

Assessment (instructor):	2017 CIVL 11 Test 2 (Frank Secretain)
Time allowed:	170 minutes
Devices allowed:	pencil, pen, calculator, scrap paper
Marks allocated:	5 questions worth 100 marks
Percentage of final grade:	worth 20% of final grade
Notes from instructor:	Be neat, Box final answers

Kinematics (linear motion)

$$\vec{X} = \vec{X}_0 + \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{x} = \vec{x}_0 + \left(\frac{\vec{V}_0 + \vec{V}}{2} \right) t$$

$$\vec{V}^2 = \vec{V}_0^2 + 2\vec{a}(\Delta\vec{x})$$

$$\vec{V} = \vec{V}_0 + \vec{a}t$$

Kinematics (circular motion)

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = \theta_0 + \left(\frac{\omega_0 + \omega}{2} \right) t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$

$$\omega = \omega_0 + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = m\vec{a}$$

Force of gravity

$$F_g = \frac{Gm_1 m_2}{r^2} = mg \text{ (on earth)}$$

Force of friction

$$F_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum M_0 = I_0 \alpha_0$$

$$M = \vec{r} \times \vec{F} = F_{\perp} r$$

Centre of mass

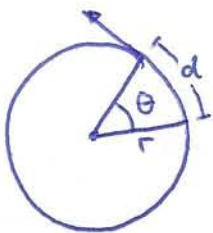
$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Conservation of Momentum (integral of Newton's 2nd Law with time)

$$\Delta \vec{p} = \int \vec{F} dt = (F_{AVE})(\Delta t)$$

$$\vec{p} = m\vec{v}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_{cent.} = -\omega^2 r = -\frac{v^2}{r}$$

Moment of Inertia Table

Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + l^2)$ $I_{yy} = \frac{1}{12} m(b^2 + l^2)$ $I_{zz} = \frac{1}{12} m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12} mb^2 + \frac{1}{3} ml^2$
Circular Cylinder		—	$I_{xx} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Sphere		—	$I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Uniform Slender Rod		—	$I_{yy} = \frac{1}{12} ml^2$ $I_{y_1y_1} = \frac{1}{3} ml^2$

(10 marks)

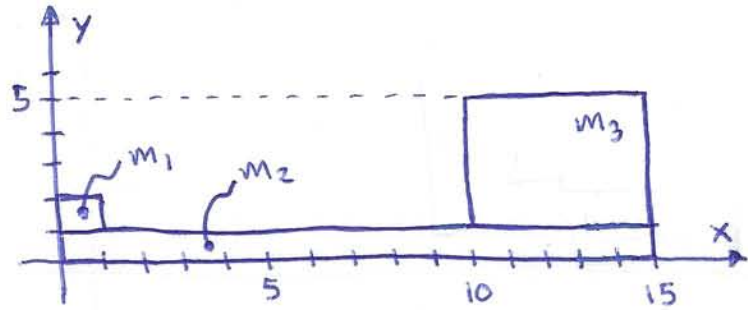
Determine the centre of mass for the three mass system and draw it on the diagram.

given;

$$m_1 = 1 \text{ kg}$$

$$m_2 = 15 \text{ kg}$$

$$m_3 = 20 \text{ kg}$$



(25 marks)

Determine the maximum velocity the car can drive around the axis of rotation before it slips.

given:

