

Test 1

Time allowed:	170 minutes
Devices allowed:	pencil, pen, calculator, scrap paper
Marks allocated:	5 questions worth 100 marks
Percentage of final grade:	worth 20% of final grade
Notes from instructor:	Be neat Box final answers

Formulas

Kinematics (linear motion)

$$\vec{X} = \vec{X}_0 + \vec{V}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{x} = \vec{x}_0 + \left(\frac{\vec{V}_0 + \vec{V}}{2} \right) t$$

$$\vec{V}^2 = \vec{V}_0^2 + 2\vec{a}(\Delta\vec{x})$$

$$\vec{V} = \vec{V}_0 + \vec{a}t$$

Kinematics (circular motion)

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = \theta_0 + \left(\frac{\omega_0 + \omega}{2} \right) t$$

$$\omega = \omega_0 + 2\alpha(\Delta\theta)$$

$$\omega = \omega_0 + \alpha t$$

Dynamics

$$\sum \vec{F} = m\vec{a}$$

Common forces

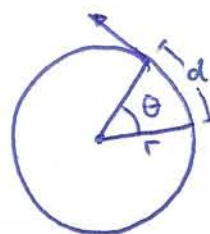
$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_f = \mu N$$

$$F_s = -k\Delta x$$

$$F_e = \frac{kq_1q_2}{r^2}$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

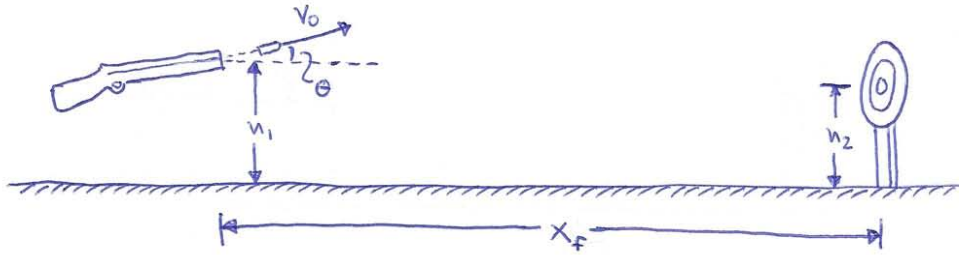
$$a_{\text{cent}} = \omega^2 r = \frac{v^2}{r}$$

Test 1

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(20 marks) A rifle fires a bullet from a distance of h_1 above the ground at a target. The bullet leaves the gun with an initial velocity of v_o and an angle with respect to the ground of θ .



Question 1-4 use: $v_o = 300$ m/s, $h_1 = 1.2$ meters, $x_f = 600$ meters, $\theta = 2$ degrees

- 1) what is the horizontal velocity when the bullet reaches the target?
- 2) what is the time it takes for the bullet to reach the target?
- 3) what is the vertical velocity when the bullet reaches the target?
- 4) at what height must the target be set (i.e. h_2) for the shooter to get a bullseye?

Question 5 use: $v_o = 300$ m/s, $h_1 = 1.2$ meters, $h_2 = 1.2$ meters, $x_f = 600$ meters

- 5) at what angle would the shooter need to shoot to get a bullseye if $h_2 = h_1 = 1.2$ m

Test 1

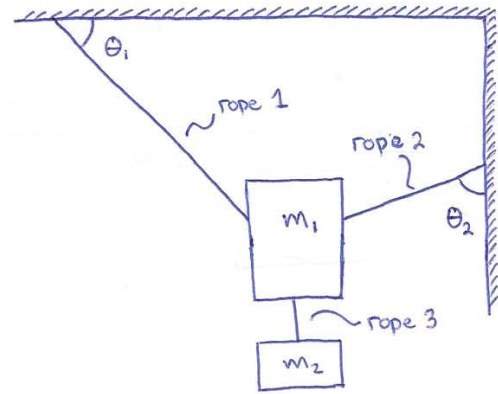
Test 1

(30 marks)

Two masses are hung by ropes as shown in the figure.

