

Assessment (instructor):	2017 CIVL 11 Final (Frank Secretain)
Time allowed:	170 minutes
Devices allowed:	pencil, pen, calculator, scrap paper
Marks allocated:	5 questions worth 100 marks
Percentage of final grade:	worth 20% of final grade
Notes from instructor:	Be neat, Box final answers

Kinematics (linear motion)

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$\vec{x} = \vec{x}_0 + \left(\frac{\vec{v}_0 + \vec{v}}{2} \right) t$$

$$\vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\Delta\vec{x})$$

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

Kinematics (circular motion)

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\theta = \theta_0 + \left(\frac{\omega_0 + \omega}{2} \right) t$$

$$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$

$$\omega = \omega_0 + \alpha t$$

Dynamics (Newton's 2nd Law)

$$\sum \vec{F} = m\vec{a}$$

Force of gravity

$$F_g = \frac{Gm_1 m_2}{r^2} = mg \text{ (on earth)}$$

Force of friction

$$F_f = \mu N$$

Moments (integral of Newton's 2nd Law with space)

$$\sum M_o = I_o \alpha_o$$

$$M = \vec{r} \times \vec{F} = F_{\perp} r$$

Centre of mass

$$\vec{x}_{cg} = \frac{\sum m_i \vec{x}_i}{\sum m_i}$$

Conservation of Momentum (integral of Newton's 2nd Law with time)

$$\Delta \vec{p} = \int \vec{F} dt = (F_{AVE})(\Delta t)$$

$$\vec{p} = m\vec{v}$$

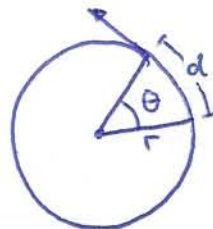
1st Law of Thermodynamics

$$\Delta E = Q - W$$

$$E_{PE} = mgh, \quad E_{KE} = \frac{1}{2} mv^2$$

$$W = F \cdot \Delta x$$

Uniform circular motion



$$d = \theta r$$

$$v = \omega r$$

$$a = \alpha r$$

$$a_{cent.} = -\omega^2 r = -\frac{v^2}{r}$$

Moment of Inertia Table

Body	Figure	Mass Center	Moments of Inertia
Rectangular Parallelepiped		—	$I_{xx} = \frac{1}{12} m(a^2 + l^2)$ $I_{yy} = \frac{1}{12} m(b^2 + l^2)$ $I_{zz} = \frac{1}{12} m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12} mb^2 + \frac{1}{3} ml^2$
Circular Cylinder		—	$I_{xx} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Semicylinder		$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy} = \frac{1}{4} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = \frac{1}{2} mr^2$
Circular Cylindrical Shell		—	$I_{xx} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Half Cylindrical Shell		$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2} mr^2 + \frac{1}{12} ml^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2} mr^2 + \frac{1}{3} ml^2$ $I_{zz} = mr^2$
Sphere		—	$I_{zz} = \frac{2}{5} mr^2$
Spherical Shell		—	$I_{zz} = \frac{2}{3} mr^2$
Hemisphere		$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$
Hemispherical Shell		$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3} mr^2$
Uniform Slender Rod		—	$I_{yy} = \frac{1}{12} ml^2$ $I_{y_1y_1} = \frac{1}{3} ml^2$

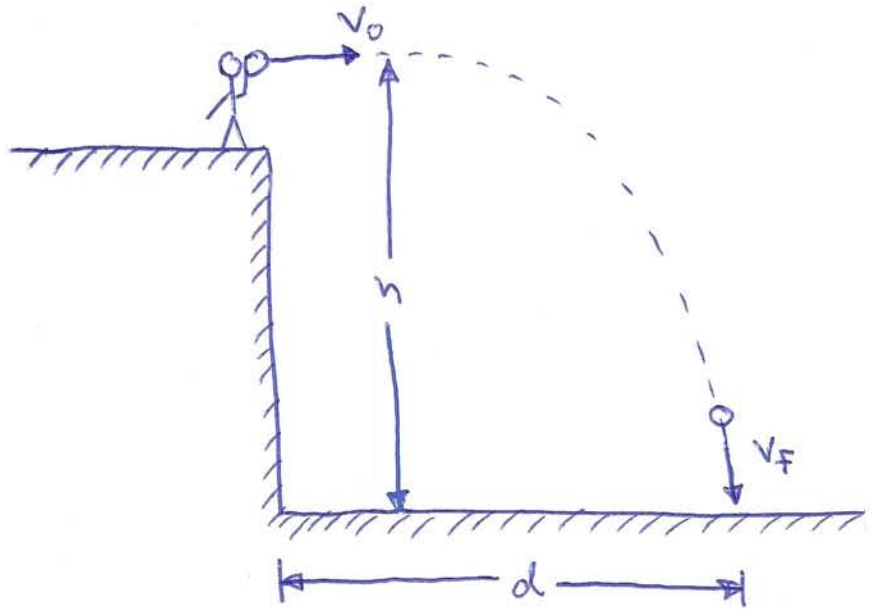
(20 marks)

Using kinematics determine the final velocity of the ball (v_f) and the horizontal distance the ball travelled (d).

given:

$$v_o = 15 \text{ m/s}$$

$$h = 10 \text{ m}$$



(20 marks)

Determine the minimum velocity the spinning hoop must have to keep the mass from sliding off of the hoop's inner surface (i.e. slides down).