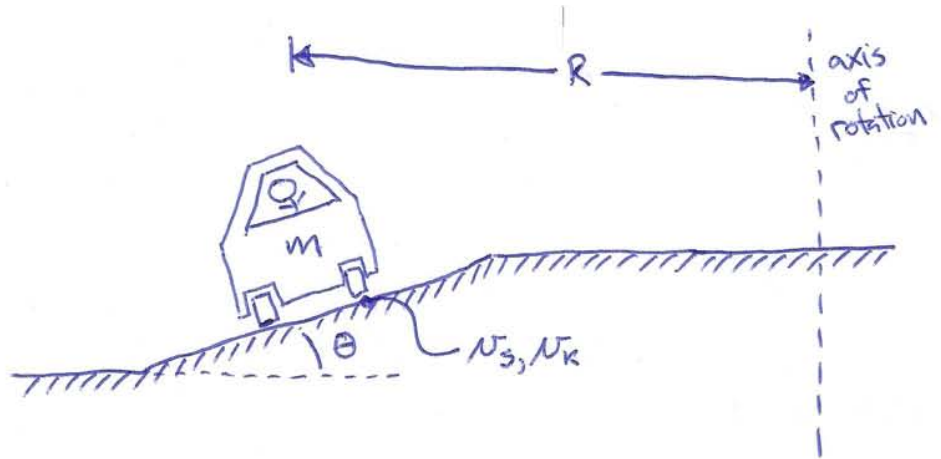
$$m = 1000 \text{ kg}$$

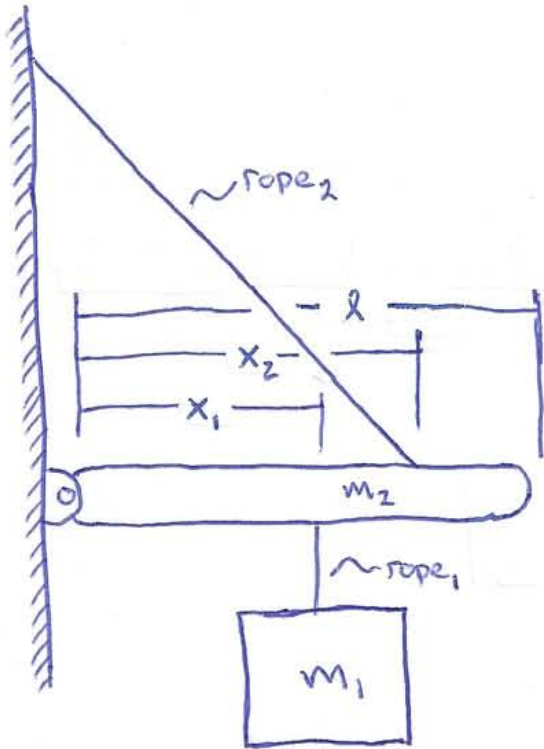
$$\theta = 10^\circ$$

$$\mu_s = 0.9$$

$$\mu_k = 0.8$$

$$R = 100 \text{ m}$$





(25 marks)

Determine the tensions in both ropes and the magnitude of the force extended on the pin attached to the beam.

given:

$$m_1 = 100 \text{ kg}$$

$$m_2 = 50 \text{ kg}$$

$$\text{length } (l) = 2 \text{ m}$$

$$x_1 = 1 \text{ m}$$

$$x_2 = 1.5 \text{ m}$$

(25 marks) Confirm that the systems will move, then determine the acceleration and the magnitude of the force on the pin through the solid cylinder pulley.

given:

$$m_1 = 100 \text{ kg}$$

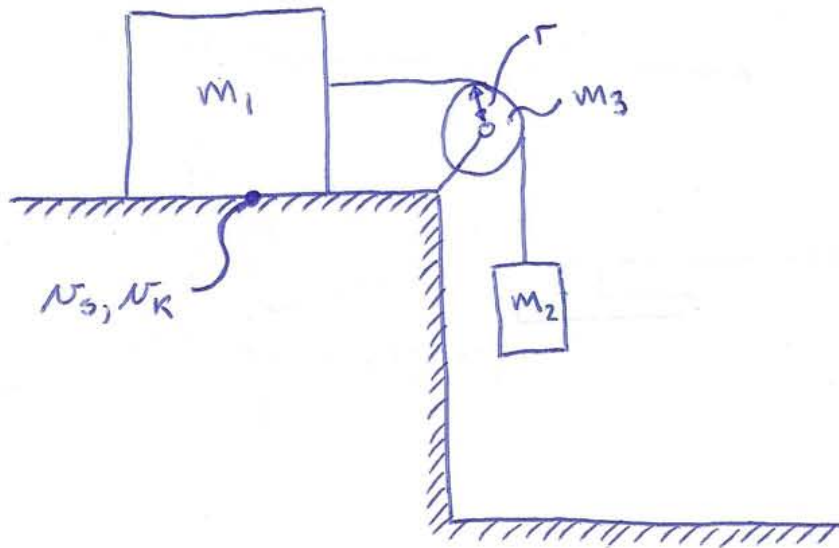
$$m_2 = 30 \text{ kg}$$

$$m_3 = 10 \text{ kg}$$

$$r = 1 \text{ m}$$

$$\mu_s = 0.2$$

$$\mu_k = 0.1$$



(15 marks)