$m_1 = 100 \text{ kg}$,
 $m_2 = 10 \text{ kg}$,
 $\theta_1 = 50^\circ$,
 $\theta_2 = 70^\circ$

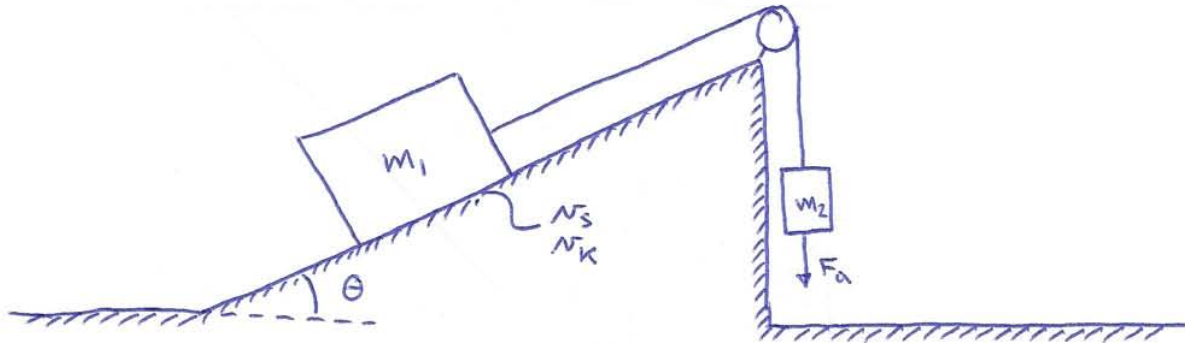
- 6) what is the tension in rope 3?
- 7) what is the tension in rope 2?
- 8) what is the tension in rope 1?



Test 1

Test 1

(30 marks) A block (the “first block”) is attached by a rope to a second block over a frictionless masses-less pulley. An applied force is directed downward to the second mass as shown in the figure.



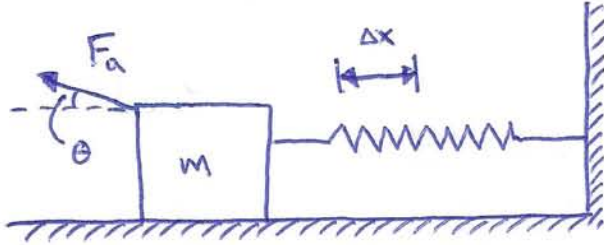
$$m_1 = 100 \text{ kg}, m_2 = 10 \text{ kg}, \mu_s = 0.2, \mu_k = 0.15, \theta = 30 \text{ degrees}$$

- 9) what is the normal force on the first block?
- 10) what is the minimum applied force that will stop the first block from sliding down the ramp?
- 11) what is the minimum applied force that will accelerate the first block up the ramp?
- 12) what would the acceleration of the blocks be?

Test 1

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(10 marks) A block on a frictionless surface is attached to a spring which is attached to a wall. A force is applied to the block away from the spring at angle of θ with respect to the ground.



$$F_a = 50 \text{ N}, m = 10 \text{ kg}, \theta = 30 \text{ degrees}, k_{\text{spring}} = 30 \text{ N/m}$$

13) what is the normal force on the block?

14) how much does the spring stretch from its equilibrium position (i.e. Δx)?

Test 1

(10 marks) Assuming that you live on a flat planet (i.e. no hills or anything that would get in the way) and neglecting air resistance at what velocity would you need to fire a bullet so that it would travel all the way around the planet (i.e. be in orbit).

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$$

15) on earth

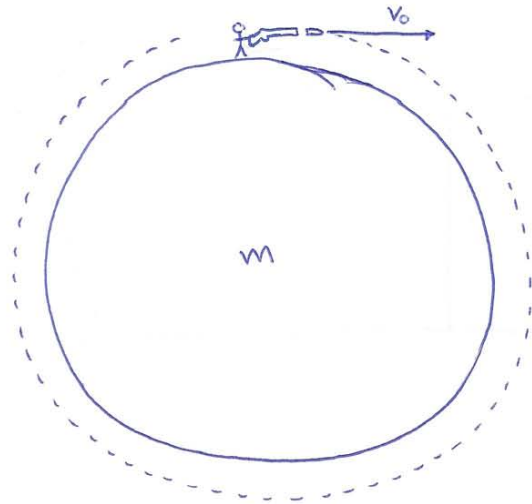
$$m_{\text{earth}} = 6.0 \times 10^{24} \text{ kg}$$