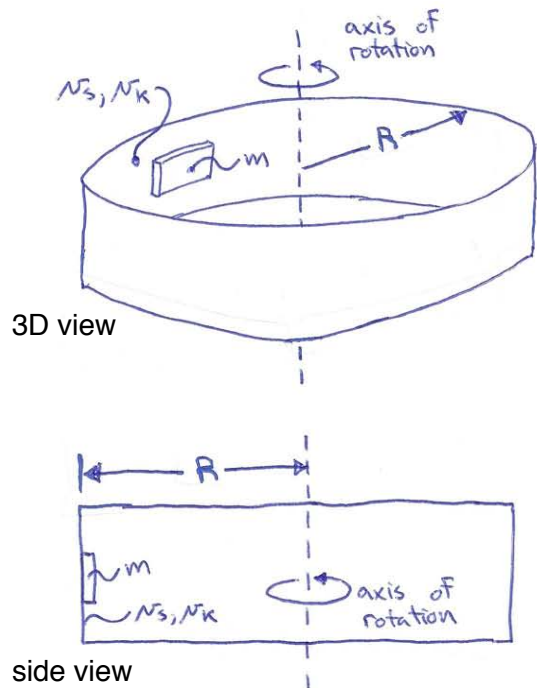
given:

$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.9$$

$$\mu_k = 0.8$$

$$R = 1 \text{ m}$$



(20 marks)

Determine the tensions in both ropes and the magnitude of the force extended on the pin attached to the beam.

given:

$$m_1 = 100 \text{ kg}$$

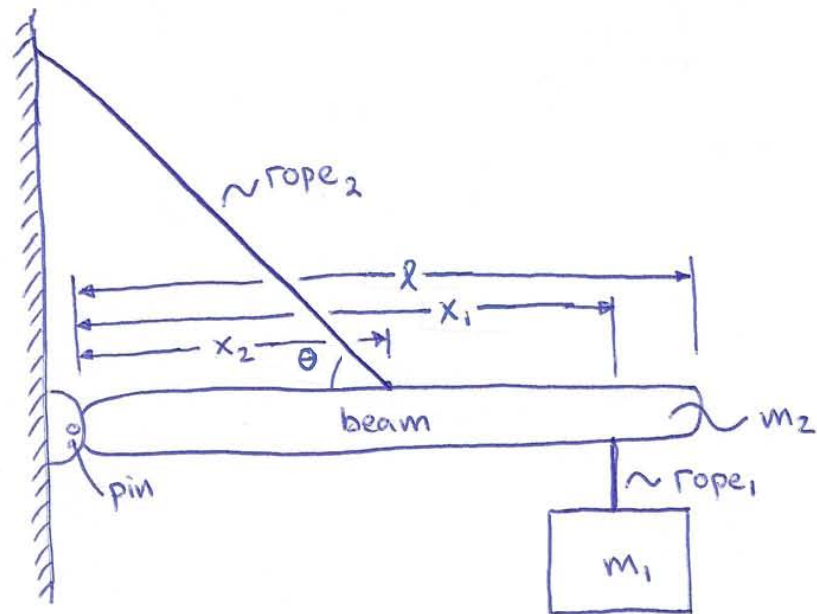
$$m_2 = 50 \text{ kg}$$

$$\text{length } (l) = 2 \text{ m}$$

$$x_1 = 1.8 \text{ m}$$

$$x_2 = 1 \text{ m}$$

$$\theta = 50^\circ$$



(20 marks)

Confirm that the systems will move, then determine the acceleration and the magnitude of the force on the pin through the solid cylinder pulley.

given:

$$m_1 = 100 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

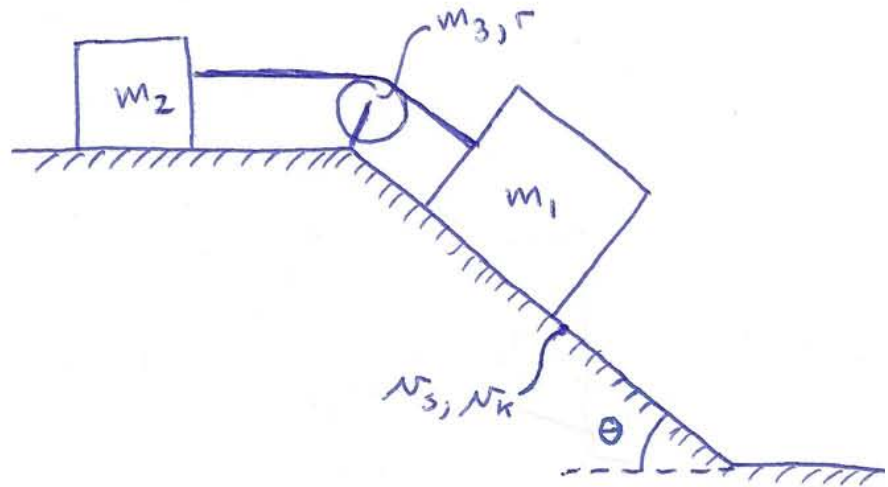
$$m_3 = 10 \text{ kg}$$

$$r = 1 \text{ m}$$

$$\mu_s = 0.2$$

$$\mu_k = 0.1$$

$$\theta = 45^\circ$$



(20 marks)