Determine the average force and acceleration of the car as it smashed through the wall.

given:

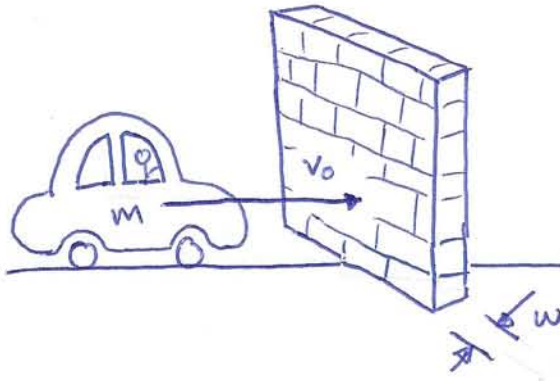
$$m = 1000 \text{ kg},$$

$$v_o = 30 \text{ m/s},$$

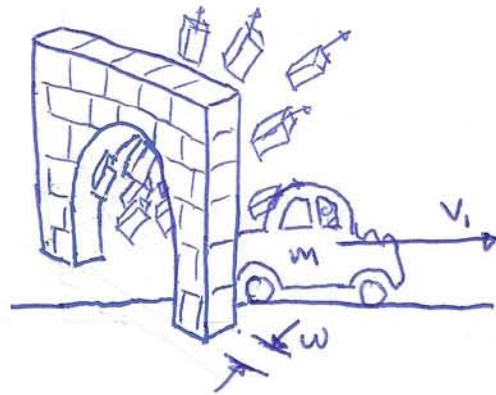
$$v_1 = 10 \text{ m/s},$$

$$w = 0.3 \text{ m}$$

initial state ($t=t_0$)



Final state ($t=t_1$)



(5 marks BONUS)

Determine the maximum velocity the van can drive around the axis of rotation before it tips over.

given:

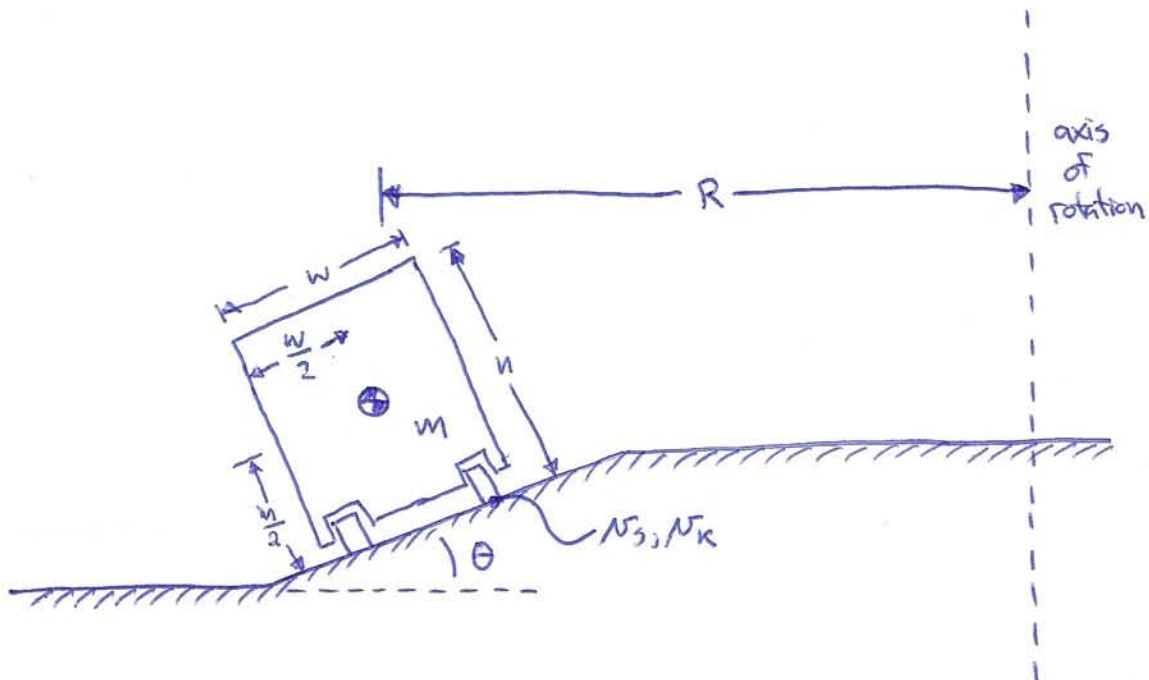
$$m = 1000 \text{ kg}$$

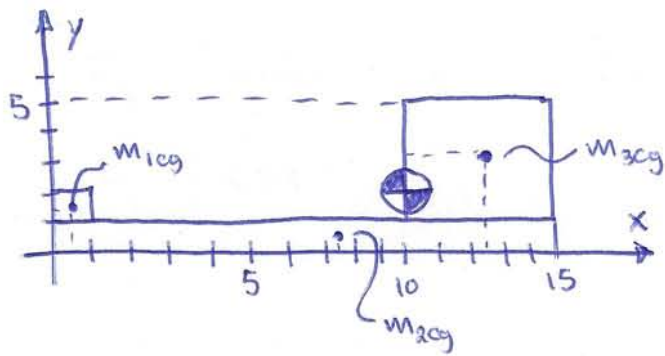
$$\theta = 10^\circ$$

$$\mu_s = 0.9$$

$$\mu_k = 0.8$$

$$R = 100 \text{ m}$$





$$m_1 = 1 \text{ kg}$$

$$m_2 = 15 \text{ kg}$$

$$m_3 = 20 \text{ kg}$$

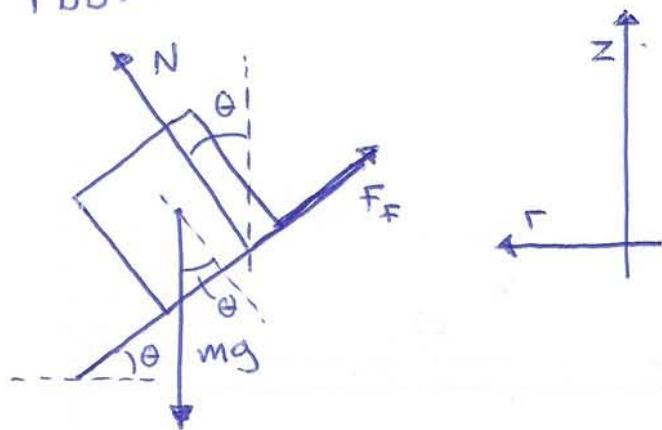
$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(1)(0.5) + (15)(7.5) + (20)(12.5)}{(1 + 15 + 20)} = 10.08\bar{3}$$

$$y_{cg} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(1)(1.5) + (15)(0.5) + (20)(3)}{(1 + 15 + 20)} = 1.91\bar{6}$$

$$x_{cg} = 10 \text{ units}$$

$$y_{cg} = 1.9 \text{ units}$$

FBD:



$$\sum F_z = m a_z = N \cos \theta + F_f \sin \theta - mg \quad (1)$$

$$\sum F_r = m a_r = N \sin \theta - F_f \cos \theta \quad (2)$$

Solve for N in eqn (1)

$$0 = N \cos \theta + \mu N \sin \theta - mg$$

$$mg = N (\cos \theta + \mu \sin \theta)$$

$$N = \frac{mg}{\cos \theta + \mu \sin \theta} \quad (1a)$$

Sub (1a) into (2) and solve for v :