$$r_{\text{earth}} = 6371 \text{ km}$$

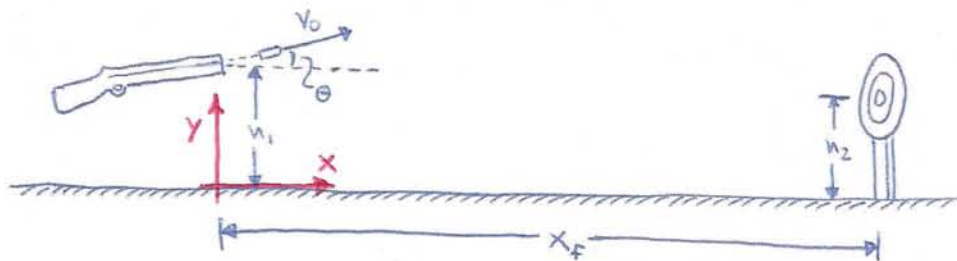
16) on the moon

$$m_{\text{moon}} = 7.3 \times 10^{22} \text{ kg}$$

$$r_{\text{moon}} = 1737 \text{ km}$$



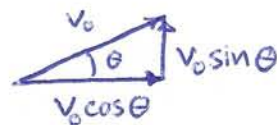
A rifle fires a bullet from a distance of h_1 above the ground at a target. The bullet leaves the gun with an initial velocity of v_o and an angle with respect to the ground of θ .



$v_o = 300$ m/s, $h_1 = 1.2$ meters, $x_f = 600$ meters, $\theta = 2$ degrees
 what is the horizontal velocity when the bullet reaches the target?
 what is the time it takes for the bullet to reach the target?
 what is the vertical velocity when the bullet reaches the target?
 at what height must the target be set (i.e. h_2) for the shooter to get a bulls-eye?

$v_o = 300$ m/s, $h_1 = 1.2$ meters, $h_2 = 1.2$ meters, $x_f = 600$ meters
 at what angle would the shooter need to shoot to get a bulls-eye if $h_2 = h_1 = 1.2$ m

$$v_{x0} = v_o \cos \theta = (300) \cos(2) = 299.8 \text{ m/s}$$



$$x_f = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \quad a_x = 0 \Rightarrow t = \frac{x_f - x_0}{v_{x0}}$$

$$t = \frac{600 - 0}{299.8} = 2.0 \text{ s}$$

$$v_{y0} = v_o \sin \theta = (300) \sin(2) = 10.5 \text{ m/s}$$

$$v_{yf} = v_{y0} + a_y t = 10.5 + (-9.81)(2.0) = -9.12 \text{ m/s}$$

$$y_f = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 = 1.2 + (10.5)(2.0) + \frac{1}{2}(-9.81)(2.0)^2 = 2.6 \text{ m}$$

$$t = \frac{x_f - x_0}{v_o \cos \theta} \quad (1), \quad y_f = y_0 + (v_o \sin \theta)t + \frac{1}{2}a_y t^2 \quad (2)$$

sub (1) into (2) and solve for θ :

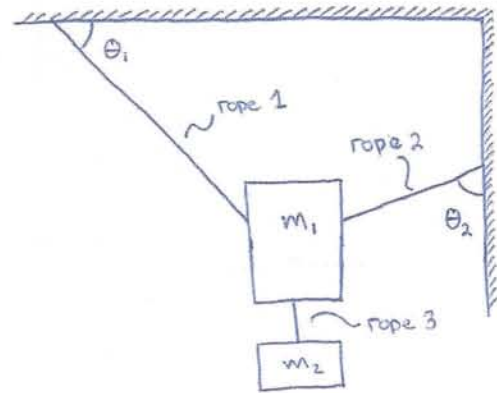
$$y_f - y_0 = v_o \sin \theta \left(\frac{x_f - x_0}{v_o \cos \theta} \right) + \frac{1}{2}a_y \left(\frac{x_f - x_0}{v_o \cos \theta} \right)^2 \Rightarrow 2 \sin \theta \cos \theta = \frac{a_y (x_0 - x_f)}{v_o^2}$$

$$2 \sin \theta \cos \theta = \sin(2\theta) \Rightarrow \theta = \frac{1}{2} \sin^{-1} \left(\frac{a_y (x_0 - x_f)}{v_o^2} \right) = 1.87^\circ$$

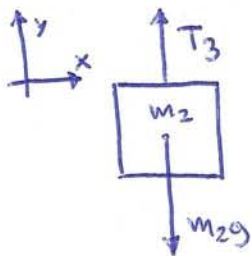
Two masses are hung by ropes as shown in the figure.