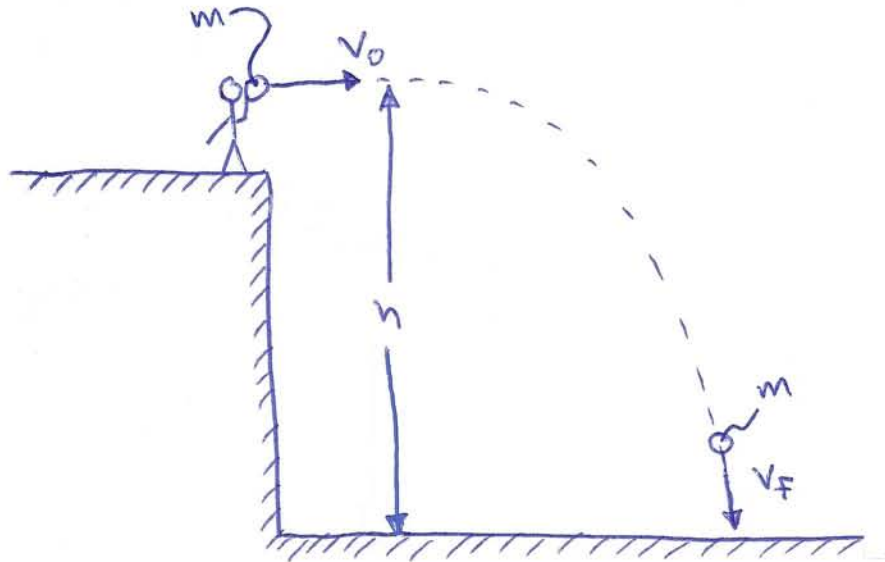
Using the 1st Law of Thermodynamics determine the final velocity of the ball (v_f).

given;

$$v_o = 15 \text{ m/s}$$

$$h = 10 \text{ m}$$

$$m = 1 \text{ kg}$$



(5 marks BONUS)

Determine the centre of mass for the six mass system and draw it on the diagram.

given:

$$m_1 = 3 \text{ kg}$$

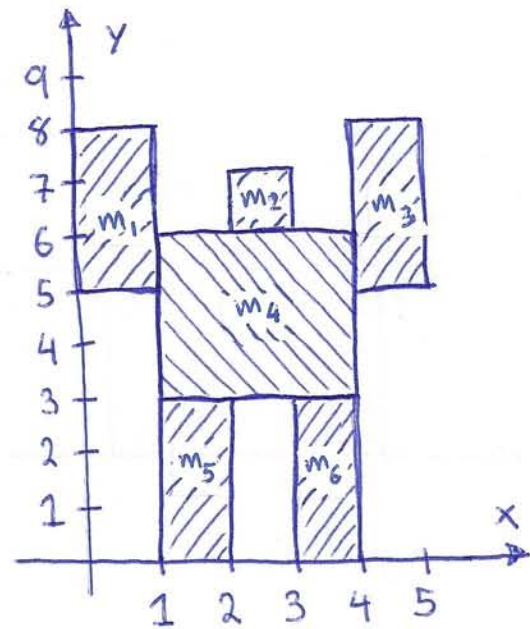
$$m_2 = 1 \text{ kg}$$

$$m_3 = 3 \text{ kg}$$

$$m_4 = 9 \text{ kg}$$

$$m_5 = 3 \text{ kg}$$

$$m_6 = 3 \text{ kg}$$



(5 marks BONUS 2)

Using the 1st Law of Thermodynamics determine the final velocity of the system after it has travelled a distance of (d) down the ramp. Confirm your answer by re-solving the problem using Dynamics and kinematics.

given:

$$m_1 = 100 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

$$m_3 = 10 \text{ kg}$$

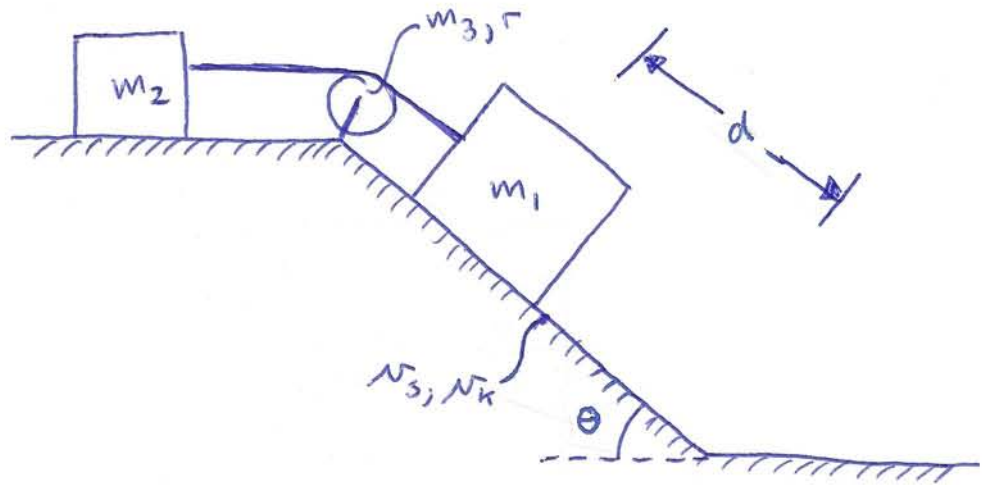
$$r = 1 \text{ m}$$

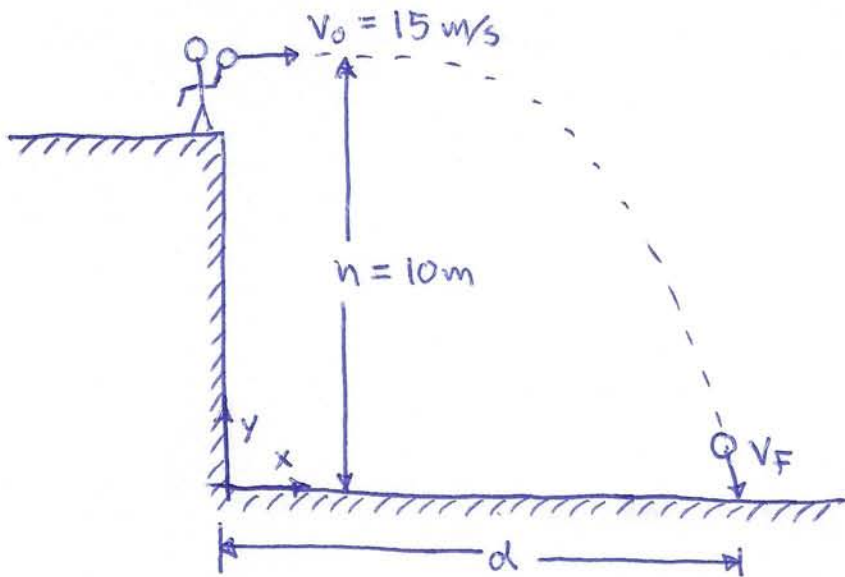
$$\mu_s = 0.2$$

$$\mu_k = 0.1$$

$$\theta = 45 \text{ degrees}$$

$$d = 5 \text{ m}$$





$$x_0 = 0$$

$$y_0 = h$$

$$x_F =$$

$$y_F = 0$$

$$v_{x0} = v_0$$

$$v_{y0} = 0$$

$$v_{xF} = v_0$$

$$v_{yF} =$$

$$a_x = 0$$

$$a_y = -9.81$$

$$t =$$

Kinematics:

$$\vec{v}_F^2 = \vec{v}_0^2 + 2\vec{a}\Delta\vec{x} \Rightarrow$$

$$v_{yF}^2 = v_{y0}^2 + 2a_y\Delta y$$

$$v_{yF} = \pm \sqrt{v_{y0}^2 + 2a_y(0-h)}$$

$$v_{yF} = \pm \sqrt{0^2 + 2(-9.81)(-10)}$$

$$v_{yF} = -14\text{ m/s}$$

$$v_F = \sqrt{v_{xF}^2 + v_{yF}^2} = \sqrt{15^2 + (-14)^2} = 20.5\text{ m/s}$$

$$\vec{v} = \vec{v}_0 + \vec{a}t \Rightarrow$$

$$v_{yF} = v_{y0} + a_y t$$

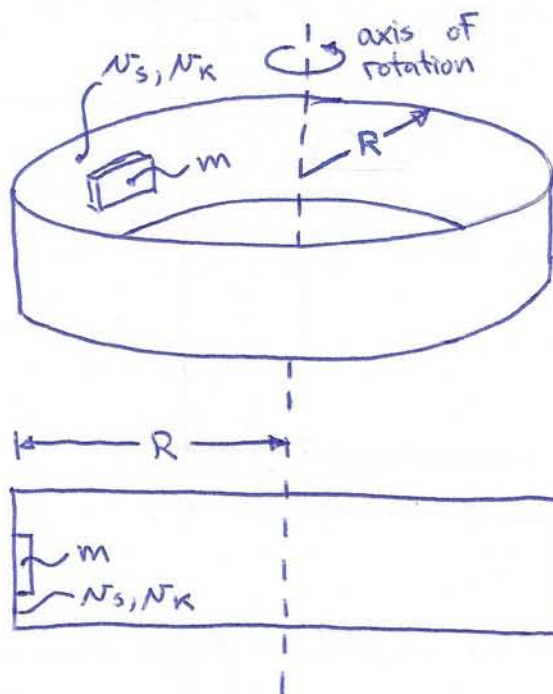
$$t = \frac{(v_{yF} - v_{y0})}{a_y} = \frac{(-14 - 0)}{-9.81} = 1.4\text{ s}$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \Rightarrow$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \rightarrow 0 \quad (a_x = 0)$$

$$x = 0 + (15)(1.4)$$

$$x = 21.4\text{ m}$$



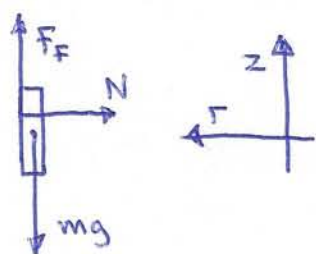
$$m = 0.5 \text{ kg}$$

$$\mu_s = 0.9$$

$$\mu_k = 0.8$$

$$R = 1 \text{ m}$$

FBD:



$$\sum F_z = \cancel{ma_z} = F_f - mg \quad (1)$$

$$\sum F_r = ma_r = -N \quad (2)$$

Solve equ. (1) for N :

$$0 = \mu N - mg \Rightarrow \boxed{N = \frac{mg}{\mu}} \quad (1a)$$

sub (1a) into (2)