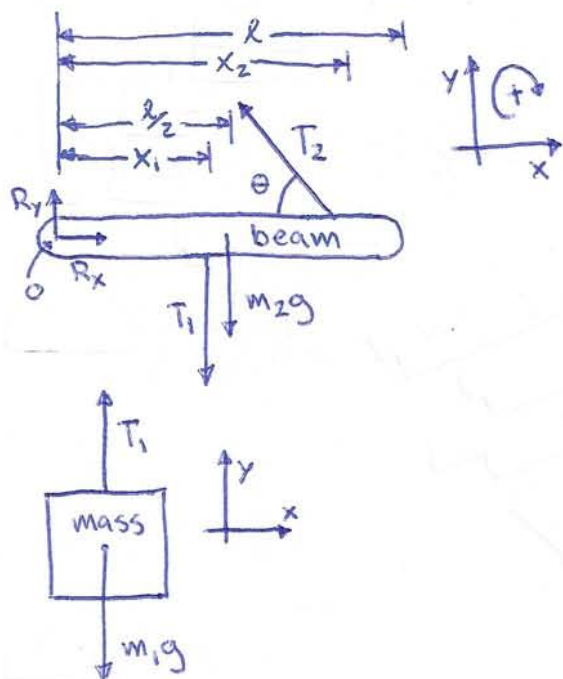
$$m \left(-\frac{v^2}{r} \right) = \left[\frac{mg}{\cos \theta + \mu \sin \theta} \right] \sin \theta - \mu \left[\frac{mg}{\cos \theta + \mu \sin \theta} \right] \cos \theta$$

$$-\frac{v^2}{r} = \left[\frac{g}{\cos \theta + \mu \sin \theta} \right] [\sin \theta - \mu \cos \theta]$$

$$v = \sqrt{(gr) \left[\frac{\mu \cos \theta - \sin \theta}{\cos \theta + \mu \sin \theta} \right]}$$

$$v = \sqrt{(9.81)(100) \left[\frac{(0.9) \cos(10) - \sin(10)}{\cos(10) + (0.9) \sin(10)} \right]} = 25 \text{ m/s} \quad (89 \text{ km/h})$$

FBD:



$$\sum F_x = m_2 a_x = R_x - T_2 \cos \theta \quad (1)$$

$$\sum F_y = m_2 a_y = R_y + T_2 \sin \theta - T_1 - m_2 g \quad (2)$$

$$\sum M_O = I_O \alpha_O = (T_1)(x_1) + (m_2 g)\left(\frac{l}{2}\right) - (T_2 \sin \theta)(x_2) \quad (3)$$

$$\sum F_y = m_1 a_y = T_1 - m_1 g \quad (4)$$

Solve for T_1 in eq. (4):

$$T_1 = m_1 g$$

Sub (4a) into (3) and solve for T_2 :

$$0 = [m_1 g] x_1 + (m_2 g)\left(\frac{l}{2}\right) - (T_2 \sin \theta)(x_2)$$

$$T_2 = \frac{(m_1 x_1 + \frac{l}{2} m_2) g}{x_2 \sin \theta}$$

Solve for R_x and R_y in eq. (1) and (2)

$$R_x = T_2 \cos \theta$$

$$R_y = T_1 + m_2 g - T_2 \sin \theta$$

$$T_1 = (100)(9.81) = 981 \text{ N}$$

$$T_2 = \frac{[(100)(1) + (\frac{2}{2})(50)](9.81)}{(1.5) \sin(60)} = 1133 \text{ N}$$

$$R_x = (1133) \cos(60) = 566 \text{ N}$$

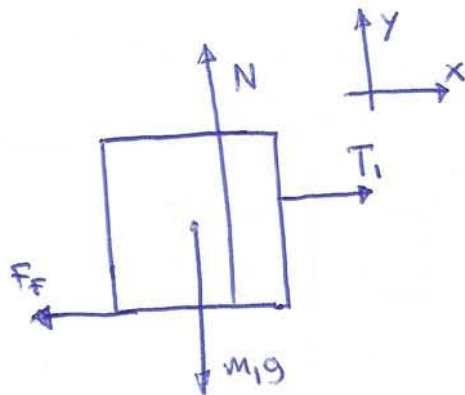
$$R_y = 981 + (50)(9.81) - (1133) \sin(60) = 490 \text{ N}$$

$$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{566^2 + 490^2} = 750 \text{ N}$$

$$\angle R = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{490}{566}\right) = 41^\circ$$