$m_1 = 100 \text{ kg}$,
 $m_2 = 10 \text{ kg}$,
 $\theta_1 = 50^\circ$,
 $\theta_2 = 70^\circ$

what is the tension in rope 3?
 what is the tension in rope 2?
 what is the tension in rope 1?



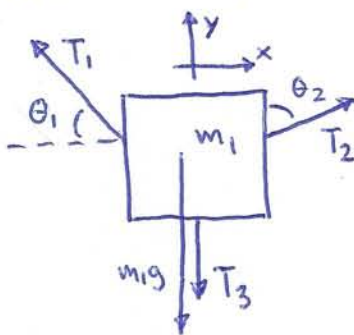
FBD on m_2 :



$$\sum F_y = m_2 a_y = T_3 - m_2 g \quad (a_y = 0)$$

$$T_3 = m_2 g = (10)(9.81) = 98.1 \text{ N}$$

FBD on m_1 :



$$\sum F_x = m_1 a_x = -T_1 \cos \theta_1 + T_2 \sin \theta_2 \quad (a_x = 0) \quad (1)$$

$$\sum F_y = m_1 a_y = T_1 \sin \theta_1 + T_2 \cos \theta_2 - m_1 g - T_3 \quad (a_y = 0) \quad (2)$$

Solve for T_1 in equ. (1)

$$T_1 = \frac{T_2 \sin \theta_2}{\cos \theta_1} \quad (1a)$$

Sub (1a) into equ. (2):

$$0 = \left[\frac{T_2 \sin \theta_2}{\cos \theta_1} \right] \sin \theta_1 + T_2 \cos \theta_2 - m_1 g - T_3$$

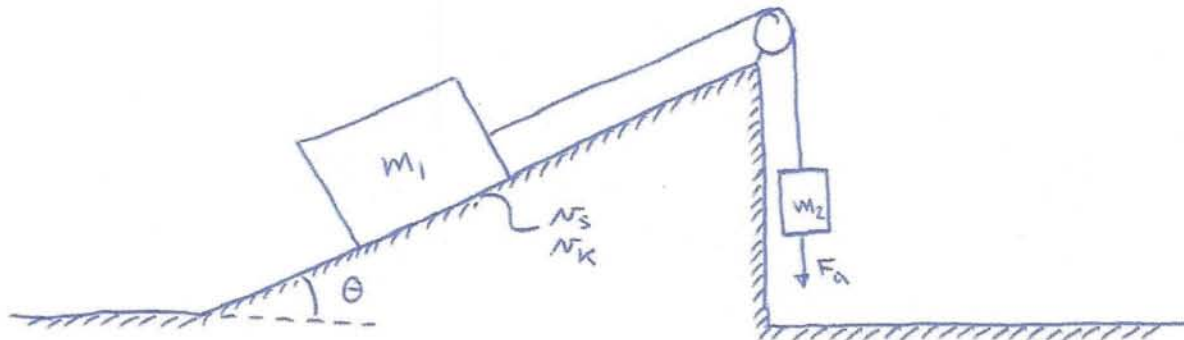
$$0 = T_2 [\sin \theta_2 \tan \theta_1 + \cos \theta_2] - m_1 g - T_3$$

$$T_2 = \frac{m_1 g + T_3}{\sin \theta_2 \tan \theta_1 + \cos \theta_2} = \frac{(100)(9.81) + 98.1}{\sin(70) \tan(50) + \cos(70)} = 738 \text{ N}$$

sub into equ. (1a)

$$T_1 = \frac{T_2 \sin \theta_2}{\cos \theta_1} = \frac{(738) \sin(70)}{\cos(50)} = 1079 \text{ N}$$

A block (the "first block") is attached by a rope to a second block over a frictionless massless pulley. An applied force is directed downward to the second mass as shown in the figure.



$$m_1 = 100 \text{ kg}, m_2 = 10 \text{ kg}, \mu_s = 0.2, \mu_k = 0.15, \theta = 30^\circ$$

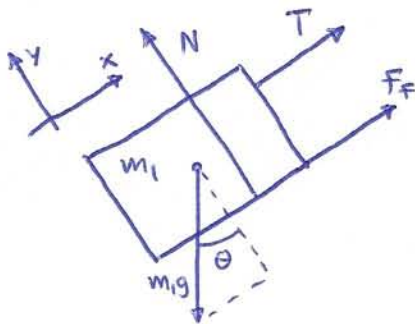
what is the normal force on the first block?

what is the minimum applied force that will stop the first block from sliding down the ramp?

