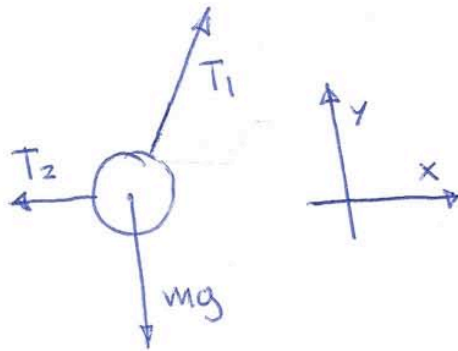
(10 marks) Determine the tension in each rope given initial conditions:

$$m = 100 \text{ kg}$$

$$\theta = 75 \text{ degrees}$$



FBD:



Newton's 2nd Law:

$$\sum F_x = m a_x = T_1 \cos \theta - T_2 \quad (1)$$

$$\sum F_y = m a_y = T_1 \sin \theta - mg \quad (2)$$

Solve eq. (2) for T_1 :

$$T_1 = \frac{mg}{\sin \theta} = \frac{(100)(9.81)}{\sin(75)} = 1016 \text{ N} \quad (2a)$$

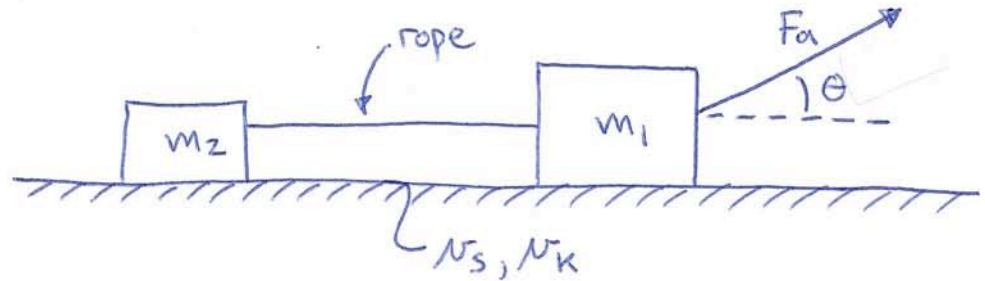
sub (2a) into (1) and solve for T_2 :

$$0 = \left[\frac{mg}{\sin \theta} \right] \cos \theta - T_2$$

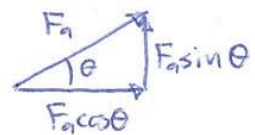
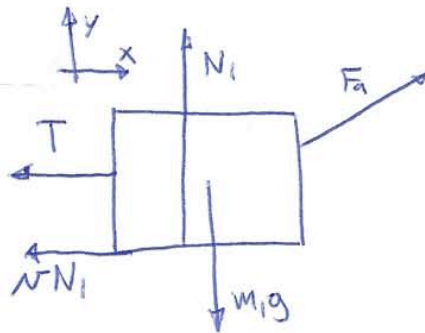
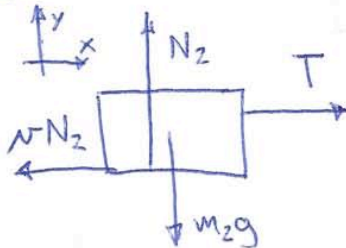
$$T_2 = mg \frac{\cos \theta}{\sin \theta} = (100)(9.81) \frac{\cos(75)}{\sin(75)} = 263 \text{ N}$$

(15 marks) Determine the tension in the rope and the acceleration given initial conditions:

$$\begin{aligned} F_a &= 500 \text{ N} \\ \theta &= 35 \text{ degrees} \\ m_1 &= 100 \text{ kg} \\ m_2 &= 50 \text{ kg} \\ \mu_s &= 0.1 \\ \mu_k &= 0.05 \end{aligned}$$



FBD:



Newton's 2nd Law:

$$\sum F_x = m_2 a_{x2} = T - \mu N_2 \quad (1)$$

$$\sum F_y = m_2 a_{y2} = N_2 - m_2 g \quad (2)$$

$$\sum F_x = m_1 a_{x1} = F_a \cos \theta - T - \mu N_1 \quad (3)$$

$$\sum F_y = m_1 a_{y1} = F_a \sin \theta + N_1 - m_1 g \quad (4)$$

assume mass-less, stretch-less rope so $a_{x1} = a_{x2} = a_x$

solve for N_1 in equ (4) and N_2 in equ. (2):

$$N_1 = m_1 g - F_a \sin \theta \quad (4a)$$

$$N_2 = m_2 g \quad (2a)$$

sub (2a) into (1) and solve for T :

$$m_2 a_x = T - \mu [m_2 g]$$

$$T = m_2 a_x + \mu m_2 g \quad (1a)$$

sub (1a) and (4a) into (3) and solve for a_x :

$$m_1 a_x = F_a \cos \theta - [m_2 a_x + \mu m_2 g] - \mu [m_1 g - F_a \sin \theta]$$