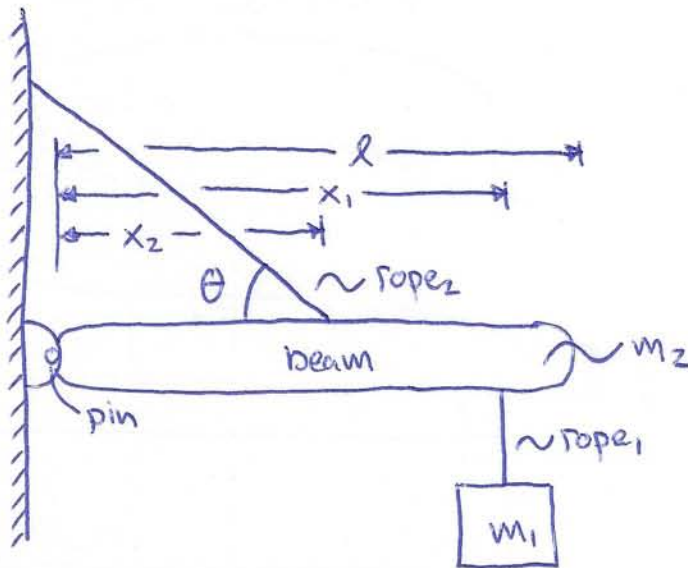
$$ma_r = -\left[\frac{mg}{\mu}\right]$$

$$\text{sub } a_r = -\frac{v^2}{r}$$

$$\mu\left[-\frac{v^2}{r}\right] = -\left[\frac{mg}{\mu}\right] \Rightarrow$$

$$\boxed{v_{\min} = \sqrt{\frac{gr}{\mu_s}}}$$

$$\boxed{v_{\min} = \sqrt{\frac{(9.81)(1)}{0.9}} = 3.3 \text{ m/s}}$$



$$m_1 = 100 \text{ kg}$$

$$m_2 = 50 \text{ kg}$$

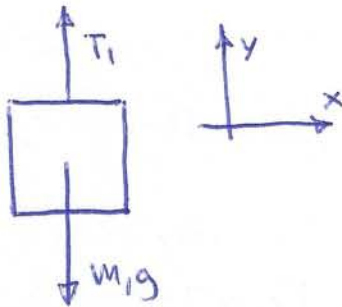
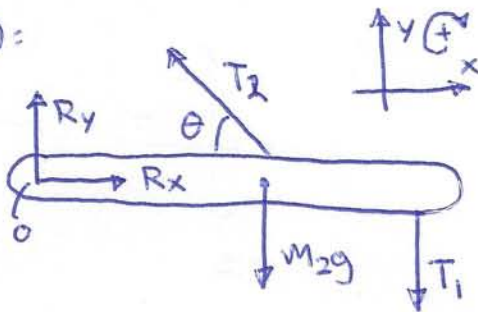
$$l = 2 \text{ m}$$

$$x_1 = 1.8 \text{ m}$$

$$x_2 = 1 \text{ m}$$

$$\theta = 50^\circ$$

FBD:



$$\sum F_x = \cancel{m_2 a_x} = R_x - T_2 \cos \theta \quad (1)$$

$$\sum F_y = \cancel{m_2 a_y} = R_y + T_2 \sin \theta - m_2 g - T_1 \quad (2)$$

$$\sum M_O = \cancel{I_O \alpha_O} = (m_2 g) \left(\frac{l}{2} \right) + (T_1)(x_1) - (T_2 \sin \theta)(x_2) \quad (3)$$

$$\sum F_y = \cancel{m_1 a_y} = T_1 - m_1 g \quad (4)$$

Solve (4) for T_1 :

$$0 = T_1 - m_1 g \Rightarrow T_1 = m_1 g = (100)(9.81) = 981 \text{ N} \quad (4a)$$

Sub (4a) into (3) and solve for T_2 :

$$0 = \frac{m_2 g l}{2} + [m_1 g] x_1 - (T_2 \sin \theta)(x_2)$$

$$T_2 = \frac{\frac{m_2 g l}{2} + m_1 g x_1}{x_2 \sin \theta} = \frac{\frac{(50)(9.81)(2)}{2} + (100)(9.81)(1.8)}{(1) \sin(50)}$$

$$\boxed{T_2 = 2945 \text{ N}}$$

(3a)

Sub (3a) into (1) and solve for R_x :

$$0 = R_x - T_2 \cos \theta \Rightarrow \boxed{R_x = T_2 \cos \theta = (2945) \cos(50) = 1893 \text{ N}}$$