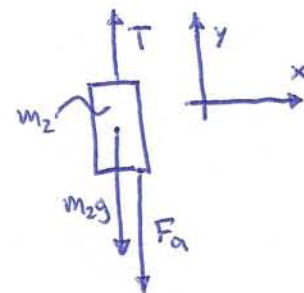
what is the minimum applied force that will accelerate the first block up the ramp?

what would the acceleration of the blocks be?

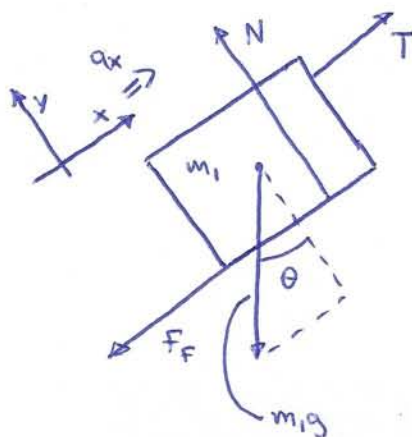
FBD on m_1 (stop from sliding down)



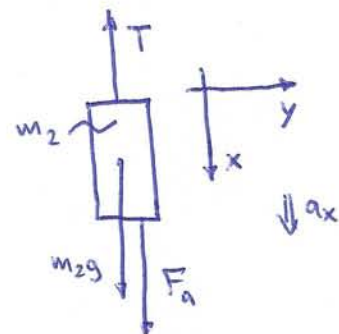
FBD on m_2 (stop from sliding down)



FBD on m_1 (sliding up)



FBD on m_2 (sliding up)



$$\sum F_y = m_1 a_y = N - m_1 g \cos \theta$$

$$N = m_1 g \cos \theta = (100)(9.81) \cos(30) = 850 \text{ N}$$

Stop from sliding down:

$$\sum_{\text{block 1}} F_x = m_1 a_x = T + \mu N - m_1 g \sin \theta$$

$$T = m_1 g \sin \theta - \mu N = (100)(9.81) \sin(30) - (0.2)(850) = 320 \text{ N}$$

$$\sum_{\text{block 2}} F_y = m_2 a_y = T - m_2 g - F_a$$

$$F_a = T - m_2 g = 320 - (10)(9.81) = 222 \text{ N}$$

Accelerate the blocks:

$$\sum_{\text{block 1}} F_x = m_1 a_x = T - \mu N - m_1 g \sin \theta$$

$$T = m_1 g \sin \theta + \mu N + m_1 a_x \quad (1)$$

$$\sum F_x = m_2 a_x = F_a + m_2 g - T \quad (2)$$

sub eqn. (1) into (2)

$$m_2 a_x = F_a + m_2 g - [m_1 g \sin \theta + \mu N + m_1 a_x]$$

$$F_a = (m_1 + m_2) a_x + (m_1 \sin \theta - m_2) g + \mu N$$

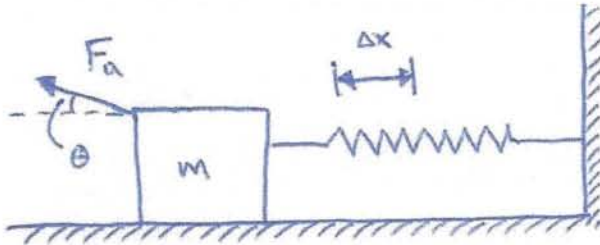
$$F_a = (100 + 10)(a_x) + (100 \sin 30 - 10)(9.81) + (0.2)(850)$$

$$F_a = 562 \text{ N} \quad \leftarrow \text{min. acceleration}$$

$$a_x = \frac{F_a - (m_1 \sin \theta - m_2) g - \mu N}{m_1 + m_2} = \frac{562 - (100 \sin 30 - 10)(9.81) - (0.15)(850)}{100 + 10}$$

$$a_x = 0.38 \text{ m/s}^2$$

A block on a frictionless surface is attached to a spring which is attached to a wall. A force is applied to the block away from the spring at angle of θ with respect to the ground.