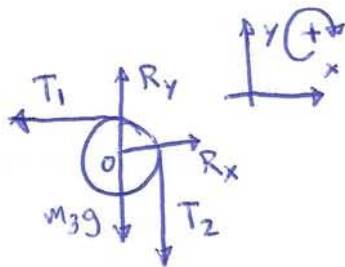


FBD



$$\sum F_x = m_1 a_x = T_1 - F_f \quad (1)$$

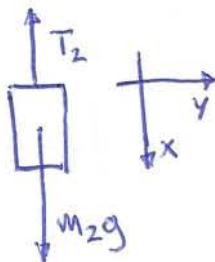
$$\sum F_y = m_1 a_y = N - m_1 g \quad (2)$$



$$\sum F_x = m_3 a_{x3} = R_x - T_1 \quad (3)$$

$$\sum F_y = m_3 a_{y3} = R_y - T_2 - m_3 g \quad (4)$$

$$\sum M_o = I_o \alpha_o = T_2 r - T_1 r \quad (5)$$



$$\sum F_x = m_2 a_x = m_2 g - T_2 \quad (6)$$

Solve eq. (2) for N:

$$N = m_1 g \quad (2a)$$

Sub (2a) into (1) and solve for T_1 :

$$m_1 a_x = T_1 - \mu(m_1 g) \Rightarrow T_1 = m_1 a_x + \mu m_1 g \quad (1a)$$

Solve (6) for T_2 :

$$m_2 a_x = m_2 g - T_2 \Rightarrow T_2 = m_2 g - m_2 a_x \quad (6a)$$

Sub (1a) & (6a) into (5) and solve for a_x :

$$I_0 \alpha_0 = [m_2 g - m_2 a_x] r - [m_1 a_x + \mu m_1 g] r$$

$$\left(\frac{1}{2} m_3 \cancel{r}\right) \left(\frac{a_x}{\cancel{r}}\right) = m_2 g \cancel{r} - m_2 a_x \cancel{r} - m_1 a_x \cancel{r} - \mu m_1 g \cancel{r}$$

$$a_x = (g) \left(\frac{m_2 - \mu m_1}{m_1 + m_2 + \frac{m_3}{2}} \right)$$

Solve for R_x in eqn. (3) and sub (1a)

$$R_x = T_1 = m_1 a_x + \mu m_1 g$$

Solve for R_y in eqn. (4) and sub (6a)

$$R_y = T_2 + m_3 g = [m_2 g - m_2 a_x] + m_3 g = m_2 (g - a_x) + m_3 g$$

$$a_{x_{ms}} = (9.81) \left[\frac{30 - (0.2)(100)}{100 + 30 + \frac{10}{2}} \right] = +0.7 \text{ m/s}^2 \quad (\text{it moves!})$$

$$a_x = (9.81) \left[\frac{30 - (0.1)(100)}{100 + 30 + \frac{10}{2}} \right] = 1.5 \text{ m/s}^2$$

$$R_x = (100)(1.5) + (0.1)(100)(9.81) = 248 \text{ N}$$

$$R_y = (30)(9.81 - 1.5) + (10)(9.81) = 347 \text{ N}$$

$$|R| = \sqrt{R_x^2 + R_y^2} = \sqrt{248^2 + 347^2} = 427 \text{ N}$$

$$\angle R = \tan^{-1} \left(\frac{R_y}{R_x} \right) = \tan^{-1} \left(\frac{347}{248} \right) = 54^\circ$$



Momentum

$$p_0 = m v_0$$

$$p_1 = m v_1$$

$$\Delta p = F_{\text{AVE}} \Delta t \quad \Rightarrow \quad F_{\text{AVE}} = \frac{m(v_1 - v_0)}{\Delta t}$$

Kinematics

$$x_1 = x_0 + \left(\frac{v_1 + v_0}{2} \right) \Delta t \quad \Rightarrow \quad \Delta t = \frac{2(x_1 - x_0)}{v_0 + v_1}$$

$$v_1^2 = v_0^2 + 2a \Delta x \quad \Rightarrow \quad a_{\text{AVE}} = \frac{v_1^2 - v_0^2}{2\Delta x}$$