$$m_1 a_x = F_a \cos \theta - m_2 a_x - \mu m_2 g - \mu m_1 g + \mu F_a \sin \theta$$

$$(m_1 + m_2) a_x = F_a (\cos \theta + \mu \sin \theta) - \mu g (m_1 + m_2)$$

$$a_x = \frac{F_a (\cos \theta + \mu \sin \theta)}{m_1 + m_2} - \mu g$$

check if it moves:

$$a_x = \frac{500 (\cos 35 + (0.1) \sin 35)}{100 + 50} - (0.1)(9.81)$$

$$= 1.94 \text{ m/s}^2 \quad (\text{it moves!})$$

calculate acceleration:

$$a_x = \frac{500 (\cos 35 + (0.05) \sin 35)}{100 + 50} - (0.05)(9.81)$$

$$= 2.34 \text{ m/s}^2$$

substitute into (1a):

$$T = m_2 a_x + \mu m_2 g$$

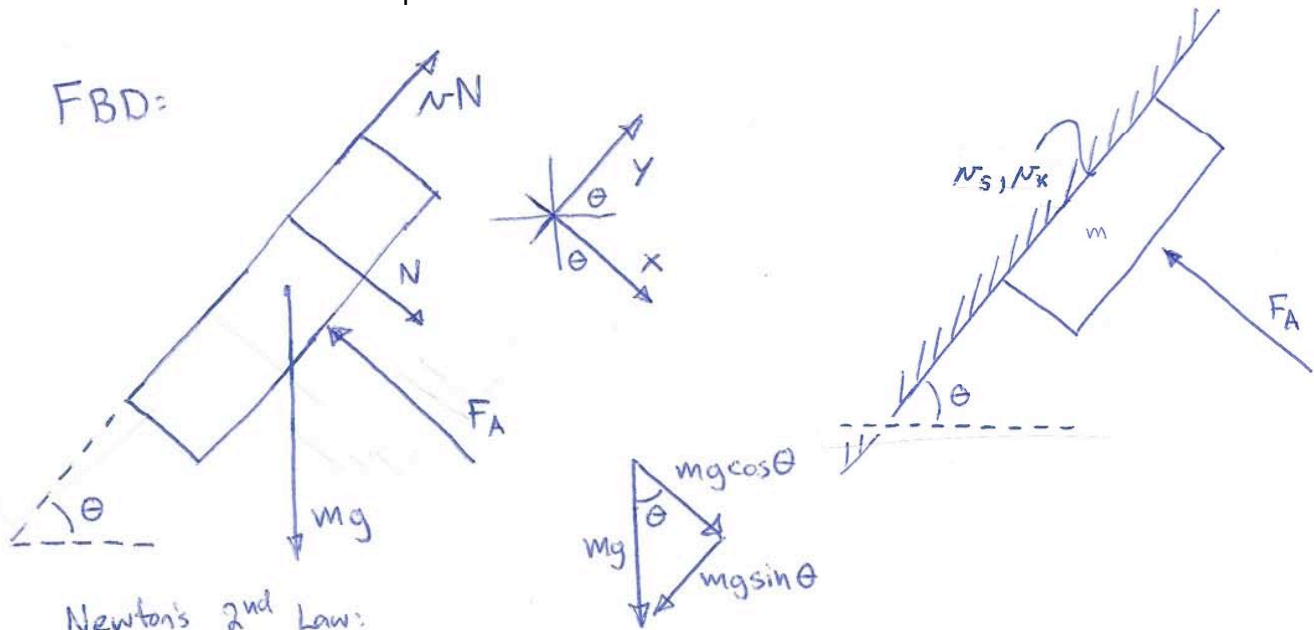
$$= (50)(2.3) + (0.05)(50)(9.81)$$

$$= 141.5$$

$$a_x = 2.3 \text{ m/s}^2, \quad T = 142 \text{ N}$$

BONUS (2 marks)

Determine the minimum force need to keep the mass from moving.
Does the mass fall or does it slip first?



Newton's 2nd Law:

$$\sum F_x = \cancel{m a_x} = N + mg \cos \theta - F_A \quad (1)$$

$$\sum F_y = \cancel{m a_y} = N \sin \theta - mg \sin \theta \quad (2)$$

solve for N in eq. (2):

$$N = \frac{mg \sin \theta}{\sin \theta} \quad (2a)$$

sub (2a) into (1) and solve for F_A :

$$0 = \left[\frac{mg \sin \theta}{\sin \theta} \right] + mg \cos \theta - F_A$$

$$F_A = mg \left(\frac{\sin \theta}{\sin \theta} + \cos \theta \right)$$

minimum force needed:

$$F_A = mg \left(\frac{\sin \theta}{\sin \theta} + \cos \theta \right)$$

Falls when N goes to 0, eq. (2a)

$$N = \frac{mg \sin \theta}{\sin \theta}$$

only @ $\sin \theta = 0$

$\theta = 0$ (slips first).