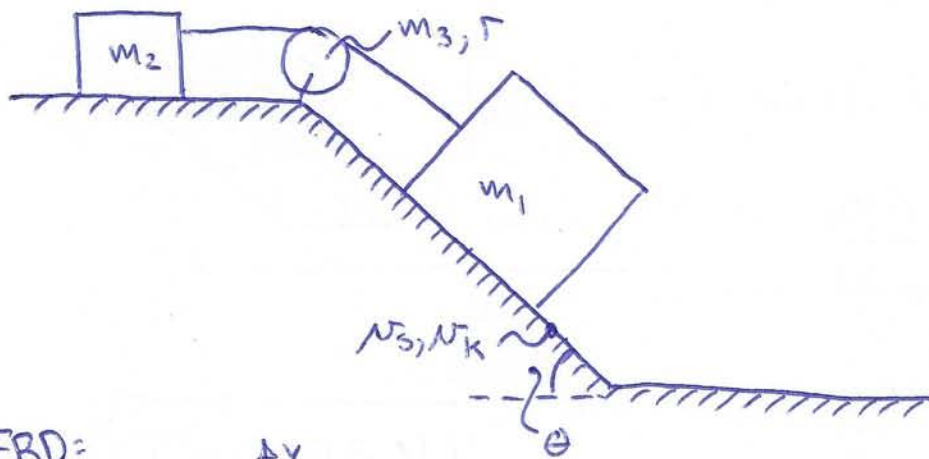
Sub (3a) & (4a) into (2) and solve for R_y :

$$0 = R_y + T_2 \sin \theta - m_2 g - T_1$$

$$\boxed{R_y = T_1 + m_2 g - T_2 \sin \theta = 981 + (50)(9.81) - 2945 \sin(50) = -785 \text{ N}}$$

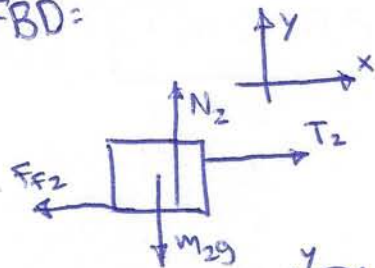
Total Reaction Force on pin

$$\boxed{R = \sqrt{R_x^2 + R_y^2} = \sqrt{(1893)^2 + (-785)^2} = 2049 \text{ N}}$$



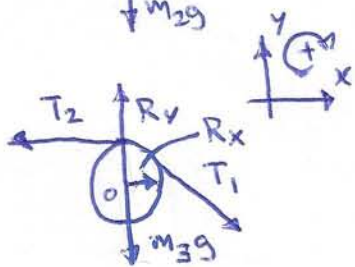
$$\begin{aligned} m_1 &= 100 \text{ Kg} \\ m_2 &= 10 \text{ Kg} \\ m_3 &= 10 \text{ Kg} \\ r &= 1 \text{ m} \\ \mu_s &= 0.2 \\ \mu_k &= 0.1 \\ \theta &= 45^\circ \end{aligned}$$

FBD:



$$\sum F_x = m_2 a_x = T_2 - F_{f2} \quad (1)$$

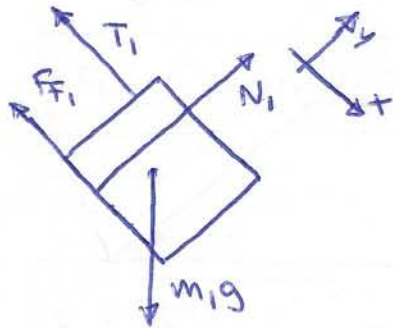
$$\sum F_y = m_2 a_y = N_2 - m_2 g \quad (2)$$



$$\sum F_x = m_3 a_{x3} = R_x + T_1 \cos \theta - T_2 \quad (3)$$

$$\sum F_y = m_3 a_{y3} = R_y - T_1 \sin \theta - m_3 g \quad (4)$$

$$\sum M_o = I_o \alpha_o = T_1 r - T_2 r \quad (5)$$



$$\sum F_x = m_1 a_x = m_1 g \sin \theta - T_1 - F_{f1} \quad (6)$$

$$\sum F_y = m_1 a_y = N_1 - m_1 g \cos \theta \quad (7)$$

Solve (2) & (7) For N_2 & N_1 :

$$0 = N_2 - m_2 g \Rightarrow \boxed{N_2 = m_2 g} \quad (2a)$$

$$0 = N_1 - m_1 g \cos \theta \Rightarrow \boxed{N_1 = m_1 g \cos \theta} \quad (7a)$$

Sub (2a) into (1) and solve for T_2 :

$$m_2 a_x = T_2 - \mu [m_2 g] \Rightarrow \boxed{T_2 = m_2 a_x + \mu m_2 g} \quad (1a)$$

sub (7a) into (6) and solve for T_1 :

$$m_1 a_x = m_1 g \sin \theta - T_1 - \mu [m_1 g \cos \theta]$$

$$\boxed{T_1 = m_1 g \sin \theta - m_1 a_x - \mu m_1 g \cos \theta} \quad (6a)$$

sub (1a) & (6a) into (5):

$$I_0 \alpha_0 = [m_1 g \sin \theta - m_1 a_x - \mu m_1 g \cos \theta] r \\ - [m_2 a_x + \mu m_2 g] r$$

sub $\alpha_0 = \frac{a_x}{r}$ and solve for a_x :

$$I_0 \left(\frac{a_x}{r} \right) = [m_1 g \sin \theta - \mu m_1 g \cos \theta - \mu m_2 g] r - [m_1 a_x + m_2 a_x] r$$

$$\left[\frac{I_0}{r^2} + m_1 + m_2 \right] a_x = [m_1 g \sin \theta - \mu m_1 g \cos \theta - \mu m_2 g]$$

$$\boxed{a_x = g \frac{m_1 \sin \theta - \mu m_1 \cos \theta - \mu m_2}{m_1 + m_2 + \frac{I_0}{r^2}}}$$

use μ_s :

$$a_x = (9.81) \frac{(100) \sin(45) - (0.2)(100) \cos(45) - (0.2)(10)}{(100) + (10) + \left(\frac{1}{2}\right)(10)} = 4.65 \text{ m/s}^2$$

use μ_k :

$$a_x = (9.81) \frac{(100) \sin(45) - (0.1)(100) \cos(45) - (0.1)(10)}{(100) + (10) + \left(\frac{1}{2}\right)(10)} = 5.34 \text{ m/s}^2$$