Newton's 2nd Law (check)

$$\sum \vec{F} = m \vec{a} \quad \Rightarrow \quad F_{\text{AVE}} = m a_{\text{AVE}}$$

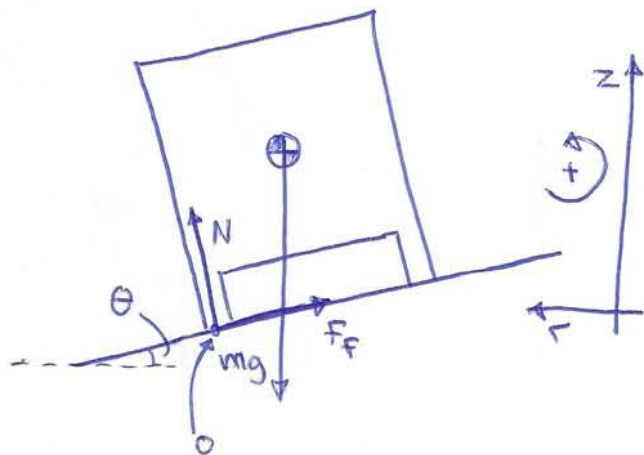
$$\Delta t = \frac{2(0.3)}{30 + 10} = 0.015 \text{ s}$$

$$F_{\text{AVE}} = \frac{(1000)(10 - 30)}{0.015} = -1333333 \text{ N}$$

$$a_{\text{AVE}} = \frac{10^2 - 30^2}{2(0.3)} = -1333 \text{ m/s}^2 \quad (136 \text{ g's})$$

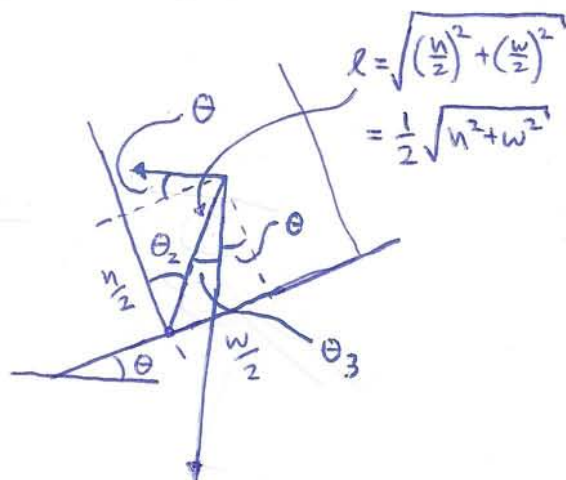
Very large #'s
"w" was estimated
too small or " $v_0 - v_1$ "
too large.

FBD:



$$\theta_2 = \tan^{-1}\left(\frac{w}{n}\right)$$

$$\theta_3 = \theta_2 - \theta$$



$$\sum F_z = m a_z = N \cos \theta + F_f \sin \theta - mg \quad (1)$$

$$\sum F_r = m a_r = N \sin \theta - F_f \cos \theta \quad (2)$$

$$\sum M_o = I_o \alpha_o = (m a_r)(l \cos \theta_3) - (mg)(l \sin \theta_3) \quad (3)$$

Sub $a_r = \frac{v^2}{r}$ into equ. (3) and solve for v:

$$0 = \cancel{m} \left(\frac{v^2}{r} \right) \cancel{\times} \cos \theta_3 - \cancel{m} g \cancel{\times} \sin \theta_3$$

$$V_{\max} = \sqrt{(gr) \frac{\sin \theta_3}{\cos \theta_3}} = \sqrt{gr \tan \theta_3}$$

$$\theta_2 = \tan^{-1}\left(\frac{2}{3}\right) = 34^\circ, \quad \theta_3 = 34 - 10 = 24^\circ$$

$$V_{\max} = \sqrt{(9.81)(100) \tan(24)} = 21 \text{ m/s} \quad (75 \text{ km/h})$$