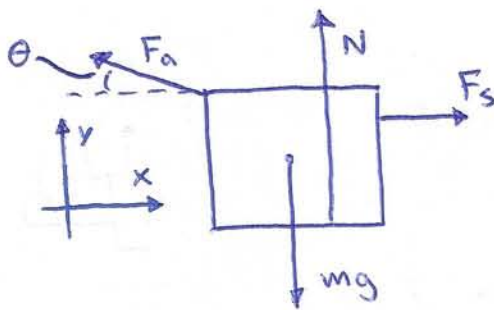


$$F_a = 50 \text{ N}, m = 10 \text{ kg}, \theta = 30 \text{ degrees}, k_{\text{spring}} = 30 \text{ N/m}$$

what is the normal force on the block?

how much does the spring stretch from its equilibrium position (i.e. Δx)?

FBD on block



$$\sum F_x = m a_x = -F_a \cos \theta + F_s \quad (1) \quad a_x = 0$$

$$\sum F_y = m a_y = N - mg + F_a \sin \theta \quad a_y = 0$$

$$N = mg - F_a \sin \theta$$

$$N = (10)(9.81) - (50) \sin(30)$$

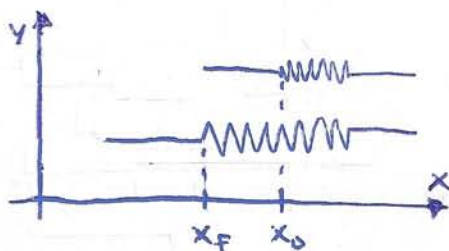
$$\boxed{N = 73.1 \text{ N}}$$

Solve eq. (1) with substitution of $F_s = -k\Delta x$:

$$0 = -F_a \cos \theta + (-k\Delta x)$$

$$\Delta x = -\frac{F_a \cos \theta}{k} = -\frac{(50) \cos(30)}{30} = -1.4 \text{ m}$$

$$\Delta x = x_f - x_0$$



Assuming that you live on a flat planet (i.e. no hills or anything that would get in the way) and neglecting air resistance at what velocity would you need to fire a bullet so that it would travel all the way around the planet (i.e. be in orbit).

$$G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$$

on earth

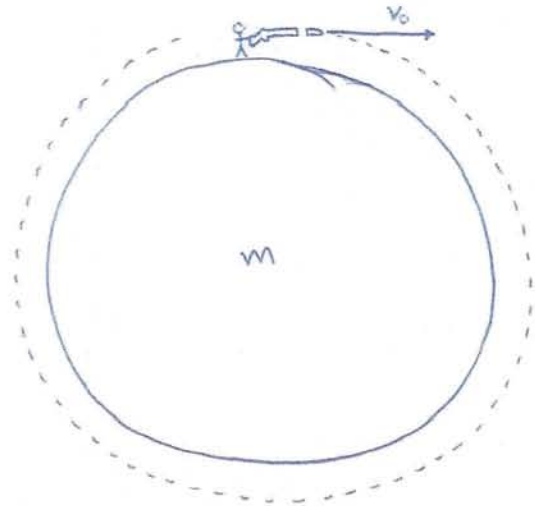
$$m_{\text{earth}} = 6.0 \times 10^{24} \text{ kg}$$

$$r_{\text{earth}} = 6371 \text{ km}$$

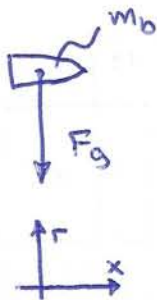
on the moon

$$m_{\text{moon}} = 7.3 \times 10^{22} \text{ kg}$$

$$r_{\text{moon}} = 1737 \text{ km}$$



FBD on bullet:



$$\sum F_r = m_b a_r = - \frac{G m_b m}{r^2}$$

$$\left(-\frac{v^2}{r}\right) = - \frac{G m}{r^2}$$

$$V = \sqrt{\frac{G m}{r}}$$

on earth:

$$V = \sqrt{\frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{6371000}} = 7926 \text{ m/s}$$

on moon:

$$V = \sqrt{\frac{(6.67 \times 10^{-11})(7.3 \times 10^{22})}{1737000}} = 1674 \text{ m/s}$$