Solve (1a) & (6a) for T_2 & T_1 :

$$T_2 = (10)(5.34) + (0.1)(10)(9.81) = 63 \text{ N}$$

$$T_1 = (100)(9.81)\sin(45) - (100)(5.34) - (0.1)(100)(9.81)\cos(45) = 90 \text{ N}$$

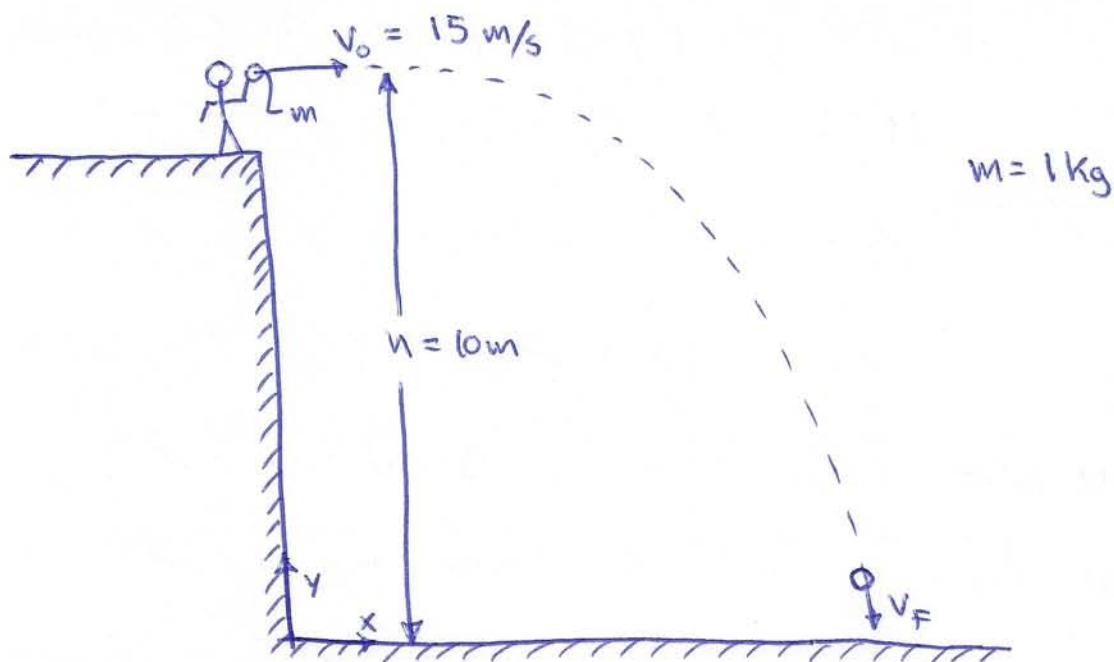
solve (3) & (4) for R_x & R_y :

$$R_x = T_2 - T_1 \cos \theta = (63) - (90)\cos(45) = -0.6 \text{ N}$$

$$R_y = m_3 g + T_1 \sin \theta = (10)(9.81) + (90)\sin(45) = 162 \text{ N}$$

magnitude of R :

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-0.6)^2 + (162)^2} = 162 \text{ N}$$



$$E_0 = \frac{1}{2} m v_0^2 + mgh$$

$$E_F = \frac{1}{2} m v_F^2$$

1st Law of Thermodynamics:

$$\Delta E = \cancel{Q} - \cancel{W} = 0 = E_F - E_0$$

$$\left[\frac{1}{2} m v_F^2 \right] - \left[\frac{1}{2} m v_0^2 + mgh \right] = 0$$

$$v_F = \sqrt{v_0^2 + 2gh}$$

$$v_F = \sqrt{(15)^2 + (2)(9.81)(10)} = 20.5\text{ m/s}$$

(5 marks BONUS)

Determine the centre of mass for the six mass system and draw it on the diagram.

given:

$$m_1 = 3 \text{ kg}$$

$$m_2 = 1 \text{ kg}$$

$$m_3 = 3 \text{ kg}$$

$$m_4 = 9 \text{ kg}$$

$$m_5 = 3 \text{ kg}$$

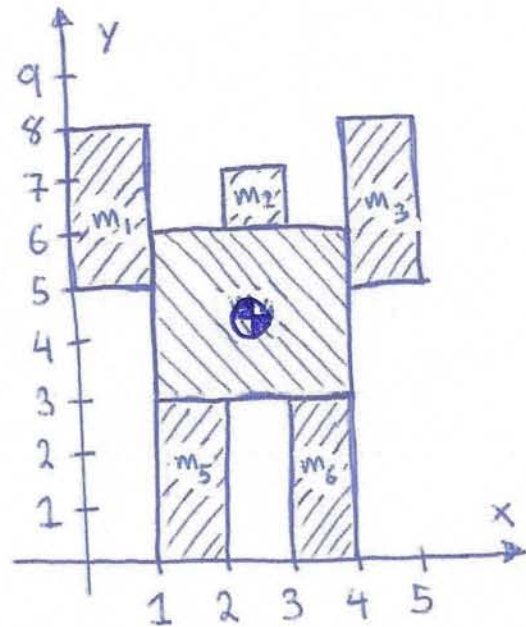
$$m_6 = 3 \text{ kg}$$

$$m_{\text{tot}} = 22 \text{ kg}$$

$$\bar{x}_{\text{cg}} = \frac{\sum x_i m_i}{\sum m_i}$$

$$x_{\text{cg}} = \frac{(0.5)(3) + (2.5)(1) + (4.5)(3) + (2.5)(9) + (1.5)(3) + (3.5)(3)}{(3) + (1) + (3) + (9) + (3) + (3)} = 2.5$$

$$y_{\text{cg}} = \frac{(6.5)(3) + (6.5)(1) + (6.5)(3) + (4.5)(9) + (1.5)(3) + (1.5)(3)}{(3) + (1) + (3) + (9) + (3) + (3)} = 4.3$$



(5 marks BONUS 2)

Using the 1st Law of Thermodynamics determine the final velocity of the system after it has travelled a distance of (d) down the ramp. Confirm your answer by re-solving the problem using Dynamics and kinematics.

given:

$$m_1 = 100 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

$$m_3 = 10 \text{ kg}$$

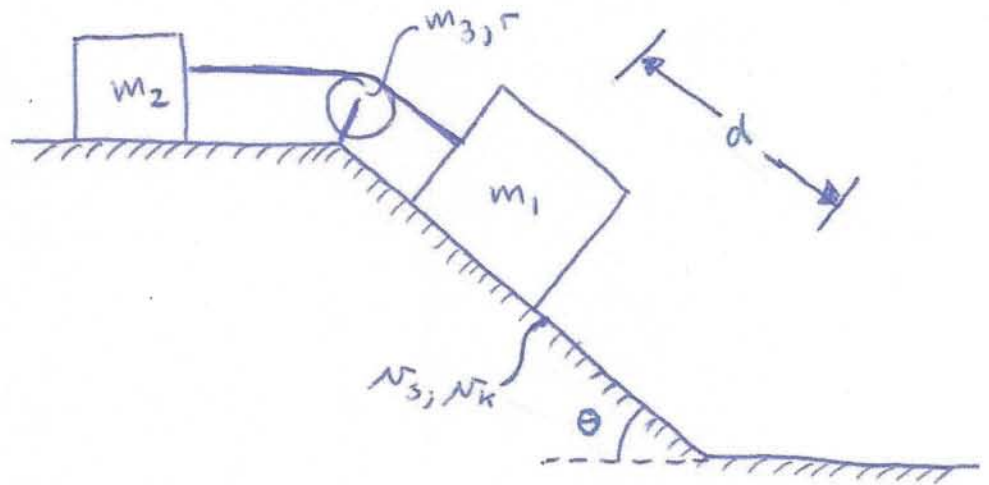
$$r = 1 \text{ m}$$

$$\mu_s = 0.2$$

$$\mu_k = 0.1$$

$$\theta = 45 \text{ degrees}$$

$$d = 5 \text{ m}$$



1st Law Thermodynamics:

$$\Delta E = \cancel{Q} - W$$

(i)

$$E_0 = m_1 g (d \sin \theta)$$

$$E_F = \frac{1}{2} m_1 v_F^2 + \frac{1}{2} m_2 v_F^2 + \frac{1}{2} I_0 \omega_0^2$$

$$W = +\mu_k N_1 d + \mu_k N_2 d$$

$$= \mu_k (m_1 g \cos \theta) d + \mu_k (m_2 g) d$$

Sub E_0 , E_F & W into (i) and sub $\omega_0 = \frac{v_F}{r}$, solve for v_F

$$E_F - E_0 = -W$$

$$\left[\frac{1}{2} m_1 v_F^2 + \frac{1}{2} m_2 v_F^2 + \frac{1}{2} I_0 \left(\frac{v_F}{r} \right)^2 \right] - [m_1 g d \sin \theta] = -[\mu_k m_1 g \cos \theta d + \mu_k m_2 g d]$$

$$[m_1 + m_2 + \frac{I_0}{r^2}] v_F^2 = 2gd [m_1 \sin \theta - \mu_k m_1 \cos \theta - \mu_k m_2]$$

$$v_F = \sqrt{\frac{2gd [m_1 \sin \theta - \mu_k m_1 \cos \theta - \mu_k m_2]}{m_1 + m_2 + \frac{I_0}{r^2}}}$$

$$V_F = \sqrt{\frac{(2)(9.81)(5) [(100)\sin(45) - (0.1)(100)\cos(45) - (0.1)(10)]}{100 + 10 + (\frac{1}{2})(10)}}$$

$$V_F = 7.3 \text{ m/s}$$

From Dynamics:

$$a_x = 5.34 \text{ m/s}^2$$

Kinematics:

$$V_{xF}^2 = V_{x0}^2 + 2a_x d$$

$$V_{xF} = \sqrt{V_{x0}^2 + 2a_x d} = \sqrt{(0)^2 + (2)(5.34)(5)} = 7.3 \text{ m/s}$$